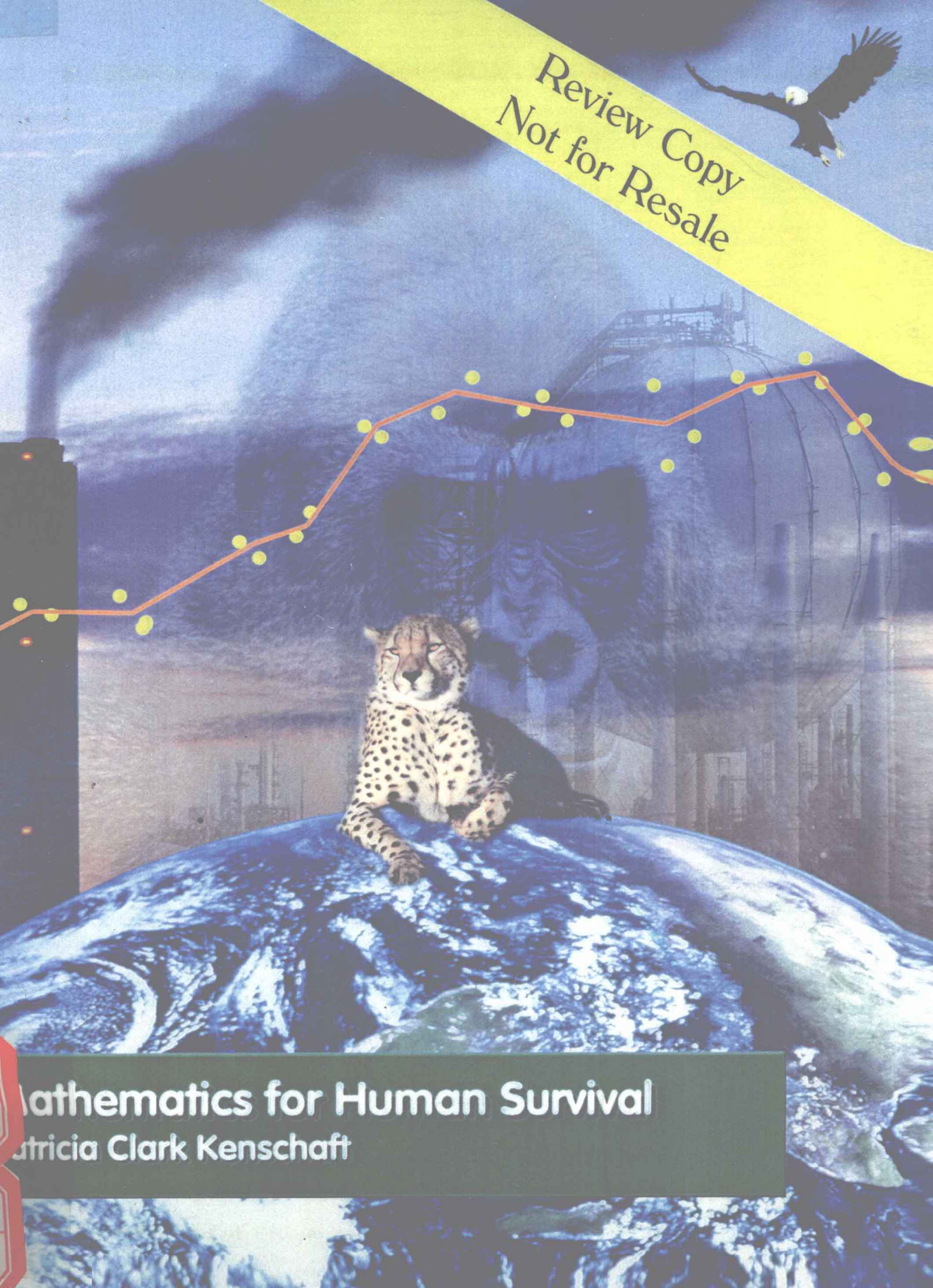

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World Scientists' Warning to Humanity

Human beings and the natural world are on a collision course. Human activities inflict harsh and often irreversible damage on the environment and on critical resources. If not checked, many of our current practices put at serious risk the future that we wish for human society and the plant and animal kingdoms, and may so alter the living world that it will be unable to sustain life in the manner that we know. Fundamental changes are urgent if we are to avoid the collision our present course will bring about.

Above is the "Introduction" to the "Warning to Humanity" signed in 1992 by over 1700 scientists, including 104 Nobel laureates, a majority of the living recipients of the award. The scientists represented 71 countries including all of the 19 largest economic powers and all of the 12 most populous nations.

