



# SUCCESS AT STATISTICS



A W o r k t e x t w i t h H u m o r

Fred Pyrczak



# Success at Statistics

A Worktext with Humor

Fred Pyrczak

California State University, Los Angeles



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P.O. Box 39731 • Los Angeles, CA 90039

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# Introduction

I wrote this book for students who need a solid grounding in basic statistical methods but who may have some anxiety about taking a math-related class. Several features should help reduce anxiety:

1. I included numerous humorous, self-checking, riddle-based worksheets. Of course, I hope the humor will bring a bit of levity into an important course that sometimes seems to run dry. Much more important, however, is the instructional value of self-checking exercises that are riddle-based. If you have incorrect answers on a worksheet, the answer to the riddle will not make sense. Thus, you get immediate feedback that *something* is wrong without being given the correct answers. Reviewing the material usually will lead to correct answers in short order. Thus, you get to learn from your mistakes by being given a chance to reconsider your answers.
2. I have divided this book into short sections—each of which is much shorter than the typical chapter in a statistics textbook. This has several benefits. First, a short section of technical material—often only two or three pages long—is less intimidating than a long chapter. Even if you have difficulty with a section, knowing that it is short and contains only a limited amount of material makes it less intimidating as you reread it to achieve mastery.

Second, your anxiety about mastering difficult material should be reduced as you experience success in learning the material. Mastering a section, even if it is only two pages long, helps reduce anxiety.

Few things in learning are more frustrating than struggling through a long chapter of technical material and then, while attempting end-of-chapter exercises, realizing that the whole chapter needs to be reread and reviewed—an intimidating prospect. Frustration and intimidation often cause anxiety and failure.

Another advantage of short sections is that they allow instructors to customize their courses by (a) assigning only those sections needed to fulfill course objectives and (b) arranging the presentation of topics in a sequence best suited to their students' needs.

### **Where to Begin**

If your basic math skills are rusty, I suggest that you begin with the Supplement in Part C of this book. It will help you review skills that you will use throughout this course.

### **History of This Book and Acknowledgments**

I developed this book over a period of three years and tried out successive drafts with hundreds of students in education, nursing, counseling, sociology, and criminal justice. My students' criticisms of the material led to improvements in the text. Their praise of the riddle-based exercises and the short sections into which the material was divided gave me the motivation to complete the book. To these students, I am very grateful.

Anne Hafner and Patricia Bates Simun, both of California State University, Los Angeles and Roger A. Stewart of the University of Wyoming reviewed large portions of this book while it was in progress. Robert Morman of the California State University, Los Angeles reviewed the entire manuscript in its final stages and offered many useful suggestions. To these colleagues I am grateful for their invaluable assistance. Errors, of course, remain my responsibility.

### **Communicating With Me**

Your criticisms of this book will be greatly appreciated. Please use the address on the title page of this book to communicate with me.

Fred Pirczak  
Los Angeles

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## Section 1: Descriptive Versus Inferential Statistics

*Descriptive statistics* summarize data.<sup>1</sup> For example, suppose you have the scores on a standardized test for 500 subjects. One way to summarize the data is to calculate an *average* score, which indicates how the typical person scored. You might also determine the highest and lowest scores, which would indicate how much the scores varied. These and other descriptive statistics are described in detail in this book.

*Inferential statistics* are tools that tell us how much confidence we can have when we generalize from a *sample* to a *population*.<sup>2</sup> You are familiar with national opinion polls in which a carefully drawn sample of only about 1,500 adults is used to estimate the opinions of the entire adult population of the United States. The pollster first calculates *descriptive statistics*, such as the *percentage* of respondents who are in favor of capital punishment and the percentage who are opposed. Having sampled, he or she knows that the results may not be accurate because the sample may not be representative; in fact the pollster knows that there is a high probability that the results are off by at least a small amount. This is why pollsters often mention a *margin of error*, which is an inferential statistic. It is reported as a warning to the audience that random sampling may have produced errors, which should be considered when interpreting results.<sup>3</sup> For example, a weekly news magazine recently reported that in a national poll 58% of the respondents believed that the economy was improving; a footnote indicated that the margin of error was  $\pm 2.3$ . This means that the pollster was confident that the true percentage for the whole population was within 2.3 percentage points of 58%.<sup>4</sup>

A *population* is any group in which an investigator is interested. It may be large, such as all adults age 18 and over who reside in the United States, or it

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<sup>1</sup>Note that the word *data* is plural.

<sup>2</sup>The word *inferential* comes from *infer*. When we generalize from a sample to a population, we are *inferring* that the sample is representative of the population.

<sup>3</sup>The measurement techniques, especially the wording of the question(s), may also produce errors. That is why sophisticated consumers of research usually want to know the exact wording of a question, especially if important decisions are to be made based on the results.

