CLASSICAL ELECTROMAGNETIC RADIATION

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Preface

This book presents an account of classical electrodynamics with emphasis on radiation problems and the wave aspects of the electromagnetic field. Designed as a text for a one-semester, three- or four-hour course for physics students at the advanced undergraduate level, the book may also be used in the electrodynamics portion of courses in mathematical physics or in theoretical physics.

The objective of the book is to provide a modern and reasonably sophisticated mathematical treatment of classical electrodynamics at the undergraduate level. Since the wave aspects of the electromagnetic field are the topics which are most important in "modern physics," Maxwell's equations are introduced after a minimum of preliminary material, and the remainder of the book develops the various implications of these equations.

It is assumed that the reader has a recent acquaintanceship with the basic principles of electromagnetism, so only a brief survey of the fundamental material is given in Chapter 1. Chapters 2 and 3 include a detailed discussion of Laplace's equation and a treatment of multipole effects, since such material is of considerable importance in the development of radiation theory, but is rarely included in basic courses. The main concern of this book, however, is not electrostatics, so Laplace's equation is treated only as a general boundary-value problem, and the solutions are developed accordingly. Thus, no mention is made of the method of images, not is the connection made between harmonic functions and the Cauchy-Riemann equations of complex analysis.

The electromagnetic field equations are developed in the time-dependent form in Chapter 4, concluding the coverage of introductory material. The subsequent chapters treat the subjects of wave propagation in space and in material media (Chapter 5), reflection and refraction (Chapter 6), the Liénard-Wiechert potentials (Chapter 7), and radiating systems (Chapter 8). In Chapter 9 the discussion turns to the interaction of electromagnetic radiation with microscopic matter, and a brief development of classical electron theory is given. The next two chapters lead up to the discussion of diffraction theory (Chapter 12) by presenting a treatment of spherical waves and by discussing at some length the subject of interference. The concluding chapter is an introduction to relativistic electrodynamics.

Because of the desire to present material upon which modern theories of matter and radiation are based, certain topics must be emphasized at the expense of others. Some aspects of electromagnetic radiation that are usually classified as "physical optics" are discussed in detail, but it was not deemed appropriate to include, for example, the topics of double refraction, optical activity, or anisotropic media. Similarly, only passing mention is made of the application of the theory to optical instruments. The subject of waveguides also lies primarily outside the domain of this book, and therefore only a brief discussion is given in connection with the properties of radiation fields within hollow, perfectly conducting pipes. (The reader may consult engineering texts for further details regarding these interesting devices.

The subject of electrodynamics is intimately connected with the theory of relativity. But, historically, essentially all the classical results had been worked out before the development of special relativity, and indeed, these investigations paved the way for the construction of relativity theory. It is possible to treat electrodynamics by first postulating special relativity and then deriving many of the results which were originally obtained from experiment in the pre-relativity era. The approach adopted here, however, is to present a more-or-less historical development and only at the end to show that relativity provides a beautiful and complete unification of the subject. This procedure places the climax appropriately at the end, and from a pedagogical standpoint seems preferable to attempting a formal deduction of the subject from a grand, all-encompassing principle laid down at the beginning.

Because this book is directed toward applications of electrodynamics in "modern physics," which employs the Gaussian system in discussions of atomic and nuclear physics, this system is used throughout the book in preference to the MKS system. Furthermore, since the velocity of light plays such an important role in electrodynamics, it does not seem pedagogically sound to set up the basic equations (Maxwell's equations) in such a way that the fundamental constant c disappears. Finally, it is comforting (at least to the author) to see a factor of 4π explicitly appear when an integration over the entire solid angle is performed.

The suggestions for further reading are frequently extensive, in order to give a sufficient number of references so that there is a reasonable probability that some source of collateral reading may easily be located.

The author wishes to express his gratitude to the University of Maryland Computer Science Center for extending to him the use of the IBM 7090/1401 computer for calculating many of the curves which appear in the illustrations of this book.

