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PHILOSOPHY OF LOGICS

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PREFACE

The century since the publication of Frege's *Begriffsschrift* has seen a tremendous growth in the development and study of logical systems. The variety of this growth is as impressive as its scale. One can distinguish four major areas of development, two in formal, two in philosophical studies: (i) the development of the standard logical apparatus, beginning with Frege's and Russell and Whitehead's presentation of the syntax of sentence and predicate calculi, subsequently supplied with a semantics by the work of e.g. Post, Wittgenstein, Löwenheim and Henkin, and studied metalogically in the work of e.g. Church and Gödel; (ii) the development of non-standard calculi, such as the modal logics initiated by C. I. Lewis, the many-valued logics initiated by Łukasiewicz and Post, the Intuitionist logics initiated by Brouwer. Alongside these one has (iii) philosophical study of the application of these systems to informal argument, of the interpretation of the sentence connectives and quantifiers, of such concepts as truth and logical truth; and (iv) study of the aims and capacities of formalisation, by those, such as Carnap and Quine, who are optimistic about the philosophical significance of formal languages, by those, such as F. C. S. Schiller and Strawson, who are sceptical of the pretensions of symbolic logic to philosophical relevance, and by those, such as Dewey, who urge a more psychological and dynamic conception of logic over the prevailing one.

I see some philosophical significance in the fact that these developments took place in parallel rather than in series; for it is salutary to remember that 'non-standard' logics have developed alongside the standard systems, and that there have always been critics, too, not only of specific formal systems, but of the aspirations of formalisation itself.

Developments in the four areas I have distinguished were not, of course, independent of each other; and I see philosophical significance, also, in the interplay between them. For example, although some of the key ideas of both modal and many-valued logics were anticipated by MacColl as early as 1880, their systematic formal development came, respectively, in 1918 after the canonical formalisation of non-modal calculi in *Principia Mathematica*, and in 1920 after the provision of truth-table semantics for 2-valued logic. However, the motivation for the development of non-standard calculi derived not only from the mathematical appeal of the prospect of extensions and modifications of classical logic, but also from philosophical criticism: in the case of modal logics, of the claim of the material conditional to represent implication, and, in the case of many-valued logics, of the assumption that every proposition is either true or else false. And one development in non-standard logic prompted another: doubts about the success of modal logics in formalising the intuitive idea of entailment led to the development of relevance logics, while the mathematical appeal of modal systems encouraged the development, by analogy, of epistemic, deontic and tense logics; or again, reflection on the philosophical motivation for many-valued logics led to the idea of supervaluations. Formal innovations, in turn, have given a new dimension to philosophical questions originally raised by standard calculi: as, for instance, issues about the interpretation of quantifiers and their relation to singular terms arose in a new and acute form when the intelligibility of modal predicate logic was challenged; or, as old worries about whether logic deals with sentences, statements or propositions turned out to be implicated in the challenge to bivalence posed by many-valued systems. Sometimes new formal systems have even challenged, explicitly or implicitly, and more or less radically, accepted assumptions about the aims and aspirations of formal logics: relevance logic, for instance, questions not only the adequacy of the material and strict conditionals but also the classical conception of validity; the distinctive character of Intuitionist logic derives in part from a challenge to the 'logicist' presumption of the priority of logic over mathematics; and fuzzy logic breaks with the traditional principle that formalisation should correct or avoid, but not compromise with, vagueness. And, as the last example reminds one, new formal developments have sometimes aspired to overcome what both supporters and critics of formal logic had taken to be its inherent limitations – such as its supposed incapacity, stressed by both Schiller and Strawson, to

deal with the pragmatic features which affect the acceptability of informal reasoning, perhaps overcome, at least in part, by the 'formal pragmatics' initiated by Montague.

My concern, in this book, is with the philosophy, rather than the history, of logic. But my strategy has been devised with an eye to the history of the interplay of formal and philosophical issues which I have just sketched. I begin with a consideration of some problems raised by the standard logical apparatus – the interpretation of sentence connectives, sentence letters, quantifiers, variables, individual constants, the concepts of validity, truth, logical truth; I turn, from ch. 9 onwards, to a consideration of the way some of these problems motivate formal innovations, 'extended' and 'deviant' logics, and to the ways in which these new formalisms lead, in turn, to a re-evaluation of the philosophical issues; and I conclude, in the final chapter, with some questions – and rather fewer answers – about the metaphysical and epistemological status of logic, the relations between formal and natural languages, and the relevance of logic to reasoning.

