# Quantum Field Theory and its Macroscopic Manifestations

## Boson Condensation, Ordered Patterns and Topological Defects

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Imperial College Press

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# Preface

In 1951 and in 1952 Van Hove observed that there are theories where no normalizable state vectors belong to the common domain of both the free Hamiltonian $H_0$ and the perturbed (full) Hamiltonian $H$. Faith in the general applicability and validity of perturbation theory was then deeply shaken and a whole conception of the physical world was put in doubt. Perturbation theory rests indeed on the assumption that perturbed and unperturbed state vectors belong to the same Hilbert space. Friedrichs' book, where the existence of a host of unitarily inequivalent irreducible representations of the canonical commutation relations was discussed at length, came out in 1953 and not much later the Haag theorem was formulated. Actually, it was in those years that the discussion on basic principles, such as Lorentz invariance, spectral conditions, locality, etc., on which a reliable quantum field theory should be founded, led to the programme of the Axiomatic Formulation of Quantum Field Theory (QFT), starting indeed from the works by Haag, Gärding, Wightman, Schweber and others. The papers by Lehmann, Symanzik and Zimmermann (LSZ), published in "il Nuovo Cimento" between 1954 and 1958, laid solid bases for future developments of QFT. The LSZ formalism is founded on the so-called asymptotic condition which requires that a field theory must have an interpretation in terms of asymptotic particles with definite quantum numbers. Such a condition has been the guiding criterion underlying most of the work done in QFT in subsequent years, from renormalization theory to the search for a unified theory of the basic interactions among the constituents of matter. The crucial problem, which has been attracting the attention of many physicists, is indeed how to map Heisenberg fields, in terms of which the dynamics is given, to the asymptotic fields, in terms of which observables are constructed. In an early unpublished note, consistent with the LSZ formalism and written in Naples by Dell'Antonio and Umezawa in 1964, it is stressed that this map can only be a weak map, i.e., a map among matrix elements computed in the Hilbert space for the asymptotic fields. In subsequent years the study of such a mapping, called the dynamical map or the Haag expansion, has been central in the research activity of Umezawa and has revealed many subtle mechanisms through which the basic dynamics manifests itself at the level of the observables. One of these mechanisms, through which the consistency between the dynamical and the phenomenological level of the theory is expressed, is the dynamical rearrangement of symmetry in spontaneously broken symmetry theories.

A very important development occurred when the dynamical generation of long-range correlations, mediated by the Nambu–Goldstone (NG) boson quanta, was discovered in the early '60s, with subsequent implications in local gauge theories, such as the Higgs mechanism, which is one of the pillars of the standard model of elementary particles. It is interesting to remark that exactly the discovery of these collective modes gave strength to non-perturbative approaches, which could then establish themselves as complementary, or even, in some cases, alternative to the perturbation theory paradigm based on the ontological postulate of the asymptotic condition. The discovery of the existence in QFT of the unitarily inequivalent representations of the canonical commutation relations, which was in some sense shocking in the previous decade, could be better appreciated. The many inequivalent representations appeared to be a richness of QFT, which was thus recognized to be, due to such a specific feature indeed, the proper frame where systems endowed with many physically different phases could be described. QFT turns out not to be simply the "extension" of Quantum Mechanics (QM) to systems with an infinite number of degrees of freedom. Instead, QFT appears to be drastically different from QM. The von Neumann theorem, known for a long time and stating the unitary equivalence of the irreducible representations of the canonical commutation relations in QM, makes QM intrinsically not adequate to describe the variety of physically (unitarily) inequivalent phases of a given system. The crucial point is that such a theorem fails to hold in QFT, indeed, due to the infinite number of degrees of freedom. Spontaneous breakdown of symmetry, thermal field theory, phase transitions in a variety of problems, the process of defect formation during the process of non-equilibrium symmetry breaking phase transitions characterized by an order parameter, could then be studied by exploiting the whole manifold of the inequivalent representations in QFT.

In these studies, the prominent role played by coherent states was recognized, and attention was more and more focused on this, especially after the discovery of laser beams in quantum optics. It appeared that the "physical differences" among inequivalent representations are the differences in the degree of coherence of the boson condensates in the respective vacua.

The developments of QFT very briefly depicted above constitute the basis on which this book rests. The existence of the unitarily inequivalent representations is, indeed, a recurrent theme in our presentation. It is explored in several Chapters and shown to be especially related with finite temperature and dissipation in QFT, to the point that QFT can be recognized to be an *intrinsically* thermal quantum theory. The possibility of defining operators such as entropy and free energy in QFT and the role played there by them has been explored. The emerging picture is that no microscopic physical system may be considered completely isolated (closed) since it is always in interaction with the background fluctuations. From a different perspective, dissipation is discussed in relation to the proposal put forward by 't Hooft, according to which classical deterministic systems with information loss at high energy (Planck scale) may exhibit quantum behavior at low energy.