The body of the "Warning" says that the atmosphere, water resources, oceans, soil, forests, and living species of this planet are "suffering critical stress." Again quoting: "We..., senior members of the world's scientific community, hereby warn all humanity of what lies ahead. A great change in our stewardship of the earth and the life on it is required, if vast human misery is to be avoided and our global home on this planet is not to be irretrievably mutilated. **What we must do:** Five inextricably linked areas must be addressed simultaneously:" (Below appear only topic sentences.)

- Bring environmentally damaging activities under control to restore and protect the integrity of the earth's systems we depend on.
- Manage resources crucial to human welfare more effectively.
- Stabilize population.
- Reduce and eventually eliminate poverty.
- Ensure sexual equality and guarantee women control over their own reproductive decisions.

Union of Concerned Scientists, Two Brattle Square, Cambridge, MA 02338-9105 www.ucsusa.org, 617-547-5552

Preface

Ask what kind of world do you want to live in? What are you good at and what do you want to work at to build that world? What do you need to know? Demand that your teachers teach you that.

"A Letter to the Young,"
Peter Kropotkin,
a nineteenth century scientist,
visionary, and activist

Surveys increasingly show that young people want a world in which they and others can live healthily and grow old. They want a world of peace, where all types of people live together in harmony, or at least in tolerance. They want a world where no person is hungry, and they know that in some sense we have enough food. They want a world in which the food we eat actually builds health, and a world without poisons in our food, water, or air. They want a world that continues to provide the abundance that has blessed humanity for its first million years, and a stable climate that does not send increasing storms, floods, and droughts. This book is written for such young people, because its author wants these things too—for us and for many generations to come.

"What can I do to help?" That is a natural question. Answering it requires thinking about magnitudes and probabilities, data and growth. A recent study of the Union of Concerned Scientists resulted in a list of the most important steps for all consumers. The most crucial is that we drive the least polluting, lowest gas mileage vehicle that can serve our needs—and drive only when we must. The second is to eat as low as possible on the food chain, and the third is to eat food grown organically, without adding poisons or chemicals to our environment (and our bodies). The analysis for this study used complicated applications of basic mathematics, most of which can be understood by anyone who knows sixth grade math—as long as they aren't scared off by numbers [*Consumer's Guide to Effective Environmental Choices*, Brower and Leon, 1999].

College students are more than consumers. They are preparing for a professional career, one in which they can provide for themselves while serving others. Almost every modern professional field uses mathematics, on some level. We all need to be able to read mathematics in context, sort out what the numbers mean, and draw conclusions. However, even relatively easy mathematics can seem forbidding if one is not accustomed to thinking mathematically.

This text prepares students to read mathematics and to become comfortable applying mathematics to both their own profession and the wider range of decisions they need to make as citizens. Except for a few examples designed to reveal the mathematics in stark simplicity, all the examples and exercises in this book involve real-world numbers related to human survival. Students learn to *use* mathematics, and they will see its connection to many vital issues. It is crucial that all American leaders, including all college graduates, understand concepts of magnitude, growth patterns, and basic probability and statistics.

When the author was young, many people worried that by the year 2000 humanity would have committed suicide by nuclear war. Back then, we thought that even a brief war might take all large mammals to extinction, including humans. But here we are! I am enjoying myself in later middle age far more than expected. I know that joyful survival is possible despite dismal prospects, and I yearn for today's young people to have similar pleasures some day.

With luck, the mathematical concepts and exercises in this book will help *you* better understand the current threats to global sustainability and develop the skills to help you address these threats. Maybe it will help readers reconsider financial and lifestyle choices, as well as public policies. Just possibly, this book will play some role in helping humanity survive on this globe. And, yes, I hope that many more people will learn to enjoy and use mathematics as they integrate it with wider, urgent concerns.

Details of Using the Book

To make the reading easier, the labels of examples, rules, and classroom activities are numbered sequentially. To find the item you are searching for, you do not need to reflect upon the category of each of the other labels. Labels are merely for convenience; I believe they need not indicate any profound mathematical pattern. Similarly, tables and graphs are labeled to reflect the example or exercise they illustrate, regardless of how many there are in that section.

As is customary, starred sections and exercises are those the author believes to be easily omitted, and may be harder than the rest. What is hard depends, of course, on the background of the students. What can be omitted depends on the passion of the instructor; the stars are merely suggestions, and should be easily ignored.

