



Tariq Abuhashim

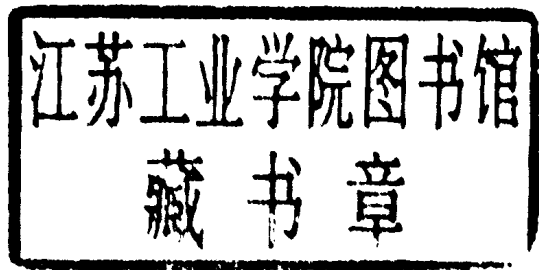
# Improving INS/GPS Integration for Mobile Robotics Applications

A Thesis in Mechatronics

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# IMPROVING INS/GPS INTGRATION FOR MOBILE ROBOTICS APPLICATIONS

Tariq Salman AbuHashim, Candidate for Master of Science in Mechatronics Engineering

American University of Sharjah, 2008

## ABSTRACT

As unmanned systems become more and more important, reliability and integrity issues become definite, specially when being implemented with low-cost (or sometimes are referred to as commercial-of-the shelf or COTS) sensors while being designed to operate in remote, hazardous and harsh environments. As a result, fault (and failure) detection and identification (FDI) is a must, and it is a crucial requirement in designing unmanned vehicles. In this thesis, integrity is defined as the ability of the system to provide reliable navigation information, to monitor the health of the aids, to detect abnormalities in their behavior, and to survive once a failure in one of its components (whether they are sensors, actuators, mathematical models, and computations) occurs. On the other hand, reliability is component dependent. A navigation system is reliable as its most unreliable component. Therefore, integrity implies reliability while reliability not necessarily implies integrity.

This thesis, mainly, discusses the issue of implementing a low-cost inertial navigation system, aided with satellite navigation system. In doing so, a fault detection and identification scheme must be involved and the performance of all the system components must be verified. The FDI system should take into account types of failures commonly occur, guarantee that all faults will be detected, assist design specifications and respond as fast as possible to faults. On the other hand, it should take into account the complexity of the implementation and its robustness in the presence of mismodelling. Innovation-based techniques, in particular the  $\chi^2$  SCT, offer tradeoffs between complexity and performance and detect a large set of failures. However, they are sensitive to filter tuning and have no fault identification ability. On the other hand, the model-based approaches, in particular the multiple model adaptive estimation (MMAE), have an outstanding decision making ability and are insensitive to filter tuning. However, they require a priori knowledge on the system and failure model and are computationally expensive. The integration of both techniques can enhance the FDI performance of both systems. In this thesis a sequential FDI algorithm is proposed. This algorithm employs an innovation-based technique for fault detection and a model-based technique for identification.

The performance of the  $\chi^2$ -MMAE sequential algorithm is simulated and tested on actual IMU and GPS data. Results showed that the sequential algorithm has a comparable identification ability as the MMAE algorithm with a substantial reduction in computational requirements, since the filters bank was only allowed to operate on segments of time where faults were detected. On the other hand, unlike the MMAE algorithm where the performance of the filter was affected during no-fault conditions, the sequential scheme guaranteed the consistency of the estimator in all of its modes of operation and didn't affect its performance during normal no-fault modes of operation.

# NOMENCLATURE

## Notation

$C_A^B$	Direction cosine matrix transforming quantities from $A$ frame to $B$ frame
$\omega_{AB}^B$	Angular rate of $B$ frame relative to $A$ frame expressed with components in $B$ frame
$\mathbf{b}^A$	Vector $\mathbf{b}$ with components in $A$ frame
$\Omega^A = [\omega^A \times]$	Skew symmetric matrix with components of $\omega$ in $A$ frame