JERRY B. MARION

The gods did not reveal from the beginning
All things to us; but in the course of time
Through seeking, men find that which is better.
But as for certain truth, no man has known it,
Nor will he know it.
—Xenophanes (6th Century B.C.)

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Chapter 11. Interference Phenomena

Chapter 12. Scalar Diffraction Theory

CHAPTER I

Fundamentals of Electromagnetics

1.1 Introduction

In this book we shall be concerned mainly with radiation phenomena associated with electromagnetic fields. We shall study the generation of electromagnetic waves, the propagation of these waves in space, and their interaction with matter of various forms. The fundamental equations which govern all of these processes are *Maxwell's equations*. These are a set of partial differential equations which describe the space and time behavior of the electromagnetic field vectors. By way of review,* we shall first examine briefly the *static* and *steady-state* properties of the electromagnetic field. In Chapters 2 and 3 we shall discuss two topics that are usually not considered at great length in introductory accounts of electromagnetism—solutions of *Laplace's equation* and *multipole effects*—since these subjects are of importance in radiation phenomena. In Chapter 4 we shall treat time-varying electromagnetic fields and will arrive at the statement of

* It is assumed that the reader has a recent acquaintanceship with this basic material, so that only a brief survey is given in the present chapter. The reader who is lacking in this background should refer to one of the books listed in the *Suggested References* at the end of this chapter.

Maxwell's equations in their most general form. Then, for the remainder of the book, we shall be concerned entirely with radiation problems.

From time to time, it will be necessary to make reference to background material which is familiar from the study of classical mechanics. Rather than duplicate the details here, references will be given to the appropriate sections or equations in *Classical Dynamics of Particles and Systems.** The reader will find that the texts listed in the Suggested References can be used as alternative sources of such material.

1.2 Units

In discussing electromagnetic phenomena, it is customary to adopt one of the many possible systems of units.† Much effort has been devoted to the defense of one or another system of units as being intrinsically better than any other. However, it is clear that the nature of the physical world can in no way be related to the choice of units. One must therefore seek the system that is most convenient for the types of problems he wishes to consider; it seems futile to attempt a justification of the choice on any other basis. The reader has no doubt been exposed to the MKS system since this system is popular for the discussion of practical or engineering problems. MKS units (volts, amperes, webers/m^2, etc.) are indeed of a convenient magnitude for the treatment of laboratory-scale effects. But in the study of the interaction of electromagnetic radiation with the fundamental constituents of matter (atoms, molecules, electrons, etc.), the arguments in favor of the MKS system lose much of their validity and it becomes more convenient to adopt the Gaussian system of units. Since we wish to emphasize in this book the relationship of electromagnetism to "modern physics," it seems reasonable to adopt a system of units that is widely used in atomic physics and allied areas. We shall therefore use the Gaussian system in which all electric quantities are measured in electrostatic units (esu) and all magnetic quantities are measured in electromagnetic units (emu). The question of units will arise from time to time as we introduce new quantities, so it seems unnecessary to give an extended discussion at this point. A summarizing list will be found in Appendix D which includes the conversion factors for passing between MKS and Gaussian units; Appendix E gives the fundamental electromagnetic equations in both systems.

The remainder of this chapter will therefore serve not only to provide the reader with a review of the fundamentals of electromagnetism but also to accustom him to the use of Gaussian units.

* Marion (Ma65a); see the Bibliography at the end of this book.

† An excellent summary of various systems of units is given by Jackson (Ja62, p. 611 ff).

1.3 The Field Vectors

In order to describe the electromagnetic field we shall use four vectors:

E ≡ *Electric intensity vector* or *electric field vector* (statvolts/cm)

D ≡ *Electric displacement vector* or *dielectric displacement vector* or, simply, *displacement vector* (statvolts/cm)

B ≡ *Magnetic induction vector* or *magnetic field vector* (gauss)†

H ≡ *Magnetic intensity vector* (oersted)†

We shall consider **E** and **B** to be the fundamental field vectors, and that **D** and **H** can be obtained from these together with the properties of the medium in which the fields occur.

