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# INTRODUCTION TO MODERN THEORETICAL PHYSICS

VOLUME 1

Classical Physics and Relativity

EDWARD G. HARRIS



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VOLUME I  
Classical Physics and Relativity

EDWARD G. HARRIS  
University of Tennessee

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## Preface

There was a time, not too long ago, when a student of physics typically began his graduate studies with a course in theoretical physics from one of the popular textbooks such as those of Joos, Slater and Frank, or Page. Then, in the rapid expansion of physics departments after World War II, such general courses were replaced by more specialized ones. The general theoretical physics courses that survived were often relegated to the status of remedial courses for the poorly prepared entering graduate student.

An unfortunate consequence was that, after the general physics course of his freshman or sophomore year, the student never again saw physics presented as a unified subject, but rather encountered it as a collection of separate subjects labeled mechanics, electrodynamics, quantum mechanics, and so forth. The connections among these have not always been apparent. Often each course is taught from a different textbook by a different professor who specializes in the particular field. Any synthesis must be provided by the student himself.

I hope that this book will be a step in the direction opposing this trend. Of course, physics has not yet reached the stage at which everything can be deduced from a small number of postulates, and I have not tried to present it as if this were the case. Instead I have attempted to organize the material in such a way that the reader can see how the concepts of particle mechanics evolved into those of continuum mechanics, how the concept of a field grew from these, thus paving the way for electrodynamics, and then how the concepts of mechanics and the gravitational and electromagnetic fields evolved into relativity and quantum mechanics, those supreme achievements of theoretical physics. A number of themes such as conservation laws, variational principles, and invariance under groups of transformations recur throughout theoretical physics. It is hoped that, a deeper appreciation of the unity of physics will be achieved by seeing these recurrences in close juxtaposition.

I have tried to take the reader closer to the frontiers of theoretical physics than is done in most other books on general theoretical physics, few of which discuss unified field theories or quantum field theory. I consider that there is good justification for exposing all candidates for the Ph.D. in physics to general relativity and quantum field theory. After all, general relativity is now more than half a century old, and the beginnings of quantum field theory go back almost as far. (Dirac quantized the electromagnetic field in 1927.) For a physicist to be ignorant of these theories would be as anomalous as for a professor of English never to have read a novel by William Faulkner. In some of the more advanced chapters such as Chapters 13 and 23, my aim has been different from that of the rest of the book. In these chapters I have not tried to impart a working knowledge of the subject, but rather to acquaint the reader with some of the more speculative ideas that have been proposed to advance physics beyond its present position.

Although I have tried to begin the presentation of each topic at the beginning, I have not really assumed that the reader would start the book in a state of complete intellectual nakedness. I have imagined the reader of my book to be a beginning graduate student in physics with the background of an undergraduate physics major. I have supposed him to know calculus, some differential equations, elementary mechanics, electricity, and magnetism and to be familiar with some of the facts about atoms, molecules, solids, and nuclei. I have not hesitated, however, to include some material that would be familiar to such a reader, when I thought such inclusion added to the continuity of the treatment. I have heard that Fermi once said, "Never underestimate the joy people derive from hearing something they already know." I think that this is useful advice for a writer of textbooks. At the same time I have tried not to bore the reader with a lot of material on wheels, pulleys, inclined planes, boundary value problems in electrostatics, and the like with which he was probably surfeited in his undergraduate years.

The organization of the book is as follows. In Part 1 I discuss some of the mathematical methods which are used repeatedly throughout the book. I begin with simple vector analysis, as this leads rather naturally to tensor analysis, which is useful in continuum mechanics, electrodynamics, and relativity, and to abstract vector spaces, which are used extensively in quantum mechanics. Another area of mathematics that is very useful to the physicist is the theory of Fourier analysis and generalized functions. Since this is not closely connected with the other material of Chapter 1, however, it has been relegated to an appendix.

Part 2 is devoted to the classical subjects of mechanics, gravitation, and electromagnetism. Since I have supposed that much of this is familiar to the reader, the treatment is rather abbreviated in some sections.

