PLASMA PHYSICS

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CONTENTS

The numbering of the equations in each chapter starts from one. When reference is made to equations in other chapters, the number of the chapter is added first (e.g., Equation (3.62) means Equation (62) in Chapter 3).

INTRODUCTION

Plasma physics is concerned with the behaviour of systems of many free electrons and ionised atoms where the mutual Coulomb interactions cannot be disregarded. In a restricted sense, such systems of particles consist of nearly equal numbers of positive and negative charges. Systems of this type are examples of a medium known as plasma which in many respects behaves differently from the solid, liquid and gaseous state of matter.

All states of matter represent different degrees of organization, to which there correspond certain values of binding energy. Thus, in the solid state the important quantity is the binding energy of molecules in a crystal; in fact, a crystal could be considered as a macro- or super-molecule. If the average kinetic energy per molecule W exceeds the binding energy U (a fraction of an eV) the crystal structure breaks up, either into a liquid or directly into a gas. A similar law operates in the case of liquids, and in order to change a liquid into a gas, a certain minimum kinetic energy per molecule is required to break the bonds of the van der Waals forces. Matter can exist as plasma, i.e., in its fourth state, when the kinetic energy W per plasma particle exceeds the ionising potential of atoms which is usually a few eV. Thus the average kinetic energy per particle determines the state in which matter exists. A precise mathematical statement of this theorem is an equation of the Saha type. However, a simple criterion can be written as

$$U_n < W < U_{n+1} \qquad (1)$$

where U_n, U_{n+1} are the respective binding energies, expressing that matter exists in the $(n+1)$st state.

The plasma state will correspond to an order-of-magnitude relationship

$$1 < W_4 < 10^6 \quad \text{(eV)}.$$

Extrapolating this principle to higher states of matter, so far unexplored, one may define the fifth state of matter as one in which

$$1 < W_5 < 500 \quad \text{(MeV)}.$$

This will be a gas of free nucleons and electrons — a "nugas". The sixth state would be, consequently, defined as

$$\tfrac{1}{2} < W_6 < 10 \quad \text{(GeV)}$$

and would contain free mesons, nucleons in various states of excitation and electrons. The fifth and sixth states of matter can be expected to exhibit an even greater variety of behaviour than a plasma owing to the action of short range internucleon forces in addition to long range Coulomb forces.

On the other hand, according to eq. (1) for W, plasma spans a broader energy band than any other state of matter; it encompasses about 20 octaves on the kinetic energy scale. This width of the kinetic energy spectrum of the plasma state is the reason for much common ground between plasma physics and many other fields of physics, such as the dynamics of single charged particles (in which many-particle interactions are not considered), or the physics of electrical discharges in gases (in which interaction between charged particles and neutral atoms and molecules is of great importance), whereas some methods of description and analysis used in plasma physics belong to the subject of hydrodynamics, particularly magneto-hydrodynamics. Another physical discipline, indispensable for the theory of a plasma, is statistical mechanics and there are yet other fields from which plasma physics draws its mathematical formulation and its terminology.

Although probably more than 99.9 % of matter in our Universe is ionised and therefore in the plasma state, on our planet plasma has to be generated by special physical processes and under special conditions. These processes are the subject of the physics of electrical discharges in gases and this is the reason for the parental relationship between the latter and plasma physics.

Using an anthropomorphic analogy one may say that whereas the physics of electrical discharges is more specifically concerned with the birth and metabolism of plasma, plasma physics concentrates mostly on the anatomy and motion of plasma.

On our planet the medium which often resembles an ideal plasma is a partially ionized gas. This medium enters in the experience of prehistoric humanity in three forms; as fire, as lightning and as Aurora Borealis. In this connection it is curious to note that a number of greek philosophers, starting with Empedocles of Agrigentum (about 490-430 B.C.), held that the material Universe is built of four "roots": earth, water, air and fire. This, in modern terminology, may be compared with four states of matter, solid, liquid, gaseous and the plasma state. The privilege of identifying the medium created in electrical discharges in gases as the fourth state of matter belongs to W. Crookes who writes (1879): "The phenomena in these exhausted tubes reveal to physical science a new world, a world where matter may exist in a fourth state...". At about this time it became obvious that this newly discovered state of matter is not very much at home on our dense and cold planet and that special conditions must be realized in order to generate a plasma-like medium in the laboratory. Investigation of these conditions were the subject of the physics of electrical discharges in gases. It was only when electrical and vacuum techniques developed to the point when long-lived and relatively stable electrical discharges were available that plasma physics emerged as a separate field of study.