<sup>4</sup>Margins of error are described in detail in several sections in Part B of this book.

might be small, such as all registered nurses employed by a specific hospital.<sup>5</sup> An investigator is free to choose a population of interest and should clearly define it when reporting results, which allows an audience to decide whether they are interested in the results. A study in which all members of a population are included is called a *census*. A census is often feasible and desirable when working with small populations (e.g., an algebra teacher may wish to pretest all students at the beginning of a course, which will determine at what level to begin instruction and how to teach the class). Inferential statistics are *not* needed when describing the results of a census because there is no sampling error.

When a population is large, it is more economical to use only a sample of the population. With modern sampling techniques, highly accurate information can be obtained using relatively small samples. Various methods of sampling are described later in this book.

Technically, descriptive tools such as averages and percentages for census data should be called *parameters* and *not statistics*. For example, an average score based on a study of a population (e.g., a census) should be referred to as a *parameter*, but an average for a sample should be referred to as a *statistic*. Here's a visual aid for remembering the difference:

**S**tatistics come from **S**amples

**P**arameters come from **P**opulations

### Terms to Review Before Attempting Worksheet 1

**descriptive statistics, average, inferential statistics,  
sample, population, percentage, margin of error,  
census, parameter**

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<sup>5</sup>Notice that all members of a population have at least one characteristic in common such as all being registered nurses employed by a specific hospital.

## Worksheet 1: Descriptive Versus Inferential Statistics

**Riddle: What is the cynical definition of "happiness"?**

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. Is an average a *descriptive* or an *inferential* statistic?
2. Do *descriptive* or *inferential* statistics help us generalize from a sample to a population?
3. What is the entire group that an investigator is interested in called?
4. Which statistic mentioned in Section 1 is an example of an inferential statistic?
5. When all members of a population in which an investigator is interested in are included in a study, the study is called a . . .
6. "National opinion polls often use a sample of about 1,500 subjects." Is this statement true or false?
7. "Populations are always large." Is this statement true or false?

## Worksheet 1 (Continued)

8. Is the term *data* singular or plural?
9. Should a percentage obtained from a census be referred to as a *statistic* or as a *parameter*?
10. Should a consumer of research allow for a *margin of error* when reading reports of census data?

### Solution section:

scores (bliss) subjects (glad) descriptive (an) parameter (of) false (the)  
inferential (agreeable) information (and) population (sensation) census (from)  
economical (bliss) no (another) pollster (but) plural (misery)  
true (contemplating) margin of error (arising) investigator (because) yes (life)

**Write the answer to the riddle here, putting one word on each line:**

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## Section 2: Scales of Measurement

*Scales of measurement* (also known as *levels of measurement*) help investigators determine what type of statistical analysis is appropriate. As a consumer of research, you can often tell from the description of the measurement techniques what level was used and then judge whether the analysis was appropriate. It is important to master this section because it is referred to in a number of others that follow.

The lowest level of measurement is **nominal** (also known as *categorical*). It is helpful to think of this level as the *naming* level. Here are some examples:

- ♦ Subjects name the political parties with which they are affiliated.
- ♦ Subjects name their gender.
- ♦ Subjects name the state in which they reside.

Notice that the categories that subjects name in these examples do not put the subjects in any particular order. There is no basis on which we could all agree for saying that Republicans are either numerically higher or lower than Democrats. The same is true for gender and state of residence.

The next level of measurement is **ordinal**. Ordinal measurement puts subjects in *order* from high to low, but it does *not* indicate how much higher or lower one subject is in relation to another. To understand this level, consider these examples:

- ♦ Subjects are ranked according to their height; the tallest subject is given a rank of 1, the next tallest is given a rank of 2, and so on.
- ♦ Three brands of hand lotion are ranked according to consumers' preferences for them.

In these examples, the measurements tell us the relative standings of subjects but not the amount of difference among subjects. For example, we know that a subject with a rank of one is taller than a subject with a rank of two, but we do not know by how much. The first subject may be only one-quarter of an inch taller *or* two feet taller than the second.

The next two levels, **interval** and **ratio**, tell us by *how much* subjects differ.

For example:

- ♦ The height of each subject is measured to the nearest inch.
- ♦ The number of times each pigeon presses a button in the first minute after receiving a reward.

Notice that if one subject is 5'6" tall and another is 5'8" tall, we not only know the order of the subjects, but we also know by how many inches the subjects differ from each other. Both *interval* and *ratio* scales have equal intervals; for instance, the difference between one inch and two inches is the same as the difference between four inches and five inches.

In most statistical analyses, *interval* and *ratio* measurements are analyzed in the same way. There is a scientific difference, however. An *interval* scale does not have an absolute zero. For example, if we measure intelligence, we do not know exactly what constitutes zero intelligence and, thus, cannot measure it.<sup>1</sup> In contrast, a *ratio* scale has an absolute zero. For example, we know where the zero point is on a tape measure when we measure height.