The book can be read in many orders. I hesitate to form a diagram, since the ordering depends both on the background of the students and the preference of the instructor. If the purpose of this course is becoming comfortable with reading and writing mathematics, and with integrating mathematics into one's daily thinking ("quantitative literacy"), then the instructor's conviction of what comes first may be more important than the students' background, especially in the ordering of the first three chapters. Furthermore, Chapter 9 could certainly be a fine introductory chapter; it is mathematically independent of the rest, and can be covered at any time, including the beginning.

Chapter 8 could also be taught at virtually any time; it has been put toward the end because the author believes it may be more meaningful after some of the other concepts have been learned. This is debatable. However, the author believes strongly that Chapter 8 is the most important chapter in the book; it may be skimmed, but should not be omitted.

Some students may find Chapter 1 entirely review, but the real-world numbers will provide enough new information to fend off boredom. Most Americans, even mathematics professors, have not thought deeply about the crucial mathematical concepts involved in the big numbers we see in our media daily. However, they are becoming ever more urgent as our world rapidly shrinks.

Chapter 2 should precede both Chapters 6 and 7 (unless the students already know the content of the first two sections), but Chapter 6 is not a prerequisite for Chapter 7. Chapter 4 is a prerequisite for Chapter 5, but neither is required for subsequent chapters.

Introduction for Students

There are many ways of learning mathematics, and at least two kinds of mathematical knowledge: procedural and conceptual. Procedural knowledge involves memorizing techniques and rules and spouting them back for tests. This is the kind of mathematical learning most common in the United States at this time, and it has its place.

People who love math, however, *play* with concepts. They keep finding new ways of looking at old ideas; they perceive mathematics as an exploration of patterns and the use of those patterns to solve problems. Bringing students closer to this way of knowing mathematics is important for long-term success in using mathematics in new situations. By the end of this course I hope you will find mathematics enjoyable and useful, or at least approachable and sometimes useful.

The numbers in this book's "word problems," with very few exceptions, are not made up, but are real-world numbers that arose from investigations of current critical issues. Many people have collected data (numbers describing issues) that reveal truths not easily expressed otherwise. The numbers, graphs, and patterns in this book describe important issues. They will stimulate you to think both about mathematics and about the important topics to which math has been applied.

To absorb the mathematical concepts, you will probably need to read the book in greater chunks than the daily lessons. This does not mean you will read every word at every reading. A light reading of each chapter as you approach it will help you

grapple with a more detailed reading later, probably in smaller units. A rereading, again not of every word, is invaluable when preparing for a quiz, test, or creative paper.

"Math is not a spectator sport," is an old adage of math teachers. You can't just *read* this book to get the most out of it. How many exercises you actually "solve" is negotiable, but reading is *not* enough. Convince yourself you can do each exercise. It is the ones that are difficult for you that are worth your time!

To absorb mathematics, you must struggle. That is part of the process. If you are so anxious that you freeze, that is not productive or fun. But the process of learning a new song, improving your basketball skills, or doing math problems just does take practice. Some practice is fun, but some is just practice. There is no substitute for "being there." Showing up is half the road to success, as one wit observed. "Showing up" in mathematics means doing exercises.

It often also means taking part in a study group. Humans were not meant to work alone. We survived because we lived in groups, and all people learn math best with a group. First try the reading and exercises yourself, and then go over them with two or three classmates. If they know the answer and you don't, you may feel more comfortable listening to them than asking in class. If you know the answer and they don't, it is great for your mathematical confidence to explain what you know. Also, you are far more likely to retain the knowledge if you have explained it aloud. (Don't take it personally that your mind is like a sieve. Everyone's is. Easy in, easy out.) If none of your friends know the answer, you will not feel embarrassed raising the question in class, because you *know* it is not a "stupid question." (I doubt that any question is a stupid question; many, many questions must be raised repeatedly if humans are to survive happily on this globe. Alas, some people are seriously thwarted by fears that asking questions might reveal stupidity.)

Doing your own projects and writing about what you learned can also be an effective way to integrate math into

your psyche and retain it. Returning to past knowledge is also valuable, so some exercises that use past sections' math keep you on your toes. The real world does not introduce its math problems with section headings. If this course is to affect your life, you must learn to spot old ideas in new contexts.

A comment about the use of "=" may be appropriate. In pure mathematical contexts, "=" means that the two quantities on opposite sides of the sign are exactly the same; in some sense they are two names for the same thing. This is what is meant by "6067 million = 6.067 billion." However, 1184/6067 = 0.195, assumes you know that "rounding" has taken place. More is said about rounding at the end of Section 1.1, but throughout the book the "=" sign may mean "approximately equal" without apology.