Quantum dynamics underlies macroscopic systems exhibiting some kind of ordering, such as superconductors, ferromagnets or crystals. Even the large-scale structures in the Universe, as well as the ordering in the biological systems, appear to be the manifestation of the microscopic dynamics ruling the elementary components of these systems. Therefore, in our discussion of the spontaneous breakdown of symmetry and collective modes, we stress that one crucial achievement has been recognizing that quantum field dynamics is not confined to the microscopic world: crystals, ferromagnets, superconductors, etc. are *macroscopic quantum systems.* They are quantum systems not in the trivial sense that they are made by quantum components (like any physical system), but in the sense that their macroscopic properties, accounted for by the order parameter field, cannot be explained without recourse to the underlying quantum dynamics. The problem is then to understand how the observed macroscopic properties are generated out of the quantum dynamics; how the *macroscopic* scale characterizing those systems is dynamically generated out of the *microscopic* scale of the quantum elementary components. Such a *change of scale* is understood to occur through the condensation of the NG boson quanta in the system ground state. Even in the presence of a gauge field, the NG boson fields do not disappear from the theory; they do not appear in the spectrum of physical particles, as the Higgs mechanism predicts; however, they do condense in the system vacuum state, thus creating a number of physically detectable properties originating from the vacuum structure so generated. Many of the physical examples we study in this book are characterized by the phenomenon of NG boson condensation. In this connection, we also consider the question of the dynamical generation of the macroscopic stability out of fluctuating quantum fields.

Moreover, a variety of phenomena are also observed where quantum particles coexist and interact with *extended macroscopic objects* which show a classical behavior, e.g., vortices in superconductors and superfluids, magnetic domains in ferromagnets, dislocations in crystals and other *topological defects*, fractal structures and so on. One is thus also faced with the question of the quantum origin of topological defects and of their interaction with quanta. This is a crucial issue for the understanding of symmetry breaking phase transitions and structure formation in a wide range of systems, from condensed matter to cosmology. We are thus led to discuss how the generation of ordered structures and of extended objects is explained in QFT. We show that topological defects are originated by non-homogeneous (localized) coherent condensation of quanta. The approach we follow is thus in some sense alternative to the one in which one starts from the classical soliton solutions and then quantizes them. Along the same line of thought, also oscillations of mixed particles, with particular reference to neutrinos, which manifest themselves on large (macroscopic) space distances appear to be connected to a (microscopic) condensation mechanism in the vacuum state.

As a general result stemming out of our discussion in this book we could say that recognizing the existence of the collective NG boson modes in spontaneously broken symmetry theories has produced a *shift of paradigm* (*á la* Kuhn): the former purely atomistic vision of the world, although necessary, turns out to be *not sufficient* to explain many physical phenomena. One needs to integrate such an atomistic vision with the inclusion of the dynamical generation of *collective* modes.

Throughout the book we have not specifically considered many important computational and conceptual questions and problems that have marked in a significant way the historical development of QFT, among these primarily renormalization problems. Neither have we discussed string theory, inflationary scenarios in cosmology and some recent theoretical and experimental achievements, such as, for example, the ones in the Bose–Einstein condensation of atoms in magnetic traps or other kinds of potential wells, and related developments in quantum optics and quantum computing. Our choice is motivated by the fact that the present book is not meant to be one on the general formalism of QFT and the whole spectrum of its applications. In any case, we apologize to the reader for neglecting many important topics and for many holes in our presentation.

We use both the operator formalism and the functional integration formalism. In the operator formalism the particle and wave-packet physical picture is more transparent, while in the functional formalism the general mathematical structure underlying the symmetry properties and the correlation functions appears more evident. From a formal point of view, the price we pay for the apparent non-homogeneous treatment is compensated by the multi-faceted understanding of the theoretical structure under study. Another price we pay for the variety of arguments treated is a non-uniform notation: our preference has been to adopt the general criterion of keeping contact with the notation of the original works.

The level of the presentation has been finalized to a readership of graduate students with a basic knowledge of quantum mechanics and QFT. Some of the presented material grew from graduate courses on elementary particle physics and/or condensed matter physics which the authors taught at the University of Salerno and Czech Technical University in Prague. The matter is organized as shown in the following table of contents and purposely several arguments and notions have been repeated in different Sections and Chapters for the reader's convenience. Much formalism is confined to the Appendices, where, however, the reader can find short discussions of conceptually and computationally important topics, such as Glauber coherent states and generalized coherent states. Some material on classical soliton theory, homotopy theory and defect classification is confined to Chapter 10, which may be skipped by the reader who is familiar with such topics.