And two recurring themes of the book also reflect this historical perspective. What seem to me to be the vital philosophical issues in logic are focussed by consideration (i) of the plurality of logical systems and (ii) of the ways in which formal calculi bear on the assessment of informal argument. More specifically, I shall be urging that, in view of the existence of alternative logics, prudence demands a reasonably radical stance on the question of the epistemological status of logic, and that the interpretation of formal results is a delicate task in which judicious attention to the purposes of formalisation is highly desirable.

I have tried to produce a book which will be useful as an introduction to the philosophical problems which logic raises, which will be intelligible to students with a grasp of elementary formal logic and some acquaintance with philosophical issues, but no previous knowledge of the philosophy of logic. But I haven't offered simple answers, or even simple questions; for the interesting issues in philosophy of logic are complex and difficult. I have tried instead to begin at the beginning, to explain technicalities, and to illustrate highly general problems with specific case studies. To this end I have supplied, for those new to the subject, a glossary of possibly unfamiliar terms used in the text, and some advice on finding one's way about the literature; while, for those anxious to go further, I have included a generous (but I hope

not intimidating) bibliography. The response of my students has encouraged me to believe that it is unnecessary, as well as undesirable, to oversimplify. I have aspired – though the result, I fear, inevitably falls short of the aspiration – to produce a book which may be of some use to the student, and at the same time of some interest to the teacher.

It is, I find, irritating to be unsure whether, or how, an author has modified views he previously put forward; but, on the other hand, it is tedious to be subjected to frequent discussions of an author's earlier mistakes. By way of compromise, therefore, I indicate here, briefly, where, and how, I have modified the ideas put forward in *Deviant Logic*. First: I have, I hope, made the distinction between metaphysical and epistemological questions about the status of logic rather clearer; and this has led me to distinguish more carefully between the question of monism versus pluralism, and the question of revisability, and to support a qualified pluralism rather than the monism somewhat confusedly assumed in *Deviant Logic*. Second: I have come to appreciate that the consequences for ontology of the substitutional interpretation of the quantifiers are somewhat less straightforward than I used to suppose; and this has led me to a more subtle, or at any rate more complex, account of the respective roles of quantifiers and singular terms. I dare say, though, that I shall have missed some old mistakes, besides making some new ones.

CONTENTS

I

'Philosophy of logics'

There is no mathematical substitute for philosophy.
Kripke, 1976

1 Logic, philosophy of logic, metalogic

The business of philosophy of logic, as I understand it, is to investigate the philosophical problems raised by logic – as the business of the philosophy of science is to investigate the philosophical problems raised by science, and of the philosophy of mathematics to investigate the philosophical problems raised by mathematics.

A central concern of logic is to discriminate valid from invalid arguments; and formal logical systems, such as the familiar sentence and predicate calculi, are intended to supply precise canons, purely formal standards, of validity. So among the characteristically philosophical questions raised by the enterprise of logic are these: What does it mean to say that an argument is valid? that one statement follows from another? that a statement is logically true? Is validity to be explained as relative to some formal system? Or is there an extra-systematic idea that formal systems aim to represent? What has being valid got to do with being a good argument? How do formal logical systems help one to assess informal arguments? How like 'and' is '&', for instance, and what should one think of 'p' and 'q' as standing for? Is there one correct formal logic? and what might 'correct' mean here? How does one recognise a valid argument or a logical truth? Which formal systems count as logics, and why? Certain themes recur: concern with the scope and aims of logic, the relations between formal logic and informal argument, and the relations between different formal systems.

The sphere of the philosophy of logic is related to, but distinct from, that of metalogic. Metalogic is the study of formal properties of formal logical systems; it would include, for instance, proofs (or disproofs) of

their consistency, completeness or decidability. Philosophy of logic likewise concerns itself with questions about formal logical systems – but with philosophical rather than purely formal questions. Take the relations between the standard, 2-valued, and many-valued sentence calculi as an example: the philosopher will want to know in what, if any, sense many-valued logics are alternatives to 2-valued logic; whether one is obliged to choose between many-valued and 2-valued calculi, and if so, on what grounds; what would be the consequences for the concept of truth if a many-valued system were adopted, and so forth. Metalogical results may well help one to answer questions of this kind: for instance, it is presumably a necessary, though not a sufficient condition of a many-valued logic's being a serious alternative, that it be consistent; and it may be pertinent to questions of their relative status that (most) many-valued logics are contained in 2-valued logic (i.e. that all their theorems are theorems in 2-valued logic, but not vice-versa). A second difference is that philosophy of logic is not wholly occupied with questions about formal logics; informal argument, and the relations between formal system and informal argument, are also within its sphere. The development of formal systems, indeed, greatly increases the depth and rigour of logical studies; but the study of informal argument is often an indispensable preliminary to such developments, and success in systematising informal arguments a test of their usefulness. It is pertinent that Frege, one of the pioneers of modern formal logic, was prompted to develop his *Begriffsschrift* (1879) because he needed a less ambiguous and cumbersome medium than German in which to give properly rigorous arithmetical proofs.

