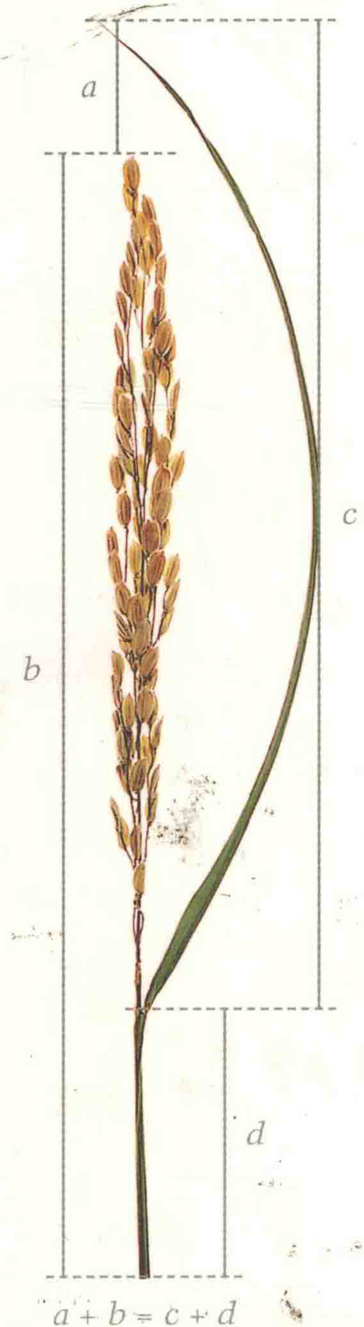
Beginning Algebra with Applications

Fourth Edition

Aufmann Barker Lockwood



BEGINNING ALGEBRA

with Applications

FOURTH EDITION

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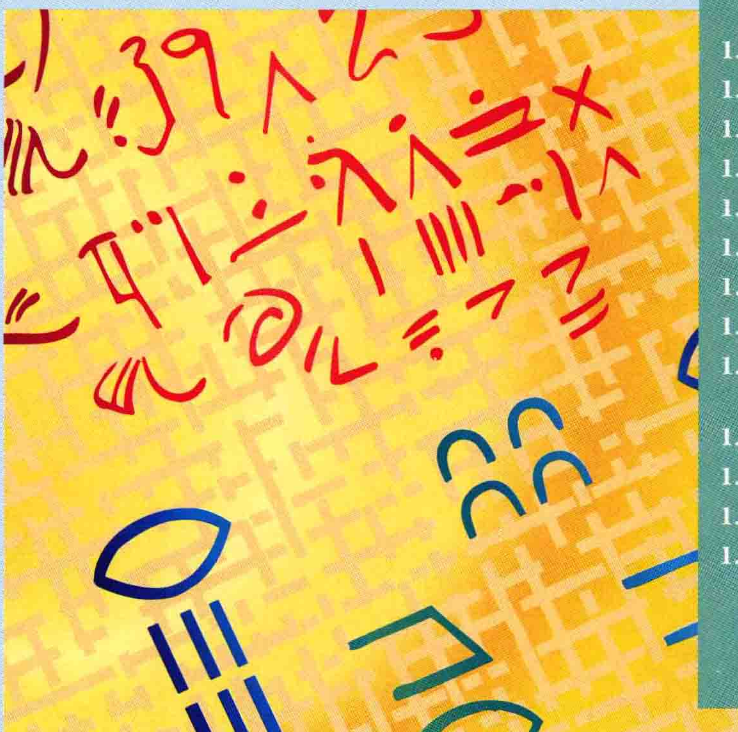

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Early Egyptian Fractions

One of the earliest written documents of mathematics is the Rhind Papyrus. This tablet was found in Egypt in 1858, but it is estimated that the writings date back to 1650 B.C.

The Rhind Papyrus contains over 80 problems. Studying these problems has enabled mathematicians and scientists to understand some of the methods by which the early Egyptians used mathematics.

