MECHANICS OF STRUCTURES

[A TEXT-BOOK FOR UNIVERSITY STUDENTS]

By

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[WITH 151 DIAGRAMS AND NUMEROUS EXERCISES]

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To
Sanjaya,
Tarini,
Shefalie
and
Indira

PREFACE

This book covers topics which could not be included in the earlier volumes without making them too bulky. Rankine's theory on pressure of frictional soils has been discussed in the chapter on "Masonry Dams and Retaining Walls" in "Mechanics of Structures" — Vol. II. It was necessary to deal with the case of soils of the cohesive and composite types. Accordingly, a chapter on Earth Pressure has been added in this book, the current notation used in Soil Mechanics being adopted for the purpose. The Reciprocal Theorem is a valuable tool in structural analysis. Most text-books, however, give a sketchy proof of this important theorem. A comprehensive and satisfying treatment with the help of "influence coefficients" is given by Sir R. V. Southwell and this has been closely followed in dealing with the subject.

I cannot adequately thank Mr. H. V. Adavi, M.E., of the College of Engineering, Poona, for undertaking the dreary task of checking all numerical work, which he has cheerfully done with great keenness and enthusiasm. I am equally indebted to Reverend Brother Hernandez, S. J. and his staff of the Anand Press, for the excellent work done by them in bringing out this volume.

Poona
March, 1962

S. B. JUNNARKAR

SECOND EDITION

In this edition, use has been made both of the British and Metric units of measurements. As far as possible, errors and misprints have been corrected.

Poona
October, 1965

S. B. J.

MECHANICS OF STRUCTURES

Vol. III

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CHAPTER I

MOHR CIRCLE DIAGRAM

1. Principal stresses and principal planes: Given the principal stresses and principal planes at a point in a system of two dimensional stresses, in art. 7 of Chapter II in Vol. I, we made use of the Mohr circle to obtain the normal and tangential stress-intensities across a given plane through the point. If the principal stresses are p_1 and p_2, the normal and tangential stresses across a plane inclined at θ to the plane of p_1 are:

$$p_n = p_1\cos^2\theta + p_2\sin^2\theta = \frac{p_1 + p_2}{2} + \left(\frac{p_1 - p_2}{2}\right)\cos 2\theta$$

$$p_t = (p_1 - p_2)\sin\theta\cos\theta = \left(\frac{p_1 - p_2}{2}\right)\sin 2\theta.$$

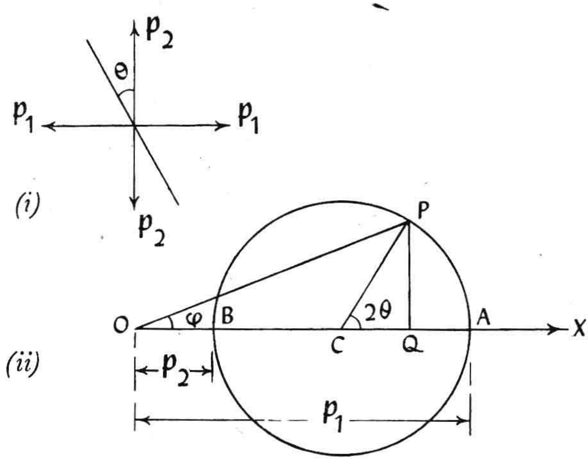

FIG. 1

Using the cartesian system of co-ordinates, along the axis OX, set off OA and OB to scale to represent p_1 and p_2 respectively. Bisect BA at C and with C as centre and BA as diameter, describe a circle. Set off CP at $\angle 2\theta$ with OX to meet the circle at P. Join OP which represents the resultant intensity of stress p on a plane inclined at $\angle\theta$ to the plane of the principal stress p_1.

From P draw PQ perpendicular to OX.