Around 1923 I. Langmuir developed the appropriate basic theory of an ionised gas and gave the medium the name "plasma" *. During the period 1923-1938 the subject developed further due to the efforts of L. Tonks, R. Seeliger, B. Klarfeld, M. Steenbeck, A. v. Engel, L.B. Loeb, W. Bennett, F.M. Penning, J. Townsend, W. Rogowski and many others.

At the beginning of this century astrophysicists became aware of the importance played by ionised matter in the processes in outer space and subsequently some of the finest contributions to plasma physics came from their ranks. Here one may mention the work of M.N. Saha, S. Chapman, T.G. Cowling, V.C. Ferraro, S. Chandrasekhar, L. Spitzer, H. Alfvén and the german astrophysical school at the Max-Planck Institute.

In 1929 F. Houtermanns and R. Atkinson suggested that the main source of energy in stars is the fusion reactions among the nuclei of the light elements. After 1945 a similar mechanism was exploited in the construction of hydrogen bombs and at the same time some physicists became interested in a controlled release of fusion energy.

However, it was appreciated that the energy output from fusion reactions depends critically on the kinetic energy of the colliding nuclei and that fusion outputs of practical interest depend on one's ability to produce temperatures of at least several million degrees Kelvin. If explosions are to be avoided, then the pressure of matter at this temperature must be balanced by external forces. This is within the power of our engineers only if the density of the nuclear fuel is substantially less than the density of our atmosphere. The search for a mechanism of a controlled release of fusion energy in the 1950's became, therefore, synonymous with the study of high temperature, low density plasmas. However, it should not be forgotten that the notion of controlled fusion is not inconsistent with controlled explosions as will be mentioned on p. 308. In accord with such an extension of the scope of

* The word plasma occurs first in the term protoplasma which was originally introduced into scientific terminology in 1839 by the czech biologist J. Purkynie for the jelly-like medium interspersed by numerous particles which constitutes the body of cells.

controlled fusion is also the recent extension of our interest to very high density plasmas.

The prospect of nuclear fusion gave a new lease of life to plasma physics which was becoming rather unfashionable and as one of my friends put it, regarded by most other physicists as a rather charming subject, full of small, colourful experiments, where there was little left to discover and whose only real justification was the amusement of those who bothered to waste their time on it. With the goal of a fusion reactor as an incentive, plasma physics became a subject of interest to many physicists and engineers. More recently many other applications of plasma physics have appeared, such as plasma rockets, direct conversion of thermal energy into electrical energy, transmission of radio and television signals through ionosphere and others. These are more than able to sustain the interest of physicists and engineers in plasmas.

When the first edition of this book was written in 1959 only very few experiments on plasma had been carried out and those that had been done were useful only for a general orientation and could not be compared with clear and precise experiments in other branches of physics. In five years this situation has changed considerably and there exist now some "classical" experiments on waves, shocks, diffusion and dynamics of plasma I have attempted, therefore, to illustrate at least some of the theoretical statements by means of related experiments. Owing to the interplay of theory and experiment in the last decade it was possible to gain a feeling for the plasma medium, appreciate its many aspects which predominate according to the values assumed by the density and by the temperature of the plasma and also according to whether or not there is a magnetic field in the plasma. In order to transmit some of this feeling the book starts with a rather lengthy chapter on the general properties of the different types of plasma.

As in many problems dealing with a large ensemble of individuals (e.g., star clusters), plasma physics uses two complementary modes of description: the analysis of the movement of a single particle and the fluid model. These two treatments are the subjects of chapters 2 and 3.