Evidence gained from the Papyrus shows that the Egyptian method of calculating with fractions was much different from the methods used today. All fractions were represented in terms of what are called unit fractions. A unit fraction is a fraction in which the numerator is 1. This fraction was symbolized with a bar over the number. Examples (using modern numbers) include

$$\overline{3} = \frac{1}{3} \qquad \overline{15} = \frac{1}{15}$$

The early Egyptians also tended to deal with powers of two (2, 4, 8, 16, . . .). As a result, representing fractions with a 2 in the numerator in terms of unit fractions was an important matter. The Rhind Papyrus has a table giving the equivalent unit fractions for all odd denominators from 5 to 101 with 2 as the numerator. Some of these are listed below.

$$\frac{2}{5} = \overline{3}\ \overline{15} \qquad \left(\frac{2}{5} = \frac{1}{3} + \frac{1}{15}\right)$$

$$\frac{2}{7} = \overline{4}\ \overline{28}$$

$$\frac{2}{11} = \overline{6}\ \overline{66}$$

$$\frac{2}{19} = \overline{12}\ \overline{76}\ \overline{114}$$

SECTION 1.1
Introduction to Integers

1 Order relations

INSTRUCTOR NOTE

Margin notes entitled POINT OF INTEREST or LOOK CLOSELY are printed in the Student Text. The POINT OF INTEREST feature provides a historical note or mathematical fact of interest. The LOOK CLOSELY feature flags important information or provides assistance in understanding a concept. The margin notes entitled INSTRUCTOR NOTE are printed only in the Instructor's Annotated Edition.

It seems to be a human characteristic to group similar items. For instance, a botanist places plants with similar characteristics in groups called species. Nutritionists classify foods according to food groups; for example, pasta, crackers, and rice are among the foods in the bread group.

Mathematicians place objects with similar properties in groups called sets. A **set** is a collection of objects. The objects in a set are called the **elements** of the set.

The **roster method** of writing sets encloses a list of the elements in braces. The set of sections within an orchestra is written {brass, percussion, strings, woodwind}.

The numbers that we use to count objects, such as the number of students in a classroom or the number of people living in an apartment house, are the natural numbers.

Natural numbers $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

The three dots mean that the list of natural numbers continues on and on and that there is no largest natural number.

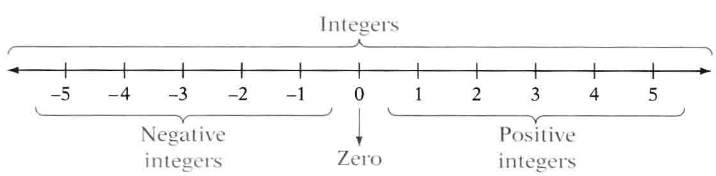
POINT OF INTEREST

The Alexandrian astronomer Ptolemy began using *omicron*, o, the first letter of the Greek word that means "nothing," as the symbol for zero in A.D. 150. It was not until the 13th century, however, that Fibonacci introduced 0 to the Western world as a placeholder so that we could distinguish, for example, 45 from 405.

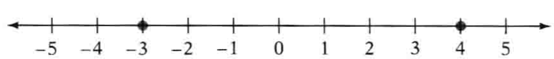
The natural numbers alone do not provide all the numbers that are useful in applications. For instance, a meterologist also needs the number zero and numbers below zero.

Integers $= \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

Each integer can be shown on a number line. The integers to the left of zero on the number line are called **negative integers.** The integers to the right of zero are called **positive integers,** or natural numbers. Zero is neither a positive nor a negative integer.



The **graph** of an integer is shown by placing a heavy dot on the number line directly above the number. The graphs of -3 and 4 are shown on the number line below.



Consider the following sentences.

> The quarterback threw the football, and the receiver caught *it*.
> A student purchased a computer and used *it* to write English and history papers.

In the first sentence, *it* is used to mean the football; in the second sentence, *it* means the computer. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable**. Variables are used in the following definition of inequality symbols.

Definition of Inequality Symbols

If a and b are two numbers and a is to the left of b on the number line, then a **is less than** b. This is written $a < b$.