In fig. 1(ii),

$$CA = CB = \frac{p_1 - p_2}{2} \text{ and } OC = \frac{p_1 + p_2}{2}.$$

Then,
$$OQ = OC + CQ = OC + CP \cos 2\theta$$
$$= \frac{p_1 + p_2}{2} + \left(\frac{p_1 - p_2}{2}\right)\cos 2\theta$$

and
$$QP = CP \sin 2\theta = \frac{p_1 - p_2}{2}\sin 2\theta.$$

OQ and QP respectively represent p_n and p_t across a plane whose inclination to the plane of p_1 is θ. OP, therefore, represents the resultant intensity of stress p on the plane, its inclination to the normal being ϕ.

In all graphical work, it is essential that a suitable convention about signs be adopted and scrupulously followed. We have adopted the usual cartesian convention. Thus, in fig. 1, the principal stresses p_1 and p_2 are *like* and have been plotted on the same side. It is customary to treat *tensile* stresses as *positive; compressive* stresses will, therefore, be treated as negative and will be plotted on the other side. A radius vector, rotating *anti-clockwise* about the origin, will be treated as describing a *positive* angle with reference to OX.

Just as we utilised the Mohr circle to obtain the normal and tangential stress-intensities across a given plane, when the principal stresses and the principal planes are given, we may utilise the construction for the inverse problem, viz., to determine the principal stresses and the principal planes at a point in a strained material, when we are given the normal and tangential stresses across two mutually perpendicular planes through the point. Before doing so, however, we must adopt a convention for representing a shear stress. A *clockwise* shear stress across a plane will be treated as *positive;* an *anti-clockwise* shear stress will, therefore, be treated as *negative.* We may summarise the conventional signs as under:

Adopting the cartesian convention, the normal stress across a plane will be plotted as an abscissa along the horizontal axis. The tangential stress across the plane will be plotted as an ordinate along the vertical axis.

A *tensile* normal stress will be treated as *positive.* A *compressive* normal stress will, therefore, be treated as *negative.*

A *clockwise* tangential stress will be treated as *positive.* An *anti-clockwise* tangential stress will, therefore, be treated as *negative.*

Consider the case of two mutually perpendicular planes through a point in a strained material, across which the normal stresses are p and p' respectively accompanied by a shear stress of intensity q. In art. 8 of Chapter II in Vol. I, we have discussed this case. The principal stresses are:

$$p_1 = \frac{p+p'}{2} + \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2}$$

$$p_2 = \frac{p+p'}{2} - \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2}.$$

The position of the principal planes is given by,

$\tan 2\theta = \dfrac{2q}{p-p'}$ where θ is the inclination of the major principal plane to the plane carrying the normal stress p.

We shall use the Mohr circle diagram to obtain these results.

Fig. 2(i) shows a rectangular block $A_1B_1C_1D_1$ in which the vertical faces A_1D_1 and B_1C_1 carry a normal tensile stress of p accompanied by a shear stress of intensity q. The horizontal faces A_1B_1 and D_1C_1 have a normal tensile stress of p' accompanied by the complimentary shear stress intensity of q. In the graphical construction, both the normal and tangential intensities of stress across the plane B_1C_1 or A_1D_1 are positive, p being tensile and q clockwise. For the plane A_1B_1 or D_1C_1, however, the normal stress p', being tensile, is positive, but the tangential stress q is negative because it is anti-clockwise.

