The Plastic Methods of Structural Analysis

THIRD (S.I.) EDITION

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Preface

Since the first edition of this book was published in 1956, there has been a widespread acceptance of the concept of limit state design. It is also generally recognized that the appropriate ultimate limit state for many steel frames is plastic collapse, so that the design of such structures is based upon an assessment of the plastic collapse load, an appropriate load factor being provided. Whereas in 1956 the case for the use of the plastic methods had to be argued, this is no longer necessary, and the presentation has accordingly been shortened. The Principle of Virtual Work has been used throughout to unify the treatment.

The book is concerned with the plastic methods of analysis for beams and plane frames, which are based upon the simplifying plastic hinge assumption. It does not discuss the conditions under which members which have entered the plastic range fail by instability. Nor does it deal with other problems of importance in design, such as the behaviour of full-strength welded joints. Nevertheless, the plastic methods, as presented here, can fairly be claimed to be an essential weapon in the armoury of any competent structural designer.

Digital computers are now used extensively to solve structural problems, both of analyis and of design. Some programs which have been developed for frames analyse their behaviour when the simplifying assumptions of the plastic methods are discarded, so that the actual properties of the members are taken into account. Others deal with the optimisation of designs subject to various forms of constraint. These developments have not been dealt with in this edition, although a few are referred to in passing. Only those techniques which are suitable for hand calculation are included; these need to be thoroughly understood as a prelude to the use of computer programs.

Earlier editions of the book contained a comprehensive bibliography. This would now be inappropriate in view of the exclusion of a full discussion of computer-based developments, and so there are few references to the important work of this nature published recently. A selection of references to the classical work which established the basic theory has been retained.

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Contents

1 Basic Hypotheses

1.1 Plastic hinge and plastic collapse concepts

The plastic methods of structural analysis are now widely used in the design of steel frames, which carry load by virtue of the resistance of their members to bending action. Multistorey, multibay rectangular frames and single or multibay pitched-roof portals are familiar examples of this type of structure, and the definition also includes simply supported and continuous beams. For such structures Baker (1949) pointed out that the most economical and rational designs are achieved by the use of the plastic methods. The plastic methods also have the advantage of simplicity.

The objective of the plastic methods is to predict the loads at which a framed structure will fail by the development of excessive deflections. It is appropriate to begin by examining the behaviour of the simplest type of structure in this category, a simply supported beam carrying a central concentrated load. Fig. 1.1 shows the results of an early test carried out by Maier-Leibnitz (1929) on an I-beam spanning 1.6 m. The beam remained elastic up to a load W of about 130 kN, when the yield stress was attained in the most highly stressed fibres beneath the load. At a load of about 150 kN, the central deflection δ began to increase very sharply for small increases in the load. The beam eventually failed catastrophically by buckling at a load of 166 kN, but before then collapse had already effectively occurred due to the development of unacceptably large deflections.

A slight idealization of the behaviour would be to assume that the deflection could grow indefinitely under a *constant* load of 150 kN, as shown by the broken line in the figure. This assumption disregards the small additional load-carrying capacity which the beam actually possesses above this load, and is therefore conservative. The assumed indefinite growth of deflection under constant load is termed *plastic collapse*, and the load 150 kN at which it occurs is the *plastic collapse load*, denoted by W_c.

This behaviour can be described on the hypothesis that a *plastic hinge* develops at the centre of the beam at the load W_c, when the central bending moment is $0.4\ W_c = 60$ kN m. The characteristic of this hinge is that it can only undergo rotation when the bending moment is 60 kN m, but while the bending moment has this value the rotation can increase indefinitely, thus permitting an indefinite growth of deflection. The bending moment required to develop a

