The Hybrid Grand Unified Theory

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Atlantis Studies in Mathematics
for Engineering and Science
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Preface

The physicist is interested in discovering the laws of inanimate nature and the mathematician uses the depth of his thought into exploring the mathematical concepts. But, the symbiotic connection between physics and mathematics and the enormous usefulness of mathematics in the natural sciences is something quite mysterious. No rational justification appears to be satisfactory to understand the uncanny success of mathematics and its role in physical theories. A possible explanation is that the laws of nature are written in the language of mathematics.

A complete mechanism, natural or man-made, is understood fully by taking it apart, studying its parts or components and their properties and learn how these parts fit to make the whole. It is really a natural way to understand any object or concept. The path of progress in knowledge is similarly a combination of both exposition of parts and their synthesis. Natural sciences and other sciences such as biology, psychology, etc. are reminders of these strides, taken in order to comprehend the world around us and there have been attempts to catch a glimpse of the higher reality and understand it. The human mind is limited and cannot go beyond certain limitations of time, space and causation. Still, the universe is experienced as a dynamic inseparable whole, including the observer in an essential way. In this experience, the traditional concepts of time and space of isolated objects and of cause and effect lose their meaning. Such an experience is very similar to that of the ancient scientists (Rishis). However, they repeatedly insisted that the ultimate reality can never be an object of reasoning, deduction or demonstrable knowledge. It can never be adequately described by words and Max Planck of the modern scientific era observed that science cannot solve the ultimate mystery of nature and that is because in the final analysis, we ourselves are part of nature and therefore part of the mystery we are trying to solve.

The scientists in physics are happy with physical concepts that can be visualized or observed directly. But most of the time they are forced to utilize mathematical concepts to describe the physical concepts. Take for example, the case of the black hole. It is not entirely observable but it is a physical concept because there is an overwhelming evidence of its existence. Although they have verified its existence in the core of galaxies including our milky way, it is not known exactly what it is and therefore, they use the mathematical concept of a singularity to describe it. However that is not amenable to computation or studying its structure. Another such important physical concept is that of the basic constituent of matter. It is very important to know what constitutes matter and it cannot be ignored because of our deep conviction in the order of the universe.

This is our attempt to provide a hybrid grand unified theory to understand the universe, both in its micro/quantum aspects as well as macro/galactic aspects. It is truly a hybrid theory as it tries to encompass both the modern and ancient theories of the universe, together with its functioning at all levels of human comprehension. During this attempt it becomes necessary to acknowledge the ambiguity and limitations of mathematics concerning the fundamental concepts of very large and very small numbers, infinity and the limiting process in general. Although the mathematical modeling is the most advanced methodology of physics and it owes all its tremendous achievements to the former, the existence of unsolved problems and unanswered questions suggest the need for improvement. It is a fact that the origin of the concepts such as natural numbers, rational and irrational numbers, zero, infinity and the place value system, among others, originated in Veda Samhitas, ancient scientific texts of India. However, because of the quirks of our narrated human history, our second hand reception of these concepts from the Arabs and the belief that everything originated in Greece, the true original source appears to have been obscured and lost. It is therefore necessary to return to the source to clarify and understand the basics to shed some light on the existing real number system.

In Chapter 1, we revisit the fundamentals of mathematics to bring out sources of ambiguity in certain basic mathematical concepts. We also raise many questions that are challenging the physicists and discuss the relevance of mathematical methodology in physics. Certain related concepts such as cosmic waves are also considered.

Chapter 2 deals with the mathematics that is essential in aiding the description of physical concepts that are discussed in Chapter 3. In particular, the existing real number system is revised with the purpose of bringing out the useful nature of decimal numeration system in the current digital era of high accuracy computation with the aid of technology. The ambiguity involved in dealing with infinity, limiting process and the computation of very large and very small numbers is minimized. The mathematics of generalized curves, generalized fractals and chaos provides the mathematical modeling of physical concepts that arise in this grand unified theory. The integrated Pontrjagin's maximum principle is briefly discussed as it has been instrumental in solving the famous n-body problem. The introduction of dark numbers as part of the refined real number system paves the way to quantize the basic constituent of matter and in general, dark matter.

In chapter 3, the qualitative modeling is employed to study the search for basic constituent of matter and its ramifications in explaining quantum gravity and macro gravity. The ten natural laws of nature are enumerated and the mathematical model of a superstring is introduced to represent the structure of the basic constituent of matter. Verification of this grand unified theory (GUT) is carried out by explaining various natural phenomena such as ultra-energetic indexcosmic waves, supernova and the rare hit on earth by asteroids inspite of being so close to the astreoid belt, etc.

