

A.N. Matveev

Optics

A large, stylized title 'Optics' in a serif font. The letter 'p' is uniquely designed as an optical illusion, featuring a rainbow-colored, three-dimensional structure that appears to be a series of parallel bars or a prism, creating a sense of depth and movement. The rest of the letters are in a plain, light-colored serif font.

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Optics

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Preface

A rapid introduction of the achievements of science in industry is the distinguishing feature of modern life. Optics is just one striking example of this trend. Intensive development of laser technology has long ceased to be just a landmark in scientific research and has revolutionized the industrial technology in many fields. Training of qualified personnel for those branches of industry where the latest achievements of science are implemented has become one of the most important problems of our time.

The application of lasers and their use in conjunction with computers have created quite favourable conditions for the development of optics. The high coherence of laser radiation permits an investigation and reproduction of a wide range of phenomena in the optical range. These phenomena cannot be studied without a highly coherent radiation. The high energy density of laser radiation opens new horizons for the investigation of nonlinear processes in optics under conditions that were hitherto unattainable. The possibility of generating short and ultrashort laser pulses has paved the way for analyzing rapid processes including intramolecular ones. The use of computers has given a tremendous impetus to optical studies by reducing them to direct computations or to numerical experiments.

All this has led to a significant advancement of optics in the last quarter of a century and has considerably widened the scope of its applications. This process was triggered by important studies leading to the creation of quantum-mechanical oscillators, viz. lasers and masers. Soviet physicists have not only carried out fundamental research in the fields of lasers and masers, but also made significant contributions towards the development of many important branches of optics, including light scattering, holography, optical systems, nonlinear optics, etc. Naturally, the basic concepts of optics have not undergone significant changes in the course of its recent development. In some cases, these concepts were clarified, while in some other cases they were enriched by the

introduction of concepts, methods and mathematical approaches from other branches of science (like the theory of random processes, physics of linear and nonlinear oscillations, matrix computational methods, etc.).

The subject matter of the book is completely reflected in Contents. Statistical properties of light and its spectral representation are covered in greater detail than usual. Diffraction of light is described in the framework of Kirchhoff's integral. The effectiveness of the matrix methods is shown in sections covering geometrical optics and interference of light in thin films. A unified approach involving Fourier optics has been adopted for describing the diffraction theory of image formation, spatial filtration of images, holography, and other allied topics. Analysis of partial coherence and partial polarization is carried out in terms of the first correlation function.

The mathematical aspect of the material presented in this book has been kept as simple as possible, and at the same time in line with the rigorous scientific approach. Wherever necessary, mathematical elucidations are presented and more detailed computations are made. The cumbersomeness of some of these presentations should not produce the impression that the mathematical apparatus used in the book is complicated. The student should muster patience and carry out these calculations independently to rid himself of such an impression.

The most significant aspect in which this book differs from the books dealing with mechanics, molecular physics and electricity is that its basic principles lie beyond the scope of this course. Because of this, considerable emphasis has been laid on the deductive method of description. The material of this course is therefore presented in deductive form and in most cases (though not always) the experimental results are analyzed to show the agreement between the theoretical results and the experimental data, or to explain the observed phenomena.

The book is based on the author's experience of teaching physics for many years at the Physics Faculty of the Lomonosov State University, Moscow. The author is grateful to his colleagues for many fruitful discussions on the content of the book.

The author is indebted to Academician A. I. Akhiezer of the Academy of Sciences of the Ukrainian SSR, as well as to Prof. N. I. Kaliteevskii and his associates at the Department of General Physics, Zhdanov State University, Leningrad, for carefully reviewing the manuscript and making valuable comments.

Moscow, May 1987.