The mathematical relations that the field vectors satisfy cannot be derived—they must be obtained from experiment. In the following sections we shall discuss the laws of electromagnetism that are valid for steady-state conditions. In Chapter 4 time-varying fields will be studied. The results of these considerations may be summarized in four partial differential equations—Maxwell's equations—which appear to be a true and accurate description of the behavior of electromagnetic fields. It must be emphasized that Maxwell's equations cannot be *derived* except by starting with four equally fundamental statements; they are mathematical representations of *empirical* facts.

1.4 Coulomb's Law

The first experimental fact which we wish to invoke is that the force between two point charges at rest is directed along the line connecting the charges, and the magnitude of the force is directly proportional to the magnitude of each charge and inversely proportional to the square of the distance between the charges. This is Coulomb's law* and in Gaussian units assumes the form

$$\boxed{\mathbf{F}_{12} = \frac{q_1 q_2}{r^2}\mathbf{e}_r} \qquad (1.1)$$

† The units *oersted* and *gauss* are identical, but historically oersted is applied to **H** and gauss to **B**.

* Named for Charles Augustin Coulomb (1736–1806) who determined by measurements with a torsion balance in 1785 that the inverse power of *r* which appears in the electrostatic

for the force exerted on q_1 by q_2. (The Gaussian unit of charge is the *statcoulomb*.) The quantity r is the distance between the charges and $\mathbf{e}_r$ is the unit vector in the direction from q_2 to q_1. If the charges carry the same sign, the force is repulsive; if the signs are opposite, the force is attractive. The force on a unit positive test charge in the field of a charge q defines the electric intensity vector according to

$$\mathbf{F} = q\mathbf{E} \tag{1.2}$$

Thus, the field due to a point charge q is

$$\boxed{\mathbf{E} = \frac{q}{r^2}\mathbf{e}_r} \tag{1.3}$$

An important property of the electric field (indeed, of the *electromagnetic* field) is that it is *linear*. That is, the principle of superposition applies and the field due to a number of charges is just the vector sum of the individual fields. Were it not for this property, the analysis of electromagnetic phenomena would be virtually impossible.

We may verify by direct differentiation that any vector which is proportional to $\mathbf{e}_r/r^2$ has an identically vanishing curl. Thus,*

$$\mathbf{curl}\,\mathbf{E} \equiv 0 \tag{1.4}$$

Now, if $\mathbf{E}$ can be represented as the gradient of some scalar function, then Eq. (1.4) will always be valid since **curl grad** is a null operator. Therefore, if we write

$$\boxed{\mathbf{E} = -\mathbf{grad}\,\Phi} \tag{1.5}$$

* In Chapter 4 we shall find that this result requires modification in the case of time-varying fields.

force law was 2 ± 0.02. A result with the same accuracy had previously been obtained (1771) by Henry Cavendish (1731–1810) but remained unknown until Lord Kelvin had the Cavendish manuscripts published in 1879. An even earlier measurement (1769) had been made by John Robison (1739–1805) who obtained 2 ± 0.06. But credit for the discovery of the inverse-square law properly belongs to Joseph Priestley (1733–1804). In 1766, acting on a suggestion from Benjamin Franklin, Priestley found that there was no electric force on a charge placed anywhere within a hollow, charged conductor. He reported in 1767: "May we not infer from this experiment that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances." (See Problem 1-1.) This brilliant deduction went unappreciated and it was not until Coulomb's experiments that the inverse-square law could be considered as established. Maxwell repeated Cavendish's experiment and reduced the uncertainty to 1 part in 21,600. The techniques used by S. J. Plimpton and W. E. Lawton in 1956 (*Phys. Rev.* **50**, 1066) were considerably more refined and they succeeded in achieving an accuracy of 2 parts in 10^9.