In Chapter 4, the theoretical consideration of n-body problem and turbulence are described. Also, physics of the mind is investigated since in this grand unified theory, mind is also an important part in the study of the universe around us.

Finally, in Chapter 5, the hybrid nature of this book is really brought to the fore as it deals with matters of mind and consciousness, evolution and involution, creation and dissolution, etc. according to the view of ancient scientists (Rishis). It is really like coming full circle to consider how much alike are the theories of ancient and modern scientists, relative to the universe around us.

Some of the important features of the book are as follows:

(i) It puts the real number system on solid foundations without inconsistency and emphasizes the appropriateness of the decimal system as a computation tool;
(ii) It improves the existing real number system by incorporating the notion of dark numbers and their duals, personal and impersonal infinity;
(iii) It introduces the superstring as the basic constituent of matter and the fractal nature of the superstring is modeled by the Cauchy representation of dark number;
(iv) It relates the dark matter of physics and dark numbers of mathematics;
(v) It employs the truly hybrid approach of combining qualitative mathematics and computation to discover the natural laws in order to explain in a unified way several natural phenomena, at micro/quantum and macro/galactic levels;
(vi) It attempts to show the common hybrid interaction between modern and ancient scientific theories of nature and search for higher reality.

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Contents

Chapter 1

Basic Problems of Mathematics and Physics

1.1 Introduction

We know that an attempt to explain all concepts can hardly be called scientific. Some concepts require acceptance without explanation. Whatever, we the scientists say, is expressed in terms of limited and approximate descriptions, which are improved in successive steps. We progress from truth to truth and from lesser truth to higher truth. We view the truth, get and absorb as much of it as the circumstances permit, color it with our feelings, understand it with our intellect and grasp with our minds. This makes the difference among human beings and sometimes generates contradictory ideas. Nonetheless, we all belong to the same universal truth.

In this preliminary chapter, we shall attempt to assemble many of the questions and paradoxes that have been raised and observed in the present real number system, set theory and theoretical physics. These are mainly due to not having a clear understanding of the concepts like infinity, large and small numbers and the set axioms. In physics, the raised questions essentially deal with non-observable dark matter, the Cosmos, the basic constituent of matter, Big Bang, and many other related notions. For example, Section 1.1 deals with the retrospection of fundamentals in analysis where a clear exposition of some important concepts is presented. Section 1.2 is concerned with the problems in physics and several basic unanswered questions are incorporated. The final section 1.3 discusses some related notions such as indexcosmic waves, generalized fractals and vibration that are useful later.

1.2 Retrospection of Fundamentals

As we are aware, much of mathematical work in the 20th century has been devoted to examining the logical foundations and structure of the subject. One of the major influences on 20th century mathematics is Cantor's introduction of infinite sets into the vocabulary of mathematics. The interest in the set theory developed rapidly until virtually every field of mathematics has felt its impact. Under its influence, a considerable unification of traditional mathematics did occur and new mathematics has been created at an explosive rate.

Another interesting development is Alan M. Turing's change of definition of numbers which concentrated on what a machine could produce using programs. Thus a computable number is a number for which there is a program to compute it in some Turing machine, to as many digits as we may specify. Thus Turing changed the concept of a number. For example, the number π is now a program which generates some six billion digits and is no longer the original infinite (non-ending) representation. This takes away our attention from actual infinite back to a finite, though large but unbounded number which is nearer to potential infinite (personal infinity, a concept we introduce in Chapter 2) of ancient times. Most mathematicians are aware of this change in the definition of a number but those involved with computation always make use of it.

The crisis in the foundation of mathematics, that is brought about by the discovery of paradoxes in the wake of Cantor's general theory of sets, has resulted in numerous attempts at the resolution. These have given rise to the three main schools of thought or philosophy. They are logistic, intuitionist and formalist schools, each of which dealt with paradoxes of general set theory in its own way.

Let us list below a few examples to illustrate the dificulty involved in defining certain sets.

(i) (Bertrand Russell) Let M be the set of all sets where each element does not belong to itself, i.e. $M = \{m : m \notin m\}$. Then it must be either $M \in M$ or $M \notin M$. If $M \in M$, then the defining conditions for M holds and $M \notin M$. On the other hand, if $M \notin M$, then M satisfies the defining condition and therefore $M \in M$.
A way out of this dilemma is to agree that M is not really a set and this kind of self reference is a source of ambiguity and should not be used in defining a set.

(ii) The famous Russell antonym: A Cretan (native to Crete) saying "All Cretan's are liars." Is he telling the truth?