The locution 'philosophy of logic' is, I think, much to be preferred to 'philosophical logic', which is apt to convey the unfortunate impression that there is a peculiar, philosophical way of doing logic, rather than that there are peculiarly philosophical problems about logic. (I observe that, unlike 'philosophical logic', 'philosophical science' and 'philosophical mathematics' have never gained currency.) My examples have already shown, however, that philosophical interest attaches to the fact that there is not just one, but a plurality of formal logics; and so 'philosophy of logics' is, I hope, better yet.

2 The scope of logic

Among the problems of the philosophy of science are questions about the scope of science: what domains of knowledge (or 'knowledge') are to count as sciences? – for example, should alchemy, or astrology, or sociology, or psychology count as *bona fide* sciences? And what grounds could be given for including or excluding a given domain of inquiry? Similarly, among the problems of the philosophy of logic are questions about the scope of logic, and hence about the scope of the philosophy of logic: what is a logic? which formal systems are systems of logic? and what makes them so?[1]

Because I have to begin somewhere, I shall take for granted an intuitive idea of what it is to be a formal system. But I shall indicate what range of formal systems I have in mind when I speak of formal *logics*.

It is relevant to distinguish, at the outset, between *interpreted* and *uninterpreted* formal systems: uninterpreted, a formal system is just a collection of marks, and cannot, therefore, be identified as a formal logic rather than, say, a formalisation of a mathematical or physical theory. The claim of a formal system to be a logic depends, I think, upon its having an interpretation according to which it can be seen as aspiring to embody canons of valid argument: I count many-valued 'logics' as logics, for example, because they have interpretations according to which their values are 'truth-values', their variables sentences, their operators negation, conjunction etc. (They also have *other* interpretations – e.g. in terms of electrical circuits; the isomorphism between the logical and the electrical interpretations is relevant to the way computers work. See Rescher 1969 p. 61 for references.) So, in speaking of various formalisms as logics, I shall be making an implicit appeal to their usual interpretations.

In deciding which formalisms to count as logics I have adopted, for the present, the hospitable policy of giving the benefit of any doubt – subsequently, though, I shall give some attention to arguments why systems I have *included* ought to be *excluded*. One reason for this policy is that it lessens the danger of dismissing a formal system as 'not really a logic', when one ought to be asking seriously whether it is a good or useful system. I fear for instance that Quine (1970 ch. 5),

[1] The significance of such questions as these will, I hope, become increasingly apparent as the book proceeds. Readers who find this section hard going may prefer to return to it at the end of the book.

who excludes second-order predicate calculus because of what he takes to be its commitment to an ontology of abstract, intensional objects – properties – may have succumbed to this danger. (Similarly, I should distrust definitions of what it is for something to be a work of art which encouraged evasion of questions about *bad* works of art.) Anyway, as formal logics I shall include:

'traditional' logic – Aristotelian syllogistic
'classical' logic – 2-valued sentence calculus
predicate calculus[1]
'extended' logics – modal logics
tense logics
deontic logics
epistemic logics
preference logics
imperative logics
erotetic (interrogative) logics
'deviant' logics – many-valued logics
Intuitionist logics
quantum logics
free logics
'inductive' logics

The intention is to distinguish between formal logics and systems of, say, arithmetic or geometry, or axiomatisations of biology, physics and so forth. The demarcation is not based on any very profound ideas about 'the essential nature of logic' – indeed, I doubt that there is any such 'essential nature'. But it is not wholly arbitrary; it corresponds reasonably well, I hope, to what writers on philosophy of logic usually have in mind when they speak of 'logics'; and it has, at least, the following pragmatic rationale.