These modes of description are subsequently applied to equilibrium configurations, i.e., to plasma statics (chapter 4), to wave-motion and instabilities in plasma (chapter 5) and to shocks in plasma (chapter 6). This brings us to consider the dynamics of a plasma (chapter 7).

In order to complete our description of plasma it is important to know how an equilibrium configuration is established. This problem can be solved only if one can find a suitable description of the various collision, diffusion and radiation processes that are operative in arriving at an equilibrium. This is the aim of chapter 8.

The eight chapters provide us with models of plasma processes which are used in chapter 9 to describe some of the applications of plasma physics to the research on the controlled fusion of light nuclei, and in chapter 10 to electronics and to other problems in applied physics and in engineering. Those who will be using this book as a text book may not want to get involved with some of the more complicated mathematical arguments. In such a case it may be advisable to only gloss over paragraphs marked by an asterisk. The c.g.s. system of units will be used unless specified otherwise.

CHAPTER I

1.1. Plasma State

Let us first restrict the applications of the word plasma to systems in which the positive ions are not bound in any lattice in space. This will exclude systems in which the positive ions belong to a conducting or semi-conducting solid body, it will also exclude liquid conductors and electrolytes in spite of the fact that the lattice in these latter cases is ever changing. Such a restriction amounts to the requirement hat the density of the kinetic energy of positive ions be much higher than the density of binding energy corresponding to a lattice. By making this restriction no offence is meant to thus excluded types of plasma, the behaviour of electron plasma in conductors (ref. 1, 2) and semi-conductors (ref. 3, 4) is of considerable theoretical and experimental interest, however, it is more fitting to discuss such systems in another book entitled perhaps "Electron plasmas in solids". For similar reasons it is advisable not to mix the physics of electron and of ion beams with that of plasma physics proper, besides several books have already been written on that subject (ref. 5, 6).

At this point it may be better not to go on deciding what is not a plasma, instead we shall study some important properties of a system consisting of many free electrons and ions and decide which are the parameters corresponding to a typical plasma *.

Let us first consider a special case in which the system is of infinite extension, with no fields of force imposed from the outside and the velocity vectors of the particles randomly distributed both with respect to their direction and their amplitude. Let the average density of either type of particle be n. Let us try to find out how the lines of force e of the electric field E of an electron Q are distributed in space. Obviously most of these lines will be attached to the nearest positive ions $P_1 \ldots P_3$, some go to the more distant ones such as P_4, P_5 and some (e') leak out and travel far before they, too, get attached to positive charges (fig. 1). It is easy to see that no e' lines would exist in a perfectly ordered lattice. There all the lines emanating from the charge Q finish on the oppositely charged nearest neighbours. Evidently the

* Lists of symbols are given at the end of each chapter.

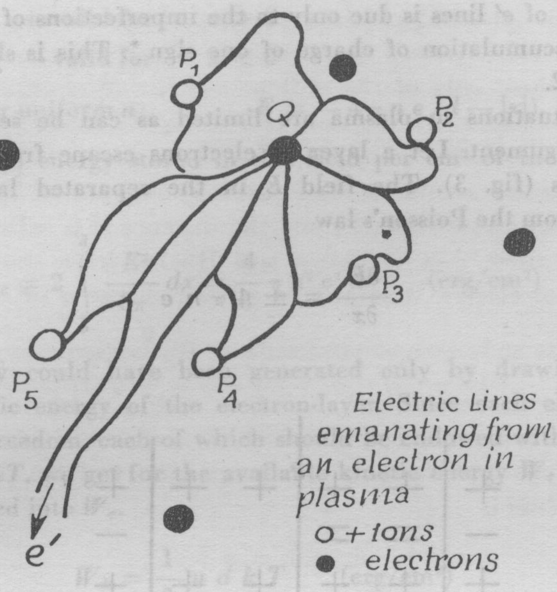

Fig. 1.