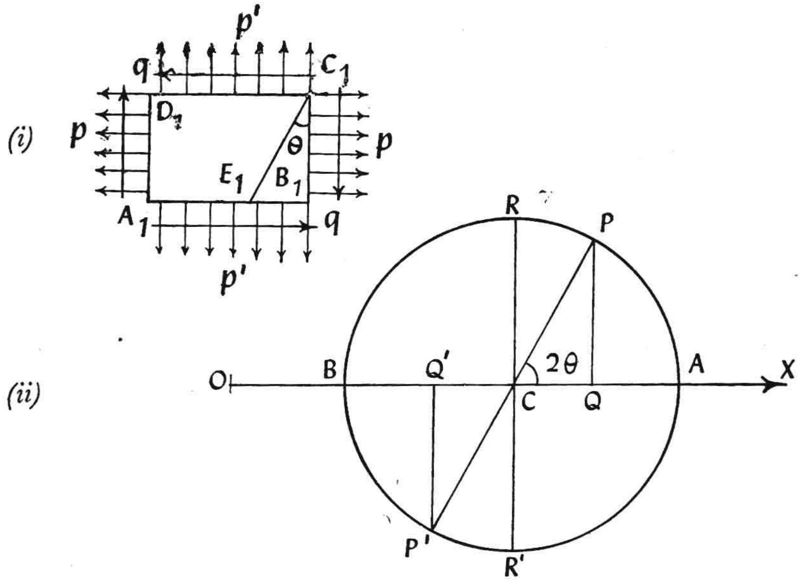

FIG. 2

We may now proceed with the construction as follows:

Select the origin O and set off the base-line OX. Along the axis OX, set off $OQ = p$. At Q, erect the ordinate $QP = q$, above the base-line. Likewise, set off $OQ' = p'$ along OX and erect the ordinate $Q'P' = q$, below the base-line. Join PP' cutting the axis OX at C. C is the centre of the Mohr circle. With centre C and radius $= CP$ or CP', describe a circle cutting the axis OX at A and B. Then OA represents the major principal stress p_1 and OB represents the other principal stress p_2.

Proof:

By construction, $QQ' = OQ - OQ' = p - p'$

$$\therefore \quad CQ = \frac{p - p'}{2} \text{ and } OC = \frac{p + p'}{2}.$$

Also, $QP = Q'P' = q$

$\therefore$ the radius $CP = \sqrt{CQ^2 + QP^2} = \sqrt{\left(\frac{p - p'}{2}\right)^2 + q^2}$

$$OA = OC + CA = \frac{p + p'}{2} + \sqrt{\left(\frac{p - p'}{2}\right)^2 + q^2} = p_1$$

and $OB = OC - CB = \frac{p + p'}{2} - \sqrt{\left(\frac{p - p'}{2}\right)^2 + q^2} = p_2.$

As regards the positions of the principal planes, let $\angle QCP$ in fig. 2(ii) be 2θ. Then,

$$\tan 2\theta = \frac{QP}{CQ} = \frac{q}{(p - p')/2} = \frac{2q}{p - p'}.$$

Solve for θ, which is the angle which the plane B_1C_1 makes with the plane of the principal stress p_1, *anti-clockwise*, if θ is positive.

Therefore, to set off the principal plane of p_1 with reference to the plane B_1C_1, draw C_1E_1 at $\angle\theta$ with B_1C_1, measured in the *clockwise* direction. C_1E_1 will, then, be the major principal plane, the other principal plane being, of course, at right angles to C_1E_1.

The construction is, therefore, proved.

Since the normal and tangential stress intensities across a plane are represented by the abscissa and ordinate, respectively, in this construction, it is evident that the maximum and minimum direct stresses are represented by OA and OB, i.e., by the principal stresses p_1 and p_2. There is no accompanying shear.

For the maximum shear stress, the ordinate in the Mohr circle diagram must be a maximum. Through the centre C, draw a perpendicular to OX cutting the circle at R and R'. The maximum shear stress across a plane is, therefore, represented by the ordinate CR or CR' and is $\pm\sqrt{\left(\frac{p - p'}{2}\right)^2 + q^2}$. The accompanying normal stress across the planes of maximum shear is represented by OC and is $\frac{p + p'}{2}$. For the position of the planes of maximum shear, $\angle ACR$ is twice the angle which the plane makes with the plane of the principal stress p_1.

But $\angle ACR = 90°$.

Therefore, the plane of maximum shear is inclined at half this angle or 45° to the principal plane. Since $\angle ACR' = 270°$, the other plane of maximum shear is inclined at 135° to the major principal plane.

Example 1.