(iii) The barber paradox: The barber of Seville shaves those and only those who do not shave themselves. Who shaves the barber?

Another source of ambiguity in mathematics are statements involving the universal or existential quantifier on an infinite set. Such statements are unverifiable. For example, to verify that every element of an infinite set has property A, we check an element to see if it has the property and keep doing the checking process for each element. Obviously, we cannot exhaust all of the elements of the infinite set and thus we cannot verify if the property A holds for all elements of the set.

In mathematics, particularly theory of numbers, there are many statements involving the infinite set of natural numbers which raise questions that remain unanswered i.e. the statements are neither proved nor disproved. Let us list the following:

(i) A perfect number has the sum of its proper factors equalling the number itself. The first few known perfect numbers are 6, 28, 496, 8128, and 33, 550, 336. The question is, are all perfect numbers even?

(ii) Twin primes are prime numbers that differ by 2, like 3 and 5 or 11 and 13. The questions is, are there twin primes that are arbitrarily large? Does there exist an infinite number of twin primes?

(iii) Goldbach's conjecture which says that every even number except 2, is the sum of two primes. For example $4 = 2+2$, $6 = 3+3$, $8 = 5+3$ etc. Question is, is the conjecture true?

Consider the statement with the existential quantifier: The decimal expansion of π has no row of one hundred threes. True or not, is not known although extensive calculation on the decimal expansion of π has not yielded such row of threes. The probability that this statement is true is $\left(1-(\frac{9}{10})^{100}\right)$ which is almost 1. This shows that even if the probability that a statement is true is near 1, it is not a total certainty.

Among the field axioms which deal with the properties of real numbers $\mathbb{R}$, the following two are important for us:

(i) trichotomy axiom which says that for any $a, b \in \mathbb{R}$, exactly one of the following is true, $a = b$, $a < b$ or $a > b$;

(ii) completeness axiom that says every non empty subset S of $\mathbb{R}$ that is bounded above (has an upper bound) has the least upper bound.

We also require the axiom of choice related to sets: Given any nonempty set A, whose members are pairwise disjoint nonempty sets, there exists a set B consisting of exactly one element taken from each set belonging to A. B is called the choice set.

Though many useful results in mathematics depend on the axiom of choice, this axiom has been seriously challenged by some mathematicians since they feel it is meaningless though not false and many others accept it since it seems reasonable. However, Banach and Tarski proved a most disconcerting result using the axiom of choice, which goes against our intuition and so is considered a paradox (the Banach-Tarski Paradox). It essentially says that if a soft ball is suitably sliced into infinitely small little pieces, then the pieces can be suitably rearranged, without distortion, and reconstructed into a ball, the size of the earth. This is a contradiction in $\mathbb{R}^3$ inherited from the reals and attributed to axiom of choice. The specific source of the problem is the Archimedian property of the reals which says given any real $\varepsilon > 0$ no matter how small and any number M, no matter how large, there exists some number N such that $N\varepsilon > M$. This allows us to form a arbitrarily large object from arbitrarily large number of arbitrarily small pieces [59].

A common feature of elements of any well defined set is to have the property of potentially exhibiting certain qualitative or quantitative activities which result in the production of certain static or dynamic structures that are essentially for the continued existence or the survival of the elements and therefore for the set itself. This implies certain order relations between elements. The order need not always be a fixed one. The temporal sequence of activities induces temporal partial ordering. For example, biochemical reactions in a cell occur in a certain order. Remember every part of a cell, every structure in it, is the result of the primary activity of the genes, which control the production and functioning of every structure in the cell. Sometimes only a subset of the set exhibits potentially possible activities and may wait for the need to arise.

Thus, we can always find at least one monotone sequence in any given set and so there exists always an algorithm how to choose an element. Consequently, we replace the axiom of choice by the strong axiom of choice and successfully construct the choice set. If we find a set whose elements are neither known to us nor have any property we can find in order to induce certain ordering, then the weak axiom of choice is employed by some inherit property of the set to find a choice set.

The concept of an irrational number was introduced into the set of fractions (quotients of the form $\frac{x}{y}$, $y \neq 0$, x, y are integers). It is known that fractions (rationals) have terminating decimal expansions or a pattern of repetition of digits. While defining irrational number as one that does not have a fractional representation, we encounter decimal expansions which are nonterminating and have no periodic repetition of digits. This fact that such numbers have an infinite number of digits in their decimal expansion make it an ambiguous, hence non-verifiable, concept. There is no way to verify if the decimal expansion of π has indeed no periodic pattern of digits, since we cannot compute all if its digits.