Summarizing, the book contains an overview of many QFT results obtained by many research groups and by ourselves. It is therefore imperative to warmly thank all those colleagues and collaborators with whom we have had the good fortune to work or to discuss some of the problems considered in this book. This is certainly not a complete list, and we apologize for that. It includes T. Arimitsu, V. Srinivasan, H. Matsumoto, S. Kamefuchi, Y. Takahashi, H. Ezawa, E. Del Giudice, T. Evans, R. Rivers, J. Klauder, E. C. Sudarshan, H. Kleinert, J. Tolar, J. Niederle, E. Celeghini, A. Widom, Y. Srivastava, R. Mańka, E. Alfinito, O. Romei, A. Iorio, A. Capolupo, G. Lambiase, A. Kurcz, F. C. Khanna, P. A. Henning, E. Graziano, A. Beige, R. Jackiw, R. Haag, P. L. Knight, G. Vilasi, G. Scarpetta, F. Mancini, D.

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Salerno, Prague, July 2010

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# Contents

## Chapter 1

# The structure of the space of the physical states

### 1.1 Introduction

Symmetry principles play a central role in the understanding of natural phenomena. However, it is not always easy to recognize symmetries in physical observations since at a phenomenological level they can manifest as distorted, "rearranged" symmetries. For example, the fundamental symmetry between protons and neutrons, the nucleons, does not manifest as an exact symmetry, but as a "broken symmetry": the charge independence of nuclear interaction is indeed violated by the electromagnetic interaction. In general, various symmetry schemes, which are quite successful, also appear to be in some way approximate symmetry schemes [79, 343, 443, 476, 617], i.e., one has to disregard some phenomenological aspects, e.g., mass differences, which violate certain symmetry requirements. A way of looking at this situation is to interpret the observed deviations from the exact symmetry as a phenomenological distortion or rearrangement of the basic symmetry. Examples of rearranged symmetries are easily found in solid state physics: crystals manifest a periodic structure, but do not possess the continuous translational invariance of the Hamiltonian of molecular gas. Ferromagnets present rotational invariance around the magnetization axis, but not the original $SU(2)$ invariance of the Lagrangian. In superconductivity and superfluidity the phase invariance is the one that disappears.

The crucial problem one has to face in the recognition of a symmetry is, then, the intrinsic two-level description of Nature: one aspect of this duality concerns original symmetries ascribed to "basic" entities, the other aspect concerns the corresponding rearranged symmetries of observable phenomena. This two-level description of Nature was soon recognized in Quantum Field Theory (QFT) as the duality between fields and particles. Without going into the historical developments of this concept, which are outside the purpose of this book, we only recall, as an example, how fundamental this duality is in the renormalization theory, where the distinction is crucial between "bare" and "observed" particles, namely the distinction between basic fields and their physical "manifestation".

In the following Sections we will focus our attention on some structural aspects of QFT in order to prepare the tools to be used in the study of the mechanisms through which the dynamics of the basic fields leads to their observable physical manifestation. Thus, the core of our interest will be the structure of the space of the physical fields, which will bring us to study that peculiar nature of QFT consisting in the existence of infinitely many unitarily inequivalent representations of the canonical (anti-)commutation relations, and thus to the analysis of the von Neumann theorem, of the Weyl–Heisenberg algebra, the characterization of the physical fields and the coherent states. Our discussion will include in a unified view, topics such as the squeezing and self-similarity transformations, fractals and quantum deformation of the Weyl–Heisenberg algebra. A glance at the table of contents shows how these subjects are distributed in the various Sections and Appendices.

## 1.2 The space of the states of physical particles

Let us consider a typical scattering process between two or more particles. By convenient measurements we can identify the kind, the number, the energy, etc., of the particles before they interact (incoming particles); there is then an interaction region which is precluded to observations and finally we can again measure the kind, the number, the energy, etc., of the particles after the interaction (outgoing particles). The sum of the energies of the incoming particles is observed to be equal to the sum of the energies of the outgoing particles. Incoming particles and outgoing particles are referred to as "physical particles", or else as "observed" or "free" particles, where the word "free" does not exclude the possibility of interaction among them; it means that the interaction among the particles can be considered to be negligible far away, in space and time, from the interaction region. The total energy of the system of free particles is given in a good approximation by the sum of the energies of the single particles. We require that the energy of the physical particles is determined as a certain function of their momenta. In solid state physics the physical particles are usually called quasiparticles.