At a certain point in a strained material, the intensities of normal stress across two planes at right angles to each other are 1000 and 400 kg/cm², tensile, and there is a shear stress of 400 kg/cm² across the planes. Locate the principal planes and evaluate the principal stresses. Also obtain the maximum intensity of shear stress and specify its planes.

Construct the Mohr circle as shown in fig. 3. Selecting the origin O, set off $OQ = 1000$ kg/cm², to scale, along OX. At Q, erect the ordinate $QP = 400$ kg/cm², to the same scale, above the base line, because the shear stress on the vertical planes A_1D_1 and B_1C_1 is clockwise. Set off $OQ' = 400$ kg/cm² along OX and erect the ordinate $Q'P' = 400$ kg/cm², below the base line, because the shear stress on the horizontal planes A_1B_1 and D_1C_1 is anti-clockwise. Join PP' cutting OX at C. With centre C and radius CP or CP', describe a circle cutting OX at A and B. Scale off $OA = 1200$ kg/cm², tension, which is the major principal stress p_1. Scale off $OB = 200$ kg/cm², tension, which is the other principal stress p_2.

Scale off $\angle PCQ = 53° \ 8'$

$\therefore \quad 2\theta = 53° \ 8'$

i.e. $\quad \theta = 26° \ 34'$.

Or, scale off $\angle PBQ$ which gives θ directly. The plane B_1C_1 is inclined at 26°34′ to the plane C_1E_1 of the principal stress p_1, *anti-clockwise*. Through C_1, set off C_1E_1 at 26°34′ with C_1B_1, *clockwise*. This will be the principal plane of the major principal stress of $p_1 = 1200$ kg/cm². The second principal plane will be at right angles to it.

For the maximum shear stress, set off the diameter RCR' perpendicular to OX. The maximum shear stress is represented by CR or CR' and is 500 kg/cm².

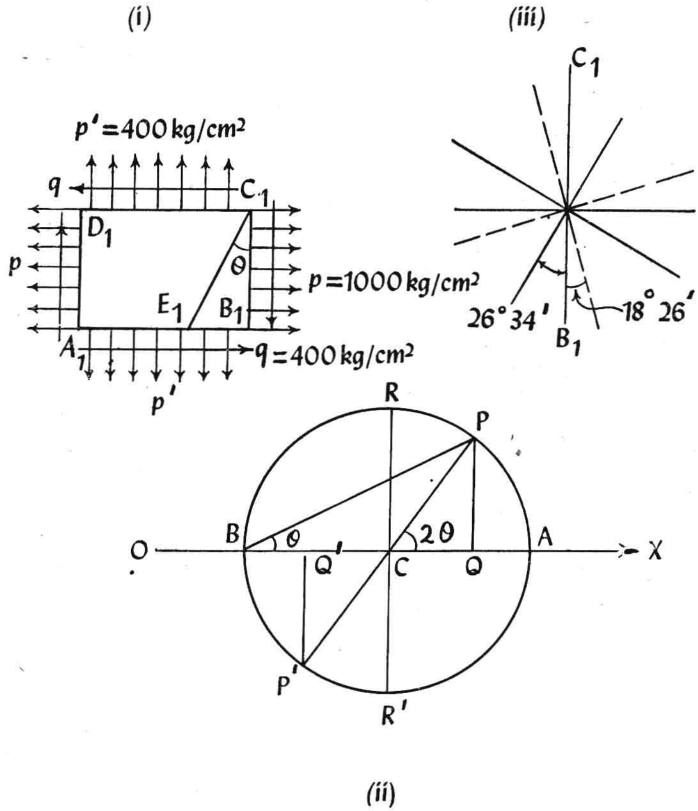

FIG. 3

For the position of the planes of maximum shear with reference to the plane B_1C_1, scale off $\angle RCP$ which is 36°52′. Therefore, one of the planes of maximum shear is inclined at 18°26′, *anti-clockwise*, with B_1C_1. The principal planes and planes of maximum shear are shown in fig. 3(iii) with reference to the vertical plane. The shear planes are shown dotted. These are, of course, inclined at 45° to the principal planes.