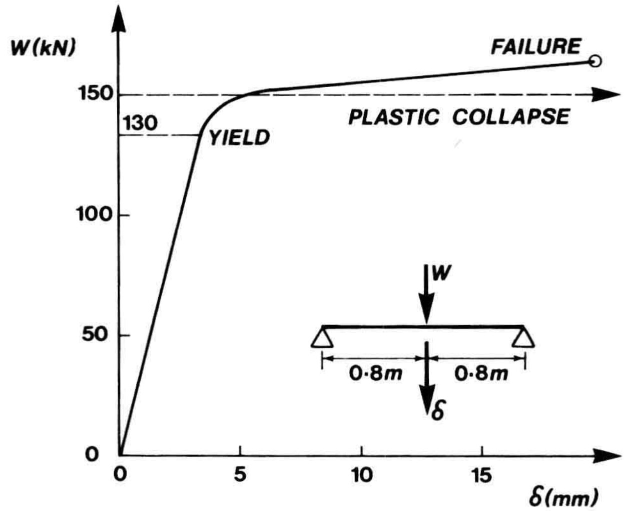

Fig. 1.1 *Test on simply supported beam (after Maier-Leibnitz)*

plastic hinge in this test, 60 kN m, is termed the *plastic moment* of the beam, and is denoted by M_p. It is related to the yield stress of the material, as will be shown in Section 1.3. The plastic methods of analysis, based on the plastic hinge assumption, enable the plastic collapse loads of quite complex frames to be found rapidly, as will be seen in Chapters 3 and 4. Their usefulness as a tool for designing steel frames depends on the fact that large deflections are unlikely to develop before the plastic collapse load is attained. However, it may be necessary to ensure that the deflections developed before collapse are acceptable, and methods for estimating these deflections are discussed in Chapter 5.

The plastic methods should only be used for design if the avoidance of plastic collapse is the governing design criterion. There will be cases in which the primary problem is to avoid other types of failure, for example by fatigue or brittle fracture. These are outside the scope of the simple plastic theory.

It is implicitly assumed throughout that no part of the structure will fail by buckling before the plastic collapse load is reached. The problems of buckling of columns under the conditions actually arising in rigid frames when the members have partially yielded, and of lateral instability and other forms of buckling under similar conditions, have been studied extensively. The pioneering work of J. F. Baker and his associates at Cambridge was presented in *The Steel Skeleton,* vol. 2(1956), and investigations carried out under the direction of Beedle at Lehigh were described in *Plastic Design in Steel* (1971). The present position has been summarized by Horne (1972) and Wood (1972). Rules are available which enable frames to be designed so that failure by certain types of buckling will not occur before the plastic collapse load is attained, but their discussion is outside the scope of this book.

1.2 Stress-strain relation for mild steel

The plastic moment of a steel beam is directly related to the yield stress, as already stated. As a preliminary, it is necessary to review the stress-strain properties of mild steel, the material which is commonly used in the construction of frames.

The relation between direct stress σ and axial strain ϵ for a specimen of annealed mild steel in tension has the typical form shown in Fig. 1.2(a). The relation is linear in the elastic range until the upper yield stress is reached at a. The stress then drops abruptly to the lower yield stress, and the strain then increases at constant stress up to the point b, this behaviour being termed *purely plastic flow.* Beyond b further increases of stress are required to produce further strain increases, and the material is said to be in the *strain-hardening range.* Eventually a maximum stress is reached at c, beyond which the stress decreases due to the formation of a neck in the specimen until rupture occurs at d. The maximum stress is of the order of 400 N/mm^2 and the strain at fracture is of the order of 0.5.

