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ROBUST MECHANISM DESIGN

The Role of Private Information and Higher Order Beliefs

Dirk Bergemann

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Foreword by Eric S. Maskin

Series Editor-in-Chief

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Foreword

Good social, political, and economic decisions are normally contingent on the preferences of individuals in society over the possible options. Indeed, the very concept of economic efficiency (Pareto optimality) demands preference contingency: an option is efficient provided there is no other option that everyone prefers. Yet, unfortunately for public decision-makers, individuals' preferences are not typically publicly known, a limitation that can seriously interfere with making the right choice.

A great accomplishment of mechanism design theory has been to show that, in many cases, this limitation can be circumvented. That is, it is often possible in principle to devise a mechanism or game whose outcome in equilibrium is the same as the one that would have been chosen had preferences been public in the first place.

Even so, a good many of the mechanisms exhibited in the literature have been justifiably criticized for depending too sensitively on details that the mechanism designer or the individuals themselves are not likely to know, at least not very precisely. For example, although the literature on Bayesian mechanism design arose to handle the case where individuals do not know one another's preferences, much of it requires that they and the mechanism designer have common knowledge of the prior probability distribution from which these preferences are drawn. This implies, in particular, that if individual A somehow learns B's preferences, then he will also know what B beliefs about A's preferences are — an implication that, in practice, is often implausible.

Dirk Bergemann and Stephen Morris have made a major contribution to mechanism design by developing the idea that, when constructing a mechanism, we should carefully model an individual's type *space*: not only his possible preferences, but also his possible *beliefs* (about others' preferences, about others' beliefs about *his* preferences, etc.). Furthermore, we should ordinarily take this type space to be *larger* (allowing for a greater variety of beliefs) than is conventionally assumed. By doing so, we make the mechanism more *robust* than those of the standard literature.

I am delighted that this volume in the Economic Theory series gathers twelve important papers that Bergemann and Morris have written with each other and with other authors on robust mechanism design. The volume also provides a detailed and beautifully lucid introduction to their work through the particular example of how to allocate a single indivisible good.

Eric Maskin
Editor-in-Chief
World Scientific Series in Economic Theory

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We would like to thank Eric Maskin, for inviting us to present the material in this series edited by him, for valuable conversations about the material and lastly for the role that his work on efficient auctions, Maskin (1992) and Dasgupta and Maskin (2000), played in the development of the present work. We would also like to thank co-authors Hanming Fang, Takahashi Kunimoto, Moritz Meyer-ter-Vehn, Karl Schlag and Olivier Tercieux for agreeing to have their joint work with us included in this volume and Andreas Blume, Tilman Börgers, Jacques Cremer, Moritz Meyer-ter-Vehn, Andy Postlewaite, Phil Reny and Olivier Tercieux for comments on this introduction. We had the opportunity to present the material in this introduction at a number of invited lectures, notably at Boston University, Northwestern University and the European and North American Econometric Society Meetings and a set of slides which cover and accompany this introduction can be found at http://dirkbergemann.commons.yale.edu/files/2010/12/robustmechanism design1.pdf.

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Robust Mechanism Design: An Introduction

Dirk Bergemann and Stephen Morris

1 Introduction

This volume brings together a number of contributions on the theme of robust mechanism design and robust implementation that we have been working on in the past decade. This work examines the implications of relaxing the strong informational assumptions that drive much of the mechanism design literature. It collects joint work of the two of us with each other and with coauthors Hanming Fang, Moritz Meyer-ter-Vehn, Karl Schlag and Olivier Tercieux. We view our work with these co-authors as thematically closely linked to the work of the two of us included in this volume.

The objective of this introductory essay is to provide the reader with an overview of the research agenda pursued in the collected papers. The introduction selectively presents the main results of the papers, and attempts to illustrate many of them in terms of a common and canonical example, the single unit auction with interdependent values. It is our hope that the use of this example facilitates the presentation of the results and that it brings the main insights within the context of an important economic mechanism, the generalized second price auction. In addition, we include an extended discussion about the role of alternative assumptions about type spaces in our work and the literature, in order to explain the common logic of the informational robustness approach that unifies the work in this volume.

The mechanism design literature of the last thirty years has been a huge success on a number of different levels. There is a beautiful theoretical literature that has shown how a wide range of institutional design questions can be formally posed as mechanism design problems with a common structure. Elegant characterizations of optimal mechanisms have been obtained. Market design has become more important in many economic arenas, both because of new insights from theory and developments in information and computing technologies, which enable the implementation of large scale trading mechanisms. A very successful econometric literature has tested auction theory in practise.