## Axis System (Reference Frames), Angles and Transformations

$i$	Inertial reference frame
$e$	Earth-fixed reference frame
$n$	Navigation reference frame
$b$	Body reference frame
$m$	Mechanisation frame
$O_{x_i y_i z_i}$	Inertial axis system
$O_{x_e y_e z_e}$	Earth-fixed axis system
$O_{x_n y_n z_n}$	Navigation axis system
$O_{x_b y_b z_b}$	Body axis system
$h$	Altitude
$\varphi$	Latitude
$\lambda$	Longitude
$\phi$	Roll angle
$\theta$	Pitch angle
$\psi$	Yaw angle

## Earth Quantities (WGS-84)

$g_n$	Normal gravitational acceleration ( $\varphi = 45^\circ$ )
$a$	Equatorial radius of the Earth (semimajor axis) = 6378137.000 m
$b$	polar radius of the Earth (semiminor axis) = 6356752.3142 m
$\Omega$	Earth turn rate with respect to $i$ frame = $7.292116 \times 10^{-5}$ rad/s
$\mathbf{g}_l$	Local gravity column matrix
$f$	Flattening (ellipticity) = $1/298.257223563$ (0.00335281066474)

$e$	Major eccentricity of Earth = 0.0818191908426
$\mu$	Earth's gravitational constant = $3986005 \times 10^8 m^3/s^2$
$M$	Mass of Earth (including the atmosphere) = $5.9733328 \times 10^{24}$ Kg

#### Dynamic Quantities

$V_n$	Normal gravitational acceleration ( $\varphi = 45^\circ$ )
$\mathbf{V}_e^n$	Kinematic velocity expressed in $n$ frame
$\omega_{nb}^b$	Angular rate of $b$ frame relative to $n$ frame expressed in $b$ frame
$\omega_{en}^n$	Angular rate of $n$ frame relative to $e$ frame expressed in $n$ frame
$\omega_{ib}^b$	Angular rate of $b$ frame relative to $n$ frame expressed in $b$ frame
$\mathbf{f}^e$	Specific force in $b$ frame
$\mathbf{f}^b$	Specific force in $n$ frame
$v_N, v_E, v_D$	The north, east and down components of $\mathbf{V}_e^n$
$f_N, f_E, f_D$	The north, east and down components of $\mathbf{f}_e^n$

#### Subscripts

$j, k, l$	Indexes for high speed computer cycle ( $j$ -cycle), moderate computer speed cycle ( $k$ -cycle) and low computer speed cycle ( $l$ -cycle) respectively
$N, E, D$	North, East and Down components of $n$ frame vector

#### Symbols

$(k k-1)$	Used to denote a quantity at time $k$ immediately before the measurement update
$(k k)$	Used to denote a quantity at time $k$ immediately after the measurement update
$z(k-1)$	A growing length measurement history vector consisting of all measurement vectors from 0 through $k-1$
$P(x)$	Probability density function of $x$
$Z^k$	Set of $k$ observations

#### Abbreviations

ACFR	Australian Center for Field Robotics
AI	Artificial Intelligence
CMA	Covariance Matching Approach
COTS	Commercial-Of-The-Shelf
CR	Confidence Region
DCM	Direction Cosine Matrix

DGPS	Differential Global Positioning System
DoD	Department of Defence
DoF	Degrees of Freedom
ECEF	Earth-Centered Earth-Fixed
ECI	Earth-Centered Inertial
EKF	EKF
FDI	Fault Detection and Identification
FMEA	Failure Modes and Effective Analysis
GA	Genetic Algorithms
GDOP	Geometric Dilution of Precision
GLONASS	Global Navigation Satellite System
GLR	Generalized Likelihood Ratio
GPS	Global Positioning System
GRMMAE	Generalized Residual Multiple Model Adaptive Estimation
IMM	Interactive Multiple Model
IMU	Inertial Measurement Unit
INS	Inertial Navigation System
IODC	Issue Of Date Clock
IODE	Issue Of Date Ephemeris
KF	Kalman Filter
LKF	Linear Kalman Filter
LOS	Line of Site
MBKF	Multiple Bank Kalman Filters
MMAE	Multiple Model Adaptive Estimation
MMSA	Minimal Mean Squared Error
MTBF	Mean Time Between Failure
NA	Navigation Algorithm
NASA	National Aeronautics and Space Administration
NED	North-East-Down
NEES	Normalized (state) Estimation Error Squared
NIS	Normalized Innovation Squared
NN	Neural Networks
PDF	Probability Distribution Function
PPS	Precision positioning System
PRN	pseudorandom noise
PSO	Particle Swarm Optimization
SA	Selective Availability
SCT	Statistical Consistency Test
SDINS	Strapdown Inertial Navigation System