Another example of encountering difficulty when dealing with ambiguous or vaguely defined concepts is the following traditional construction of an irrational number as the limit of a sequence of rationals. This also provides a modification of Felix Brouwer's counter example to the trichotomy axiom [5].

Let C be a given irrational number (whose decimal representation is known up to only first n digits). We want to isolate C in an interval such that all the decimals to the left of C are less than C and all decimals to the right of C are greater than C. We do this by constructing a sequence of smaller and smaller rational intervals such that each interval in the sequence is contained in the preceding one and such a sequence is called a nested sequence. An interval is rational if their left and right end points are rationals. In the construction, we skip those rationals that do not satisfy the condition of providing the end points of intervals of this nested sequence.

Given two rationals x, y we can tell if $x < y$ or $x > y$. Even then, we cannot line up all the rationals on the real line under the ordering $<$, since there is an infinite number of rationals between any two given rationals and this is due to the undefinable nature of the concept infinity. However, we can proceed with the following scenario: Start with a certain rational interval $[A, B]$ with $A < C < B$, and find a nested sequence of rational intervals $[A_n, B_n]$, with

$$A < A_n < C < B_n < B, \quad \text{for each } \; n = 1, 2, 3, \ldots, n.$$

At each stage, we want to make sure that

$$A_n \leqslant C - 10^{-n}, \quad B_n \geqslant C + 10^{-n}, \quad \text{and} \quad [A_n, B_n] \subset [A_{n-1}, B_{n-1}] \subset [A, B].$$

Since the irrational number C is defined to be the limit of sequences of rationals, we can choose the end points A_n, B_n, of intervals $[A_n, B_n]$ as members of two sequences $\{A_n\}, \{B_n\}$ where $\{A_n\}$ is a monotonic increasing sequence and $\{B_n\}$ is a monotonic decreasing sequence of rationals satisfying

$$A \leqslant A_1 \leqslant A_2 \leqslant \cdots \leqslant A_n < C < B_n \leqslant \cdots \leqslant B_2 \leqslant B_1 \leqslant B,$$

and for each n,

$$C - A_n \geqslant 10^{-n} \; \text{ and } \; B_n - C \geqslant 10^{-n}$$

This process can be continued as long as we are able to identify A_n, B_n to be such that $A_n < C < B_n$ i.e. as far as we know the representation of C with its n decimal digits. It cannot be taken further since we are unable to find A_{n+1}, B_{n+1} with error of $10^{-(n+1)}$ and establish $A_{n+1} < C < B_{n+1}$, with C being known only to n places. No matter how large the number n is, we still have the disadvantage of not getting the next interval $[A_{n+1}, B_{n+1}]$.

Consequently we have to acknowledge the inherent trouble involved with understanding and dealing with irrational numbers and with the concept of infinity. This example shows that the real number system has no ordering under the relation $<$ and the trichotomy axiom which says, given two real numbers x, y, only one of the following holds: $x < y$, $x = y$, $x > y$, is unverifiable.

Next we shall give a proof to show that rational and irrational numbers in the reals are not dense. Let $p \in \mathbb{R}$ be any irrational number and $\{q_n\}$ be a sequence of rationals converging to p from the left in the natural ordering of reals. Let d_n be the distance from q_n to p and take an open ball of radius $d_n/10^n$, with center at q_n. Note that q_n tends to p but distinct from it for any n. Take an open ball of the same radius $d_n/10^n$, centered at p and take the union of open balls, centered at q_n, as $n \to \infty$ and call it U. If r is any real, rational or irrational, to the left of p, then r is separated from p by two disjoint open balls, one in U and the other in its complement, center at p. If p is rational, then we take $\{q_n\}$ as a sequence of irrationals that tend to p, which is allowed by the Axiom of Choice. The same result would hold for any r distinct from and to the right of p.

Mathematics is a universal language form that is well suited to talk about concepts that are abstractions of the human mind as well as concepts that relate to and attempt to describe the physical universe around us, in terms of several laws of nature. For purposes of logical rigor and consistency, we have to start with certain set of symbols, concepts and premises (these may change as available knowledge advances) and proceed to develop mathematics as a deductive system. Its obvious tools are measurement and computation which are limited. In order to represent the external world through mathematics, just measurement and computation alone would not be adequate. It requires abstraction, intuition, imagination, visualization, trial and error in order to sift out what is more appropriate, thought experimentation, creativity, thinking backwards and the art of making inferences and drawing conclusions. These activities are collectively present in qualitative mathematics, which is complementary to measurement and computation.