However, there has been an unfortunate disconnect between the general theory and the applications/empirical work: mechanisms that work in theory or are optimal in some class of mechanisms often turn out to be too complicated to be used in practise. Practitioners have then often been led to argue in favor of using simpler but apparently sub-optimal mechanisms. It has been argued that the optimal mechanisms are not "robust" — i.e., they are too sensitive to fine details of the specified environment that will not be available to the designer in practise. These concerns were present at the creation of the theory and continue to be widespread today.[1] In response to the concerns, researchers have developed many attractive and influential results by imposing (in a somewhat ad hoc way) stronger solution concepts and/or simpler mechanisms motivated by robustness considerations. Our starting point is the influential concern of Wilson (1987) regarding the robustness of the game theoretic analysis to the common knowledge assumptions:

> "Game theory has a great advantage in explicitly analyzing the consequences of trading rules that presumably are really common knowledge; it is deficient to the extent it assumes other features to be common knowledge, such as one agent's probability assessment about another's preferences or information."
>
> "I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality."

[1]Hurwicz (1972) discussed the need for "non-parametric" mechanisms wich are independent of the distributional assumptions regarding the willingness-to-pay of the agents. Wilson (1985) states that trading rules should be "belief-free" by requiring that they "should not rely on features of the agents' common knowledge, such as their probability assessments." Dasgupta and Maskin (2000) seek "detail-free" auction rules "that are independent of the details — such as functional forms or distribution of signals — of any particular application and that work well in a broad range of circumstances."

Wilson emphasized that as analysts we are tempted to assume that too much information is common knowledge among the agents, and suggested that more robust conclusions would arise if researchers were able to relax those common knowledge assumptions. Harsanyi (1967–1968) had the original insight that relaxing common knowledge assumptions is equivalent to working with a type space which is larger if there is less common knowledge. A natural theoretical question then is to ask whether it is possible to explicitly model the robustness considerations in such a way that stronger solution concepts and/or simpler mechanisms emerge endogenously. In other words, if the optimal solution to the planner's problem is too complicated or too sensitive to be used in practice, it is presumably because the original description of the planner's problem was itself flawed. We would like to investigate if improved modelling of the planner's problem endogenously generates the "robust" features of mechanisms that researchers have been tempted to assume. Our research agenda in robust mechanism design is therefore to *first* make explicit the implicit common knowledge assumptions and then *second* to weaken them.

Thus, formally, our approach suggests asking what happens to the conventional insights in the theory of mechanism design when confronted with larger and richer type spaces with weaker requirements regarding the common knowledge between the designer and the agents. In this respect, a very important contribution is due to Neeman (2004) who showed that the small type space assumption is of special importance for the full surplus extraction results, as in Myerson (1981) and Cremer and McLean (1988). In particular, he showed that the full surplus extraction results fail to hold if agents' private information doesn't display a one-to-one relationship between each agent's beliefs about the other agents and his preferences (valuation). The extended dimensionality relative to the standard model essentially allows for a richer set of higher order beliefs.

Similarly, the analysis of the first price auction in Chapter 8 (by Hanming Fang and Morris) looks at the role of richer type spaces by allowing private values but multidimensional types. There, each bidder observes his own private valuation and a noisy signal of his opponent's private valuation. This model of private information stands in stark contrast to the standard analysis of auctions with private values, where each agent's belief about his competitor is simply assumed to coincide with the common prior. In the presence of the multidimensional private signal, it is established in Chapter 8 that the celebrated revenue equivalence result between the first and the second price auction fails to hold. With the richer type space, it is not even possible to rank the auction format with respect to their expected revenue.

2 Leading Example: Allocating a Private Good with Interdependent Values

It is the objective of this introduction to present the main themes and results of our research on robust mechanism design through a prominent example, namely the efficient allocation of a single object among a group of agents. We are considering the following classic single good allocation problem with interdependent values. There are I agents. Each agent i has a "payoff type" $\theta_i \in \Theta_i = [0,1]$. Write $\Theta = \Theta_1 \times \cdots \times \Theta_I$. Each agent i has a quasi-linear utility function and attaches monetary value $v_i : \Theta \to \mathbb{R}$ to getting the object, where the valuation function v_i has the following linear form:

$$v_i(\theta) = \theta_i + \gamma \sum_{j \neq i} \theta_j.$$

The parameter γ is a measure of the interdependence in the valuations. If $\gamma = 0$, then we have the classic private values case. If $\gamma > 0$, we have positive interdependence in values, if $\gamma < 0$, we have negative interdependence. If $\gamma = 1$, then we have a model of common values.

In this setting, a social choice function must specify the allocation of the object and the (expected) payments that agents make as a function of the payoff type profile. Thus a social choice function f can be written as $f(\theta) = (q(\theta), y(\theta))$ where the allocation rule determines the probability $q_i(\theta)$ that agent i gets the object if the type profile is θ, with $q(\theta) = (q_1(\theta), \ldots, q_I(\theta))$; and transfer function, $y(\theta) = (y_1(\theta), \ldots, y_I(\theta))$, where $y_i(\theta)$ determines the payment that agent i makes to the planner.

If $\gamma < 1$, then the socially efficient allocation is to give the object to an agent with the highest payoff type θ_i. Thus *an* efficient allocation rule is given by:

$$q_i^*(\theta) = \begin{cases} \dfrac{1}{\#\{j : \theta_j \geq \theta_k \text{ for all } k\}}, & \text{if } \theta_i \geq \theta_k \text{ for all } k; \\ 0, & \text{otherwise;} \end{cases}$$