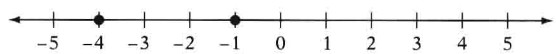
If a and b are two numbers and a is to the right of b on the number line, then a **is greater than** b. This is written $a > b$.

POINT OF INTEREST

The symbols for "is less than" and "is greater than" were introduced by Thomas Harriot around 1630. Before that, ⊏ and ⊐ were used for $<$ and $>$, respectively.

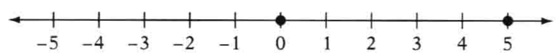
Negative 4 is less than negative 1.

$-4 < -1$



5 is greater than 0.

$5 > 0$



There are also inequality symbols for **is less than or equal to** ($\leq$) and **is greater than or equal to** ($\geq$).

$7 \leq 15$ 7 is less than or equal to 15. This is true because $7 < 15$.

$6 \leq 6$ 6 is less than or equal to 6. This is true because $6 = 6$.

INSTRUCTOR NOTE

One of the main pedagogical features of this text is the paired Examples and Problems. The Problem is for the student to work, using the Example above as a model. A *complete solution* can be found on the page referenced in the Solution of the Problem. Thus, students work on skills as they are being taught and obtain immediate feedback on and reinforcement of a skill being learned.

Example 1 Use the roster method to write the set of negative integers greater than or equal to -6.

Solution $A = \{-6, -5, -4, -3, -2, -1\}$ ► A set is designated by a capital letter.

Problem 1 Use the roster method to write the set of positive integers less than 5.

Solution See page A11. $A = \{1, 2, 3, 4\}$

Example 2 Given $A = \{-6, -2, 0\}$, which elements of set A are less than or equal to -2?

Solution ► Find the order relation between each element of set A and -2.

$-6 < -2$

$-2 = -2$

$0 > -2$

The elements -6 and -2 are less than or equal to -2.

Problem 2 Given $B = \{-5, -1, 5\}$, which elements of set B are greater than -1?

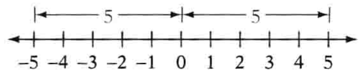
Solution See page A11. 5

2 Opposites and absolute value

Two numbers that are the same distance from zero on the number line but are on opposite sides of zero are **opposite numbers,** or **opposites**. The opposite of a number is also called its **additive inverse.**

The opposite of 5 is -5.

The opposite of -5 is 5.



The negative sign can be read "the opposite of."

$-(2) = -2$ The opposite of 2 is -2.

$-(-2) = 2$ The opposite of -2 is 2.

Example 3 Find the opposite number. A. 6 B. -51

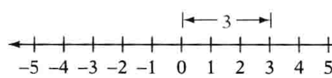
Solution A. -6 B. 51

Problem 3 Find the opposite number. A. -9 B. 62

Solution See page A11. A. 9 B. -62

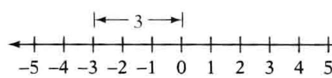
The **absolute value** of a number is its distance from zero on the number line. Therefore, the absolute value of a number is a positive number or zero. The symbol for absolute value is two vertical bars, $|\;|$.

The distance from 0 to 3 is 3. Therefore, the absolute value of 3 is 3.



$$|3| = 3$$

The distance from 0 to -3 is 3. Therefore, the absolute value of -3 is 3.



$$|-3| = 3$$

Absolute Value

The absolute value of a positive number is the number itself. The absolute value of zero is zero. The absolute value of a negative number is the opposite of the negative number.

POINT OF INTEREST

The definition of *absolute value* given in the box is written in what is called rhetorical style. That is, it is written without the use of variables. This is how *all* mathematics was written prior to the Renaissance. During that period from the 14th to the 16th century, the idea of expressing a variable symbolically was developed. In terms of that symbolism, the definition of absolute value is

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

Example 4 Evaluate. A. $|-4|$ B. $-|-10|$

Solution A. $|-4| = 4$

B. $-|-10| = -10$ ▶ The absolute value sign does not affect the negative sign in front of the absolute value sign. You can read $-|-10|$ as "the opposite of the absolute value of negative 10."