You are expected to use a calculator routinely. Maybe you will also use a computer. In any case, it is crucial to develop habits of skepticism, learning that while you *can* learn, all specific information can be questioned, and cheerful skepticism is generally appropriate.

Your comments about how to make this book *more* useful, interesting, and accurate are welcome. Despite the best efforts for accuracy, it is a rare text that has no errors. Please help this one with your suggestions.

Best wishes in your new ventures! I hope this text tickles your imagination and makes you ask new questions. Education is a lifelong activity. You will not learn all the math you will need for a lifetime from one book. But you may indeed learn enough to help you plan your personal finance more effectively, to reconsider some of your lifestyle decisions, and contemplate more thoughtfully your part in meeting the world's challenges that will determine future quality of future human life.

Good luck! Have fun! Happy learning!

Acknowledgments

All significant human endeavors involve the contributions of many people. This book is no exception. Special thanks are due to Richard Schwartz whose book, *Mathematics and Global Survival,* inspired this one, and who generously gave many, many ideas and corrections in its early stages and who continues to give support and suggestions.

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Table of Contents

** Easily Omitted (There is more than enough material for one semester, and many units can be taught independently of earlier chapters. Possible order of the chapters are described in the introductory piece, "Details of Using the Book.)*

Chapter 1: Understanding Big Numbers

Section 1.1 Definitions, Units and Averages

We Americans have just now been treated to the finest national election that $3,000,000,000 can buy.

Yes—you read that number right. That's three billion dollars.

How much is that?

It's how much you'd have if you went on the "Millionaire" show seven days a week for the next eight years and won the jackpot every time.

Nick Nyhart,
Executive Director of
Public Campaign
in a mailing of early 2001

Big numbers are so common that we all tend to confuse them. How many people really distinguish between billions and millions? Alas, our eyes tend to gloss over at those large numbers that are on the front page of almost every newspaper—not just the eyes of those with math anxiety, but those with plenty of math love as well. Yet these numbers are key to understanding issues related to the world's future—and sometimes our own. This chapter is dedicated to helping us *all* understand large numbers better so that we can apply them to the major problems that we face as a society and as individuals.

It is crucial that residents of the indisputably most powerful country on earth *try* to grapple with numbers that everyone concedes are large. For example, public awareness of the following large numbers may be crucial for human survival:

- The world population surpassed six billion in October 1999.
- Weather-related disasters such as storms and floods caused $93 billion of damage in 1998, compared to less than $3 billion in 1980 [VS, 77].
- Medical expenditures in the United States were about $1 trillion in 1997.
- The United States debt was $5.5262 trillion in 1998 [*World Almanac 2000*, 110].

How do we understand such off-putting, large numbers? First, we must know their formal names:

A thousand is ten hundreds (i.e. a thousand ones):	$1{,}000 = 10^3$
A million is a thousand thousands:	$1{,}000{,}000 = 10^6$
A billion is a thousand millions:	$1{,}000{,}000{,}000 = 10^9$
A trillion is a thousand billions:	$1{,}000{,}000{,}000{,}000 = 10^{12}$
A quadrillion is a thousand trillions:	$1{,}000{,}000{,}000{,}000{,}000 = 10^{15}$
A quintillion...	$1{,}000{,}000{,}000{,}000{,}000{,}000 = 10^{18}$
A sextillion...	$1{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 10^{21}$

And so forth! Obviously, we need to be able to go back and forth among these numbers to begin to feel comfortable with them.

The numbers on the right, 10^n, are written in what is called **scientific notation**. Some people find this convenient in handling numbers of the magnitude in this book. They prefer to say "10^6" instead of "one million." Others prefer to work directly with the numbers and their relationship to each other. Either approach can be satisfactory for the purposes of this course. Your calculator, however, uses scientific notation, and there may be times that you want to read its messages.

The skill of going back and forth among the words above is indispensable for enjoying this book. Sometimes quantities are expressed in one form, and it is easier to think about them in another. For example, the populations on the "World Population Data Summary" (WPDS) at the end of the book is expressed as millions, but you may want to read them as billions or thousands. Although the world population shows there as 6,067 million, it is often easier to use the fact that 1000 millions = 1 billion and say that there are 6.067 billion people in the world, or (rounding) simply 6 billion. Similarly, the WPDS expresses the population of the smallest country in Northern Africa as 0.3 million. Using the fact that 1000 thousands = 1 million, this might be easier to say, "The population of Western Sahara is about three hundred thousand (or 300,000)." It's important to be able to comfortably convert millions to billions or thousands.