Kendall Atkinson
Weimin Han

TEXTS IN APPLIED MATHEMATICS 39

Theoretical Numerical Analysis

A Functional Analysis Framework

Third Edition

理论数值分析 第3版

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Dedicated to

DAISY AND CLYDE ATKINSON
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and

DAQING HAN, SUZHEN QIN
HUIDI TANG, ELIZABETH AND MICHAEL

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics* (*TAM*).

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TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences* (*AMS*) series, which will focus on advanced textbooks and research-level monographs.

Pasadena, California — J.E. Marsden
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Preface

This textbook has grown out of a course which we teach periodically at the University of Iowa. We have beginning graduate students in mathematics who wish to work in numerical analysis from a theoretical perspective, and they need a background in those "tools of the trade" which we cover in this text. In the past, such students would ordinarily begin with a one-year course in *real and complex analysis*, followed by a one or two semester course in *functional analysis* and possibly a graduate level course in *ordinary differential equations, partial differential equations*, or *integral equations*. We still expect our students to take most of these standard courses. The course based on this book allows these students to move more rapidly into a research program.

The textbook covers basic results of functional analysis, approximation theory, Fourier analysis and wavelets, calculus and iteration methods for nonlinear equations, finite difference methods, Sobolev spaces and weak formulations of boundary value problems, finite element methods, elliptic variational inequalities and their numerical solution, numerical methods for solving integral equations of the second kind, boundary integral equations for planar regions with a smooth boundary curve, and multivariable polynomial approximations. The presentation of each topic is meant to be an introduction with a certain degree of depth. Comprehensive references on a particular topic are listed at the end of each chapter for further reading and study. For this third edition, we add a chapter on multivariable polynomial approximation and we revise numerous sections from the second edition to varying degrees. A good number of new exercises are included.

The material in the text is presented in a mixed manner. Some topics are treated with complete rigour, whereas others are simply presented without proof and perhaps illustrated (e.g. the principle of uniform boundedness). We have chosen to avoid introducing a formalized framework for *Lebesgue measure and integration* and also for *distribution theory.* Instead we use standard results on the completion of normed spaces and the unique extension of densely defined bounded linear operators. This permits us to introduce the Lebesgue spaces formally and without their concrete realization using measure theory. We describe some of the standard material on measure theory and distribution theory in an intuitive manner, believing this is sufficient for much of the subsequent mathematical development. In addition, we give a number of deeper results without proof, citing the existing literature. Examples of this are the *open mapping theorem, Hahn-Banach theorem, the principle of uniform boundedness*, and a number of the results on *Sobolev spaces.*

The choice of topics has been shaped by our research program and interests at the University of Iowa. These topics are important elsewhere, and we believe this text will be useful to students at other universities as well.

The book is divided into chapters, sections, and subsections as appropriate. Mathematical relations (equalities and inequalities) are numbered by chapter, section and their order of occurrence. For example, (1.2.3) is the third numbered mathematical relation in Section 1.2 of Chapter 1. Definitions, examples, theorems, lemmas, propositions, corollaries and remarks are numbered consecutively within each section, by chapter and section. For example, in Section 1.1, Definition 1.1.1 is followed by an example labeled as Example 1.1.2.

We give exercises at the end of most sections. The exercises are numbered consecutively by chapter and section. At the end of each chapter, we provide some short discussions of the literature, including recommendations for additional reading.

During the preparation of the book, we received helpful suggestions from numerous colleagues and friends. We particularly thank P.G. Ciarlet, William A. Kirk, Wenbin Liu, and David Stewart for the first edition, B. Bialecki, R. Glowinski, and A.J. Meir for the second edition, and Yuan Xu for the third edition. It is a pleasure to acknowledge the skillful editorial assistance from the Series Editor, Achi Dosanjh.

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