Problem 4 Evaluate. A. $|-5|$ B. $-|-9|$

Solution See page A11. A. 5 B. -9

STUDY TIPS

KNOW YOUR INSTRUCTOR'S REQUIREMENTS

To do your best in this course, you must know exactly what your instructor requires. If you don't, you probably will not meet his or her expectations and are not likely to earn a good grade in the course.

Instructors ordinarily explain course requirements during the first few days of class. Course requirements may be stated in a *syllabus,* which is a printed outline of the main topics of the course, or they may be presented orally. When they are listed in a syllabus or on other printed pages, keep them in a safe place. When they are presented orally, make sure to take complete notes. In either case, understand them completely and follow them exactly.

INSTRUCTOR NOTE

The Concept Review feature of this text appears before every exercise set. After presenting the material in the section, you might elect to use these exercises as oral classroom exercises or as a basis for class discussion on the topics of the lesson.

CONCEPT REVIEW 1.1

Determine whether the statement is always true, sometimes true, or never true.

1. The absolute value of a number is positive. Sometimes true
2. The absolute value of a number is negative. Never true
3. If x is an integer, than $|x| < -3$. Never true
4. If x is an integer, then $|x| > -2$. Always true
5. The opposite of a number is a positive number. Sometimes true
6. The set of positive integers is $\{0, 1, 2, 3, 4, \ldots\}$. Never true

7. If a is an integer, then $a \leq a$. Always true

8. If a and b are integers and $a > b$, then $a \geq b$. Always true

9. If x is a negative integer, then $|x| = -x$. Always true

10. If a and b are integers and $a < b$, then $|a| < |b|$. Sometimes true

EXERCISES 1.1

1 Place the correct symbol, $<$ or $>$, between the two numbers.

1. $-2 > -5$	**2.** $-6 < -1$	**3.** $-16 < 1$	**4.** $-2 < 13$
5. $3 > -7$	**6.** $5 > -6$	**7.** $0 > -3$	**8.** $8 > 0$
9. $-42 < 27$	**10.** $-36 < 49$	**11.** $21 > -34$	**12.** $53 > -46$
13. $-27 > -39$	**14.** $-51 < -20$	**15.** $-131 < 101$	**16.** $127 > -150$

Use the roster method to write the set.

17. the natural numbers less than 9
{1, 2, 3, 4, 5, 6, 7, 8}

18. the natural numbers less than or equal to 6
{1, 2, 3, 4, 5, 6}

19. the positive integers less than or equal to 8
{1, 2, 3, 4, 5, 6, 7, 8}

20. the positive integers less than 4
{1, 2, 3}

21. the negative integers greater than -7
{−6, −5, −4, −3, −2, −1}

22. the negative integers greater than or equal to -5 {−5, −4, −3, −2, −1}

Solve.

23. Given $A = \{-7, 0, 2, 5\}$, which elements of set A are greater than 2? 5

24. Given $B = \{-8, 0, 7, 15\}$, which elements of set B are greater than 7? 15

25. Given $D = \{-23, -18, -8, 0\}$, which elements of set D are less than -8? −23, −18

26. Given $C = \{-33, -24, -10, 0\}$, which elements of set C are less than -10? −33, −24

27. Given $E = \{-35, -13, 21, 37\}$, which elements of set E are greater than -10? 21, 37

28. Given $F = \{-27, -14, 14, 27\}$, which elements of set F are greater than -15? −14, 14, 27

29. Given $B = \{-52, -46, 0, 39, 58\}$, which elements of set B are less than or equal to 0? −52, −46, 0

30. Given $A = \{-12, -9, 0, 12, 34\}$, which elements of set A are greater than or equal to 0?
0, 12, 34

31. Given $C = \{-23, -17, 0, 4, 29\}$, which elements of set C are greater than or equal to -17?
−17, 0, 4, 29

32. Given $D = \{-31, -12, 0, 11, 45\}$, which elements of set D are less than or equal to -12?
−31, −12