SLAM	Simultaneous Localization and Mapping
SPRT	Sequential Probability Ratio Test
SPS	Standard Positioning System
SV	Satellite Vehicle
TOW	Time Of Week
UAV	Unmanned Aerial Vehicle
UGV	Unmanned Ground Vehicle
UKF	Unscented Kalman Filter
UT	Unscented Transformation

## GLOSSARY OF TERMS

- Conning:** The cyclic motion of one axis due to rotational motion of the other two axes (Sukkarieh, S., 2000). Such motion can result due to oscillatory motion such as vibration being undetected by the inertial navigation system.
- Dead reckoning:** Types of navigation systems which rely on the continuous updating of the position data derived from inputs of velocity components or speed and heading generated from a known start position. Inertial navigation systems are considered as sort of dead reckoning systems.
- Disturbance:** An unknown and uncontrolled input acting on a system (Gustafsson, F., 2000).
- Ellipsoid:** A three-dimensional shape formed by rotating an ellipse about its minor axis.
- Ergodicity:** A process is considered ergodic if all of its statistical parameters, mean, variance, and so on, can be determined from arbitrary chosen member functions. A sampled function is considered ergodic if its time-average statistics equal the ensemble averages (Grewal, M. S., & Andrews, A. P., 2001).
- Error:** Deviation between a measured or computed value of an output variable and the true, specified or theoretically correct value (Gustafsson, F., 2000).
- Failure:** Permanent interruption of a system ability to perform a required function under specified operating conditions (Gustafsson, F., 2000).
- Fault:** Unpermitted deviation of at least one characteristic property of parameter of the system from acceptable / usual / standard condition (Gustafsson, F., 2000).
- Fault detection:** Determination of faults present in a system and time of detection (Gustafsson, F., 2000).
- Fault isolation:** Determination of kind, location and time of detection of fault. Follows fault detection (Gustafsson, F., 2000).
- Fault identification:** Determination of the size and time-variant behavior of a fault. Follows fault isolation (Gustafsson, F., 2000).

- Fault diagnosis:** Determination of kind, size, location and time of fault. Follows fault detection and includes fault isolation and identification (Gustafsson, F., 2000).
- Monitoring:** A continuous real time task of determining the conditions of a physical system, by recording information recognizing and indicating anomalies of the behavior (Gustafsson, F., 2000).
- Perturbation:** An input acting on a system which results in a temporary departure from current state (Gustafsson, F., 2000).
- Residuals:** Processed measurements. Kalman filter residuals, which are the differences between state estimates predictions and the measurements predictions are called innovations. They can be used as fault indicators, based on deviation between measurements and model-equation-based computations.
- Sagnac Effect:** When computations for the satellite position are made in an ECEF coordinate system, and during the propagation time of the satellite vehicle signal transmission, a clock of the surface of the Earth will experience a finite rotation with respect to an ECI coordinate system (Kaplan, E. D., & Hegarty, C. J., 2006).
- Sculling:** A combination of linear and angular oscillatory motions of equal frequency in orthogonal axes.
- Symptom:** Change of an observable quantity from normal behavior (Gustafsson, F., 2000).
- Time-Invariant System:** A system is time-invariant if a time shift in the input result in a corresponding time shift in the output. The output of a time-invariant system depends on time differences and not on absolute values of time (Stremmer, F. G., 1990).

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*To Knowledge.*

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