Author

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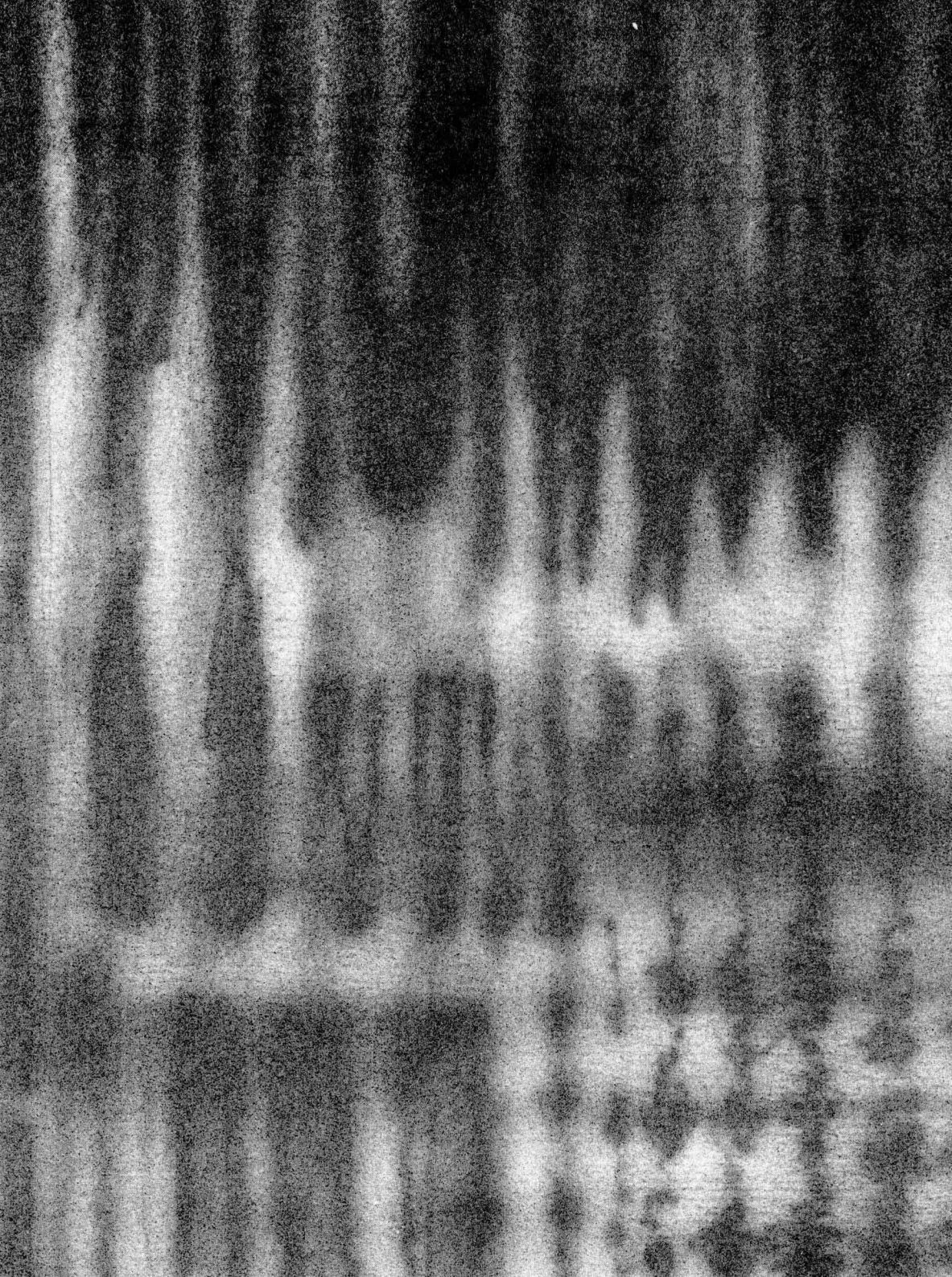
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Electromagnetic Waves

Basic idea:

On the basis of the electromagnetic nature of light, the properties of monochromatic light waves are investigated with the help of Maxwell's equations.

SEC. 1. OPTICAL RANGE OF ELECTROMAGNETIC WAVES

The factors that make the waves of the visible range most suitable for vision are analyzed and the properties of the optical range are discussed.

WAVELENGTHS OF THE VISIBLE RANGE. The visible range includes electromagnetic waves that can be perceived by the human eye. The limiting range of these waves depends on the individual properties of the eye and varies approximately in the interval

$$\lambda = 0.38\text{--}0.76 \mu\text{m}. \quad (1.1)$$

WAVE FREQUENCIES IN THE VISIBLE RANGE. In optics, one encounters the circular (cyclic) frequency

$$\omega = 2\pi/T, \quad (1.2)$$

where T is the oscillation period of the wave, as well as the frequency

$$\nu = 1/T. \quad (1.3)$$

These frequencies are connected through the obvious relation

$$\omega = 2\pi\nu. \quad (1.4)$$

The frequency is expressed in hertz, while the cyclic frequency is expressed in reciprocal seconds. Considering that

$$\lambda = cT, \quad (1.5)$$

where $c = 3 \times 10^8$ m/s is the velocity of light in vacuum, we obtain the following limiting values for the visible range:

$$\nu = (4-8) \times 10^{14} \text{ Hz}, \quad (1.6)$$

$$\omega = (2.5-5.0) \times 10^{15} \text{ s}^{-1}. \quad (1.7)$$

OPTICAL AND OTHER RANGES OF ELECTROMAGNETIC WAVES. It is possible to imagine theoretically that all frequencies from $\nu = 0$ to $\nu = \infty$ do exist. However, certain restrictions are imposed on these frequencies by the corpuscular properties of radiation. It is shown in the quantum theory that the electromagnetic radiation exists in the form of “packets” of energy (quanta). The energy of a quantum of radiation is connected with its frequency through the relation

$$E = \hbar\omega = h\nu, \quad (1.8)$$

where $h = 6.62 \times 10^{-34}$ J·s is called Planck's constant. The quantity $\hbar = h/(2\pi) = 1.05 \times 10^{-34}$ J·s is also called Planck's constant (new).

Both these quantities are encountered equally often in physics according to circumstances. It is obvious from (1.8) that the constant \hbar is convenient for use with the cyclic frequencies, while the constant h should be used for frequencies ν .

It is clear from (1.8) that infinite frequencies $\nu = \infty$ are not possible, since the corresponding radiation quanta would have infinite energies. The same relation also imposes a restriction on the lower values of the frequency if there exists the smallest possible value E_0 for the energy of a quantum. This means that the frequency cannot be lower than $\nu_0 = E_0/h$. At present, there are no indications in physics that the energy of the photons of electromagnetic radiation is restricted from below. Consequently, there is no lower limit to the frequency of the electromagnetic waves. The lowest frequency (about 8 Hz) has been observed for the standing electromagnetic waves between the ionosphere and the Earth's surface. This leads to the conclusion [see (1.8)] that the lowest energy of the electromagnetic radiation quanta is in any case below 10^{-33} J.

All possible frequencies of electromagnetic waves are subdivided into the following ranges (see the table below).

Each range has its own characteristic properties. As the wave frequency increases, the corpuscular properties of the radiation become more pronounced. Waves of different ranges