The heart of the book consists of Part 3 and 4, where relativity and quantum theory are treated. In Part 3 the ideas of relativity theory are developed, starting with the Galilean transformation and continuing through some of the speculative unified field theories. Part 4 is intended to be a complete graduate level course in quantum mechanics.

I could have included thermodynamics, classical statistical mechanics, and kinetic theory among the classical subjects of Part 2. This would have had the virtue of maintaining the rough historical order in which topics are presented. However, these topics are not necessary prerequisites for relativity and quantum mechanics. Moreover, it seemed advantageous to postpone their presentation until after the study of quantum mechanics, so that classical and quantum-statistical mechanics and kinetic theory could be discussed together. Thus Part 5 is devoted to the physics of large complex systems for which a statistical treatment is necessary.

My original intention was to squeeze all of theoretical physics into one volume, but it became apparent that the result would be an inconveniently large tome. The publisher suggested that two volumes would be preferable and pointed out that the end of Chapter 13 was a natural dividing point. Thus classical physics and relativity are covered in the first volume, and quantum theory and statistical physics in the second. I have tried to make the two volumes as independent as possible without abandoning my goal of presenting physics as a unified subject.

To cover all of theoretical physics in two volumes, it has been necessary to severely abbreviate the discussion of some topics. The reader will often encounter the expression "It may be shown that...." Generally, I have eliminated mathematical derivations and proofs that were not in themselves very instructive and have only quoted the results. I have attempted to compensate for this abbreviated treatment by giving adequate references to the literature where the interested reader may find more thorough discussions. Throughout the book my intention has been to stress the physical concepts involved and not the applications or the mathematical techniques. It is hoped that, as a result of stripping discussions of excess verbiage, these concepts have been illuminated rather than obscured.

It is obviously impossible to provide an exhaustive bibliography for a book on theoretical physics. Generally, I have given as references the books and papers which I have found particularly helpful or interesting. I apologize to the authors of many fine works that are not included among the references.

Most sections are followed by sets of problems. Since some of these problems extend the ideas discussed in the text, it is suggested that the reader at least read the problems even if he does not attempt to solve them. For some of the more difficult problems references are given to the literature where solutions may be found.

In a way, I consider myself fortunate in being employed by one of the less affluent universities. In this university it has not been practicable to let each faculty member teach his specialty alone. Consequently there has been considerable shifting about of the faculty among the graduate courses. This has been wonderful for my education and has made the writing of this book possible. Over the years I have taught in one course or another almost all of the topics covered herein. It is a pleasure to acknowledge the help of my students, who have cheerfully endured my teaching of unfamiliar subjects and by their questions added greatly to my understanding.

EDWARD G. HARRIS

Knoxville, Tennessee  
April 1975

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# Vectors, Tensors, and the Structure of Space

## 1. VECTOR ALGEBRA

Although it is assumed that the reader has previously encountered the notions of scalars and vectors and has learned to use them in elementary physics, it is convenient to begin with a review of these notions in an elementary form as a preparation for the more abstract concepts that follow.

Many quantities dealt with by mathematicians and physicists are adequately characterized by the specification of one number, the magnitude. Examples of physical quantities of this type are mass, density, temperature, and volume. We call such quantities *scalars*.

Other quantities, called *vectors*, can be adequately characterized only by specifying both a magnitude and a direction. Examples of such quantities are position, velocity, force, and electric and magnetic field intensities. For instance, consider the position vector  $\mathbf{x}$ , which specifies the position of a particle  $P$  relative to some origin  $O$  (Fig. 1-1). We can represent this vector by a line segment extending from  $O$  to  $P$  with an arrowhead on the end at  $P$ . The magnitude of the vector is the length of the line. The direction of the vector is given by the orientation of the line segment in space and the arrowhead on one end of the segment. In a similar way other vectors can be represented geometrically by directed line segments whose length (in appropriate units) is the magnitude of the vector and whose orientation and sense gives the direction.

Much economy in writing equations and also in thinking about vectors can be achieved if rules are devised for manipulating vectors as entities in themselves. In what follows we will denote vectors by boldface symbols:  $\mathbf{A}$ ,