If you are having trouble mastering levels of measurement, first memorize this phrase:

**No One Is Ready**

The first letters of the words (NOIR) are the first letters in the names of the four levels of measurement in order from lowest to highest. Then read this section again and associate definitions with each type.

## Terms to Review Before Attempting Worksheet 2

**nominal, ordinal, interval, ratio**

<sup>1</sup>Most applied researchers treat the scores from standardized tests (except percentile ranks and grade-equivalent scores) as *interval* scales of measurement, even though there is some controversy as to whether, for example, the difference between an IQ of 100 and 110 is the same as the difference between 110 and 120.

## Worksheet 2: Scales of Measurement

***Riddle: According to Josh Jenkins, how do you know when you are making too many errors?***

DIRECTIONS: For each example of measurement, circle the scale of measurement that it exemplifies. The word in parentheses to the right of the correct answer to the first question is the first word in the answer to the riddle, the word in parentheses to the right of the second correct answer is the second word, and so on.

1. Weight measured in pounds and ounces using an accurate scale.  
nominal (only) ordinal (make) interval (some) ratio (when)
2. Gender measured by asking each subject whether he or she is male or female.  
nominal (the) ordinal (if) interval (of) ratio (errors)
3. Verbal aptitude measured by the College Board's Scholastic Aptitude Test: Verbal (assuming that each point from 200 to 800 represents an equal amount of aptitude).  
nominal (calculate) ordinal (stumble) interval (eraser) ratio (become)
4. Cheerfulness measured by having a teacher give a rank of 1 to the student judged to be the most cheerful, a rank of 2 to the student judged to be the next most cheerful, etc.  
nominal (too) ordinal (wears) interval (for) ratio (out)
5. The amount of juice in a bottle measured in fluid ounces.  
nominal (big) ordinal (weak) interval (inside) ratio (out)

## Worksheet 2 (Continued)

6. Length of a telephone conversation measured by recording the number of seconds from the beginning to the end of the conversation.

nominal (count) ordinal (person) interval (mistakes) ratio (ahead)

7. Height measured by having ten subjects take off their shoes and line up according to height with the tallest person at the front of the line and giving that person a score of 10, giving the next person a score of 9, etc.

nominal (are) ordinal (of) interval (the) ratio (made)

8. Writing skills measured by having all subjects write for 15 minutes on the same topic and having an English teacher put the essays in order from best to worst.

nominal (when) ordinal (the) interval (person) ratio (wrongly)

9. Birthplace measured by having each subject write the name of the country in which she or he was born.

nominal (pencil) ordinal (seeing) interval (caught) ratio (also)

**Write the answer to the riddle here, putting one word on each line:**

_____	_____	_____	_____
_____	_____	_____	_____



### Section 3: Frequencies, Percentages, and Proportions

A *frequency* is the number of subjects or cases; its symbol is  $f$ .<sup>1</sup> Another symbol,  $N$ , meaning *number of subjects*, is also used to stand for frequency.<sup>2</sup> Thus, if you see in a report that  $f = 23$  for a score of 99, you know that 23 subjects had a score of 99.

A *percentage*, whose symbol is  $P$  or %, indicates the number per hundred who have a certain characteristic. Thus, if you are told that 44% of the subjects in a town are registered as Democrats, you know that for each 100 registered voters, 44 are Democrats. To determine how many (the *frequency*) are Democrats, multiply the total number of registered voters by .44. Thus, if there are 2,313 registered voters,  $.44 \times 2,313 = 1,017.72$  are Democrats. In a report in the general media, this would probably be rounded to 1,018. In a report in an academic journal, thesis, or dissertation, however, the answer is usually reported to two decimal places.

To calculate a percentage, use division. Consider this example: if 22 of 84 gifted children in a sample report being afraid of the dark, determine the percentage by dividing the number who are afraid by the total number of children and then multiply by 100; thus,  $22 \div 84 = .2619 \times 100 = 26.19\%$ . This result means that, based on the sample, if you questioned *100 subjects* from the same population, you would expect about 26 of them to report being afraid of the dark. Notice that only 84 subjects were actually studied, yet the result still is based on 100.

A *proportion* is part of one (1). In the previous paragraph, the proportion of children afraid of the dark is .2619 or .26—the answer obtained before multiplying by 100. This means that *twenty-six hundredths of each child* is afraid of the dark. As you can see, proportions are harder to interpret than percentages; thus, percentages are usually preferred to proportions in all types of

<sup>1</sup>Note that  $f$  is italicized. If you do not have the ability to type in italics, underline the symbol. This applies to almost all statistical symbols. Also, pay attention to the case. A lower-case  $f$  stands for *frequency*; an upper-case  $F$  stands for another statistic described later in this book.

<sup>2</sup>An upper-case  $N$  should be used when describing a population; a lower-case  $n$  should be used when describing a sample.