# QUANTUM DYNAMICS ECLASSICAL SYSTEMS

WITH APPLICATIONS OF THE NUMBER OPERATOR



FABIO BAGARELLO

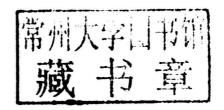


# QUANTUM DYNAMICS FOR CLASSICAL SYSTEMS

With Applications of the Number Operator

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A JOHN WILEY & SONS, INC., PUBLICATION

Cover Design: Michael Rutkowski

Cover Image: © Dmitrii Brighidov/iStockphoto

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey Published simultaneously in Canada

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#### Library of Congress Cataloging-in-Publication Data:

Bagarello, Fabio, 1964- author.

Quantum dynamics for classical systems: with applications of the number operator / Fabio Bagarello.

pages cm

Includes bibliographical references and index.

ISBN 978-1-118-37068-1 (hardback)

1. Social sciences-Mathematics. 2. Business mathematics. 3. Quantum theory-Mathematics. I. Title.

H61.25.B34 2013 300.1'53012-dc23

2012020074

Printed in the United States of America

ISBN: 9781118370681

10987654321

## QUANTUM DYNAMICS FOR CLASSICAL SYSTEMS

# Science never solves a problem without creating ten more.

### **George Bernard Shaw**

The most exciting phrase to hear in science, the one who heralds new discoveries, is not *Eureka* but *That's funny...* 

Isaac Asimov



## **PRFFACE**

In 2005 or so, I started wondering whether that particular part of quantum mechanics that theoretical physicists call second quantization could be used in the analysis of some particular, somehow discrete, classical system. In particular, I started considering stock markets, or a reasonably simplified version of these, since at that time this was a very fashionable topic: econophysics was in its early years, and the general feeling was that there was still room, and need, for many other contributions from physicists and mathematicians. I got the idea that the analysis of the huge amount of information going around a real market was only part of what was interesting to do. I was much more interested in considering the viewpoint of the single trader, who is more likely interested in having some control of his own portfolio. Therefore, I constructed a model of a simplified market, just to see if this strange approach could be interesting for such a hypothetical trader, and I suddenly realized that "yes, it might make sense to carry on in this line of research, but, wow, it is hard to have such a paper accepted in a good journal." However, after a few weeks, I also realized that this topic seemed to be interesting not only for me, but also for a large community of scientists, and that this community was increasing very fast, producing more and more contributions on the ArXiv. People started citing my first paper, and I was contacted by people interested in what I was doing and who wanted to discuss my point of view. This pushed me in the direction of considering more sophisticated

models for stock markets, using my knowledge of quantum mechanics for systems with infinite degrees of freedom in this other, and apparently completely different, field. I thought that this was essentially the end of the story: quantum versus economics. Unexpectedly, a few years ago during a conference in Acireale where I gave a talk on my quantum stock markets, I had a discussion with an old friend of mine, Franco Oliveri, and he suggested using the same general strategy in a different classical context. I remember that in our first discussion, we were thinking of foxes and chicken, a topic that was not very exciting for me. After a while, we realized that what we were discussing could also have been used to describe something completely different: a love story. And that was the beginning of a long story that still continues. Since then, we have constructed several models for different classical systems, playing with our understanding of these systems and looking for some phenomenological description. It turned out that these models quite often produce nontrivial and, in my opinion, quite interesting features that are not fully explored yet. Moreover, what is also very intriguing to me is that the same general framework can be used in many different contexts, giving rise to a sort of unifying setting.

This book might be considered as a first attempt to summarize what I have done so far in this field. My idea was to make the book reasonably simple and self-contained. This is because I expect that some not necessarily mathematical-minded readers might be intrigued by the title, and I do not want to lose these readers. However, a complex system can be made easy only up to a certain extent, and this is exactly what I have tried to do in these notes. Even the love story I will consider in Chapter 3, which from the purely dynamic point of view is surely the simplest situation, is not simple at all. This is not a big surprise, as almost every lover knows very well from personal experience. I should also clarify that it is not my main interest to discuss the psychological aspects behind a love story, a migration process, or the choices of traders in a stock market. I am not even interested in giving any abstract, or too general, description of these systems. Here I want to be quantitative! I want to deduce formulas from general ideas, and I want to see what these formulas imply for the system I have in mind, and if they have some predictive power. However, this ultimate goal implies some effort to the reader, who is required to create his own background on quantum mechanics (if needed) by reading Chapter 2. Dear reader, if you can understand Chapter 2, you can understand the rest! On the other hand, if Chapter 2 is too technical for you, do not worry: you could still try to read the book, simply jumping over this chapter. Of course, if you are not a physicist, you will lose a lot. But you can still get the feeling of what is going on. It is up to you! I really hope you enjoy reading this book!

FABIO BAGARELLO

## **ACKNOWLEDGMENTS**

It is always a pleasure to thank old friends of mine such as Camillo Trapani and Franco Oliveri for their very precious help in so many different situations that I cannot even remember myself. This could be seen as (the first) evidence of the death of my neurons. But, do not worry! I have still enough neurons left in my brain to remember that if something in functional analysis is not particularly clear, Camillo is the right one! And I also have a post-it on my desk that says "Any numerical problem? Call Franco!" What is more funny is that they still answer my questions, even when they are very, very busy! Franco is also responsible, in part, for what is discussed in these notes, and I also thank him for this scientific collaboration and for his enthusiasm.

It is also a pleasure to thank the various editors and referees whom I have *met* along these years, including the ones who referred this book (before it became a book!). Most of them have contributed significantly to the growth of my research, with many useful, and sometimes unexpected, suggestions. Particular thanks goes to Wiley, for the enthusiasm shown for my manuscript.

I dedicate this book to my beloved parents Giovanna and Benedetto, to my brother Vincenzo, and, *dulcis in fundo*, to Federico, Giovanna, and Grazyna. When I look at them I often ask myself the same question, a question for which I have no answer, yet: how can they resist with so much mathematics and physics going around?

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