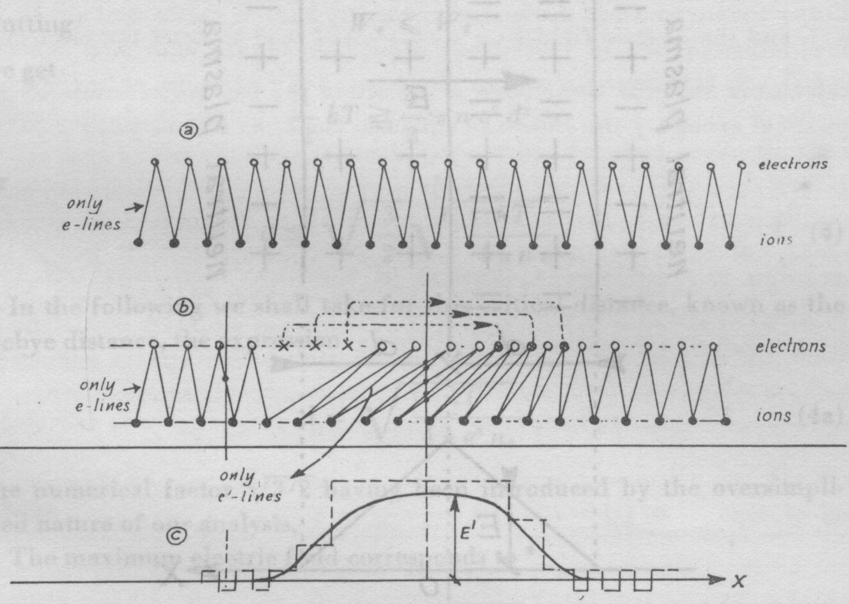

Fig. 2.

a) perfect lattice.

b) e-lines transformed into e′-lines as a result of disturbance in the electron lattice resulting from the shift of 3 electrons.

c) The intensity of the electric field in the disturbed lattice, equivalent to the number of electric lines at any position χ.

112166

reaching out of e' lines is due only to the imperfections of a lattice, i.e. to a local accumulation of charge of one sign *. This is shown graphically in fig. 2.

Such fluctuations in plasma are limited as can be seen from the following argument. Let a layer of electrons escape from a layer of positive ions (fig. 3). The field E in the separated layers can be calculated from the Poisson's law

$$\frac{\partial E}{\partial x} = \pm\, 4\, \pi\, n\, e \tag{1}$$

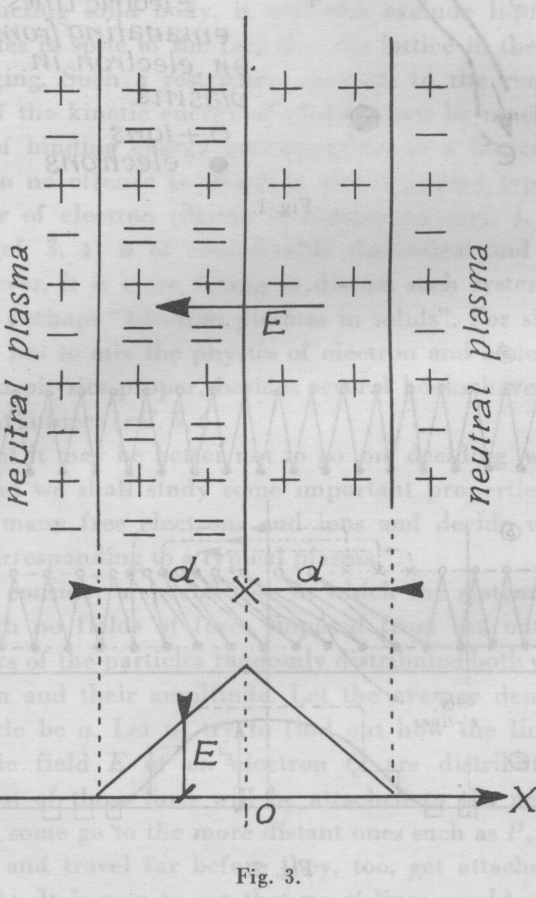

Fig. 3.