Those formal systems which are known as the 'standard' or 'classical' logic (and taught in courses in elementary formal logic) must surely count as logics if anything does. It then seems appropriate to admit also as logics those formal systems which are analogous to these. Among such 'analogous' systems I include: extensions of classical logic, systems, that is, that add new logical vocabulary ('necessarily' and 'possibly' in modal logics, 'it used to be the case

[1] In accordance with the 'benefit of the doubt' policy, I take this to include identity theory (i.e. axioms or rules for '=') and second-order predicate calculus (i.e. quantification binding 'F'...etc. as well as 'x'...etc.) besides first-order predicate calculus.

that' and 'it will be the case that' in tense logics, 'ought' and 'may' in deontic logics, 'knows' and 'believes' in epistemic logics, 'prefers' in preference logics) along with new axioms or rules for the new vocabulary, or which apply familiar logical operations to novel items (imperative or interrogative sentences); deviations of classical logic, i.e. systems with the same vocabulary but different (usually more restricted) axioms or rules; and inductive logics, which aim to formalise a notion of support analogous to, but weaker than, logical consequence. Their similarity to classical logic – not just formal similarity, but also similarity in purpose and intended interpretation – makes it natural to regard these systems as logics. (Alternatively, I could have begun with traditional Aristotelian logic, of which the modern 'classical' logic is an extension, and proceeded from there by a similar process of analogy.)

However, the idea of a system's being sufficiently similar to the classical logic is obviously pretty vague; and one might reasonably wonder whether the scope of logic could be delimited in some less pragmatic, and more precise, fashion.

The traditional idea that logic is concerned with the validity of arguments as such, irrespective, that is, of their subject-matter – that logic is, as Ryle neatly puts it, 'topic-neutral' – could be thought to offer a principle on which to delimit the scope of logic. On this account those systems which are *applicable to reasoning irrespective of its subject-matter* would count as logics. This idea is one with which I sympathise; I doubt, though, that it is really appreciably more precise than the notion of analogy to classical logic with which I began. What does it mean, first, to say that a formal system is 'applicable' to reasoning on such-and-such subject-matter? Presumably, that its principles are intended to be true of such reasoning. And now what is one to understand by 'irrespective of its subject-matter'? It could be suggested that while sentence and predicate calculi are indifferent to subject-matter, arithmetic, for example, is not topic-neutral because it is specifically about numbers; but this raises awkward questions about 'about' (is first-order predicate calculus 'about individuals'?). It is suggested, again, that logic applies to reasoning irrespective of its subject-matter because it is concerned with the *form* of arguments rather than their *content*. Again, I think, the idea is helpful, though it is still imprecise. How is one to distinguish between the form of an argument and its content? Tense logic is applicable to tensed sentences, imperative logic to imperative sentences, and the tense or

mood of a sentence could, not implausibly, be regarded as a matter of its form rather than its content; but other cases are less straightforward – the idea of form would need refinement before it was clear that a sentence's being about belief was a matter of form, but its being about numbers a matter of content, for example.

However, the vagueness of the idea of topic-neutrality and the related distinction between form and content isn't necessarily objectionable; as I said, I am doubtful that logic has a precisely specifiable 'essential character'. When I judged that modal logics, for example, are enough like classical logic to be included within the scope of logic, I was implicitly relying on the idea that the adverbs 'necessarily' and 'possibly' are topic-neutral enough to count as 'new *logical* vocabulary'. So the idea of topic-neutrality can certainly help to fortify one's intuitions about what formal systems are relevantly analogous to classical logic. It is also significant that where to draw the line between logics and other formal systems is more doubtful and more controversial in some cases than in others. For example: some mathematical theories, notably set theory, are very general in application, and seem to have strong affinities to logic; while epistemic or preference logics seem more specific as to subject-matter than the standard logical formalisms, and not to have quite so strong a claim to inclusion. Briefly, one gets more doubtful about the exclusion of a 'mathematical' formalism, the more general its application, and more doubtful about the inclusion of a 'logical' formalism, the less general its application; this suggests that topic-neutrality is vague *in the right way*.

These ideas will prove important subsequently. The distinction between form and content will receive some closer scrutiny when, in the next chapter, I discuss the thesis that the validity of an argument depends upon its form; and the idea that logic is characteristically topic-neutral will be relevant when, in ch. 12, I tackle the question of monism versus pluralism in logic, i.e. whether there is, so to speak, one correct logic, or whether different logics might each be appropriate to different areas of discourse.

Sometimes a purely formal, metalogical criterion is suggested to demarcate logical from other formal systems. Kneale, for instance, urges that only *complete* systems be allowed within the scope of logic. The upshot of adopting such a criterion would be to restrict my hospitable list; since second-order predicate calculus is not complete in the usual sense, it would, by these standards, be excluded. This proposal has the advantage of precision; one is entitled to ask,