The skill of thinking backwards involves figuring out what we need as premises or boundary conditions to obtain a desired result. As example, consider the famous inverse (ill-posed) problem known as gravitational n-body problem: Given n bodies in cosmos at time T, of given masses, positions and velocities and subject to their mutual gravitational attraction, find their positions, velocities and paths at later time $t > T$. This problem is ill-posed because the bodies have cosmological history and initial, boundary conditions belong to the past and we do not know what they were.

In spite of the fact that mathematics is known for its universality, the power of its abstractions and growing usefulness in all fields of sciences and arts, we have to admit that there are certain sources of uncertainty or ambiguity in mathematics. These essentially deal with the ideas of infinity and infinitesimal and consequently, with very large and very small numbers. Scientists and practitioners of computation have tried to cope with this problem by approximation and use of scientific notation to represent any order of magnitude, large or small, by using powers of ten. For example, the radius of our visible universe is of order of magnitude 10^{10} light years while the order of magnitude of the basic constituent of matter (a non-agitated superstring) is less than 10^{-14}.

The ambiguity involved in approximation can be minimized by emphasizing the order of magnitude of the error, at every level of approximation. Even then certain concepts like infinity and infinitesimal remain mainly intuitive and ideal. We need to employ some new concepts to denote the pragmatic level at which these can be handled, in computations and deliberations. Also, we have to make sure to avoid vacuous concepts and statements because these invariably lead to contradictions. For example, consider the statement "The largest integer is 1" and its proof: Let N be the largest integer and by ordering axiom, $N < 1$, $N = 1$ or $N > 1$. First option is ruled out because of the definition of "largest": Take $N > 1$ which gives $N^2 > N$, contradicting the assumption that N is largest. Therefore N can only be equal to 1. In this formulation of Perron paradox, the culprit is the vacuous concept "largest integer".

Having pointed out certain challenges and ambiguities that are present in mathematics, we shall revisit the real number systems in Chapter 2, with a purpose to deal with some of the questions raised here and discuss the development of the decimal system of numeration which is more suitable for the present era of high accuracy computation with the aid of technology.

1.3 The Problems of Physics

Report from the Hubble says: matter forms in the supposedly empty space between cosmological bodies at the staggering rate of one star per minute capped by the recent discovery of two baby galaxies and indication of more in the last four years [4], [108], [105]. First cosmic dust forms then it gets entangled into cosmological vortices and collects at their cores (core: collected mass around the eye) and become cosmological bodies like galaxy, planet and moon. There are places in the cosmos called star nurseries that produce stars at quite a rapid rate [105]. While these findings resolve the puzzle of the missing 95% of matter in the Cosmos, that, after all, it is there but for cannot be detected, physics is faced with the unprecedented challenge of how to study matter whose existence has no direct evidence whatsoever. And yet it exists in view of the first law of thermodynamics that says energy, therefore, also matter, cannot be created or destroyed. Since then that missing matter has been called dark matter because light, our medium for observation, cannot detect it and the question is how to deal with the nonobservable like dark or invisible matter.

At the same time, there are long-standing unsolved problems of physics such as the gravitational n-body and the turbulence problems as well as some old fundamental questions that remain unanswered to this day. Physics has abandoned some of them, for example, the gravitational n-body problem and the structure of the electron, but is in hot pursuit of others such as the 5,000-year-old quest for the basic constituent of matter and the turbulence problem. The pursuit of the former has absorbed staggering amount of resources during the atom-smashing frenzy of the last half century and for good reason. Unless we know that basic constituent we really do not know what matter is and physics has correctly assessed that this question is the key to the resolution of its fundamental questions and longstanding unsolved problems. Let us list down the others.

(1) What is gravity?

(2) What is a black hole?

(3) What is the so-called elementary particle and what is its structure?

(4) What is superconductivity?

(5) What are cosmic waves and what are their source and medium?

(6) What is charge?

(7) Explain magnetic levitation (the basis of the development of the magnetic train).

(8) What was the Big Bang?

(9) How do galaxies and other cosmological bodies form?

(10) Why do they spin?

(11) What is the destiny of our universe?

The questions spill over to the applications of physics. In biology we have these questions:

(1) What distinguishes living from non-living organism?

(2) How does the brain work?

(3) What is cosmic energy and what is the nature of its interaction with the brain?

(4) How does a mutant spread in the body?

In psychology, there is a need for physics-based theory of intelligence to explain:

(1) Intelligence,

(2) Learning and creativity, and

(3) Cognition, i.e., the human ability to know the real world and express that knowledge as physical theory.

Then there are astonishing and paranormal phenomena some of which now have physical explanation. Paranormal refers to natural phenomena having no physical explanation yet. They include:

(1) Kerlian photography,

(2) Human aura,

(3) Human levitation, and

(4) Telekinesis.