* It is also due to the $\frac{1}{r^2}$ dependence of electrical forces. A short-range force would cause the group of charges to behave more like a gas of neutral atoms.

The sign $\left.\begin{array}{l} + \text{ valid for } \quad x > 0 \\ - \text{ valid for } \quad x < 0 \end{array}\right\}$

which gives for uniform n: $\quad E = -4\,\pi\,n\,e\,(d - |x|)$.

The electrical energy stored in this field per cm² of the surface of the layer is

$$W_e = 2\int_o^d \frac{E^2}{8\pi}\,dx = \frac{4}{3}\,\pi\,n^2\,e^2\,d^3 \qquad (\text{erg/cm}^2) \tag{2}$$

This energy could have been generated only by drawing on the random kinetic energy of the electron-layer. Since each electron has 3 degrees of freedom, each of which should be endowed with an energy equal to $1/2\;kT$, we get for the available kinetic energy W_t capable of being converted into W_e.

$$W_t = \frac{1}{2}\,n\;d\;k\;T \qquad (\text{erg/cm}^2) \tag{3}$$

Putting $\quad W_e \leqslant W_t$

we get

$$\frac{1}{2}\,kT \geqq \frac{4}{3}\,\pi\,n\,e^2\,d^2$$

or

$$d \leqq \sqrt{\frac{3}{2}}\sqrt{\frac{kT}{4\,\pi\,n\,e^2}} \tag{4}$$

In the following we shall take for this critical distance, known as the Debye distance, the expression

$$d = \sqrt{\frac{kT}{4\,\pi\,e^2\,n_e}} \tag{4a}$$

the numerical factor $\sqrt{3/2}$ having been introduced by the oversimplified nature of our analysis.

The maximum electric field corresponds to *

$$E_{max} = \sqrt{4\pi nkT} \tag{5}$$

* Defining the energy densities $w_e = \dfrac{E^2{}_{\max}}{8\pi}$ and $w_t = \dfrac{3}{2}\,nkT$ respectively, the equation (5) can also be written $w_e = \dfrac{1}{3}\,w_t$.

Considering a spherical rather than plane geometry we obtain using similar arguments a critical distance $d_{sph} \sim d$ and $E_{\text{maxsph}} \sim E_{\text{max}}$. The E field is composed only of the e' lines and the distance d is then the longest distance to which the field of a charge in plasma can penetrate before being screened.

These ideas are similar to those used by Debye and Hückel in the theory of electrolytes (ref. 7)) and the distance d is called the Debye distance. In their theory, Debye and Hückel have shown that the distribution of electric field around a fixed charge q in an electrolyte corresponds to a "screened potential" $\phi = \frac{q}{r} \exp\left(-\frac{r}{d}\right)$. Using a simplified argument we shall show how such a potential distribution arises around a fixed charge q in a plasma of density n and temperature T. In absence of charge q the charge density of the electronic and positive ion fluid is

$$\bar{n}_e = \bar{n}_i = n \tag{6}$$

Introducing the charge $+q$ creates a spherical, positive potential wall $\phi(r)$, and the electric field $E = -\frac{\partial \phi}{\partial r}$ will tend to bend the electron trajectories towards the charge and deflect the ion trajectories. In a spherical geometry the Poisson's equation for ϕ, n_e and n_i reads

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = 4\pi (n_e - n_i) e \tag{7}$$

According to a well-known theorem of statistical mechanics (ref. 8) the distribution of electrons (temperature T) in thermodynamic equilibrium in such a potential well is given by

$$n_e(r) = n_e \cdot \exp\left[\frac{+e\phi(r)}{kT}\right] \tag{8a}$$

and of ions $$n_i(r) = n_i \cdot \exp\left[\frac{-e\phi(r)}{kT}\right] \tag{8b}$$

For radii r so large that $\frac{e\phi(r)}{kT} \ll 1$, the electron and ion distribution will not be greatly perturbed and we can write

$$n_{e,i}(r) = n\left[1 \pm \frac{e\phi(r)}{kT}\right] \tag{8c}$$

The potential ϕ can now be determined from