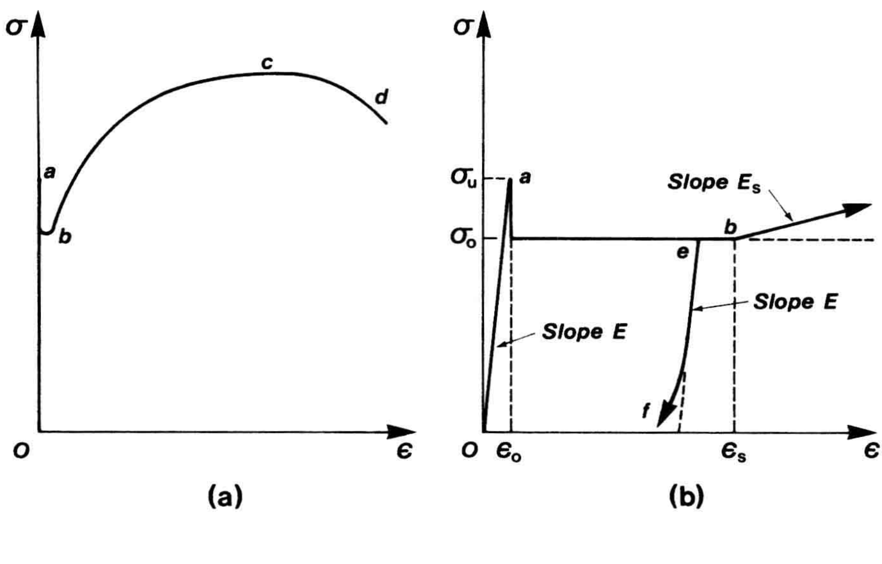

Fig. 1.2 *Stress-strain relation for mild steel in tension*
(a) Behaviour up to rupture
(b) Yield range

The yield range Oab is of the most interest from the point of view of plastic theory. Since the strain at b is generally of the order of 0.01–0.02, the yield range can be examined more conveniently if the strain scale is enlarged, as in Fig. 1.2(b). In this figure the upper and lower yield stresses are defined as σ_u and σ_0, respectively, the slope of the initial elastic line Oa is Young's modulus E, and the slope of the initial portion of the strain-hardening line beyond b is defined as E_s. The strains at the yield point a and at the onset of strain hardening b are defined as ϵ_0 and ϵ_s, respectively. If the stress is reduced after yield a relation such as ef is observed, the initial slope being Young's modulus. The deviation from linearity in such an unloading relation is associated with the Bauschinger (1886) effect.

If the stress is increased again after a reduction of this sort, yield occurs at the lower yield stress along eb. This indicates the effect of cold-working in destroying the upper yield stress, which only reappears after further heat treatment.

The values of the constants defined in Fig. 1.2(b) depend markedly on the composition of the steel and its heat treatment, except for the value of Young's modulus, which shows very little variation. Data derived by Roderick and Heyman (1951) from the results of bending tests on four annealed steels of different carbon content are as shown in Table 1.1.

Table 1.1 *Effect of carbon content on properties of steel*

% C	σ_0 (N/mm^2)	$\frac{\sigma_u}{\sigma_0}$	$\frac{\epsilon_s}{\epsilon_0}$	$\frac{E_s}{E}$
0.28	340	1.33	9.2	0.037
0.49	386	1.28	3.7	0.058
0.74	448	1.19	1.9	0.070
0.89	525	1.04	1.5	0.098

It will be seen that the effect of increasing the carbon content is to increase the lower yield stress σ_0 while decreasing the ductility as measured by the ratio ϵ_s/ϵ_0. For structural steel ϵ_s is of the order 10 ϵ_0, and E_s is of the order 0.04 E, so that the stress-strain relation is very flat after yield.

It is difficult to determine the actual tensile stress-strain relation of mild steel in the elastic range near the yield point, because of unavoidable eccentricities of loading which cause significant bending stresses. However, Morrison (1939) showed that the initial departure from linearity usually observed below the yield point could be ascribed to yielding in the most highly stressed fibres caused by the eccentricity of loading. He therefore concluded that the yield point, proportional limit and elastic limit were all coincident. The tests also showed that the values of the upper yield stress showed no more variation from specimen to specimen of the same material than those of the lower yield stress. The unpredictable variations in the values of the upper yield stress reported by other observers were therefore concluded to be due to variations in the eccentricity of loading. It was also shown that for a given steel the stress-strain relation in compression is practically identical with that for tension up to the point b where strain-hardening begins.

The yield phenomenon for mild steel is accompanied by the formation of Lüders' lines making an angle of about 45° with the axis of the tensile specimen, showing that plastic flow occurs on those planes where the shear stress is greatest. The material within the Lüders' lines has undergone a considerable amount of slip, corresponding to a jump in the strain from a to b in Fig. 1.2(b). The longitudinal strain in a yielded fibre therefore varies discontinuously along the fibre, and a stress-strain relation such as that shown in Fig. 1.2(b) only represents average strains over a finite length.

The stress-strain relation is often idealized by the neglect of strain-hardening and the Bauschinger effect on unloading, leading to the relation shown in Fig. 1.3(a). Although the upper yield effect is a very real one, it disappears on cold-working and is usually not exhibited by the material of rolled steel sections. Moreover, it will be seen in Section 1.3 that it has no effect on the value of the plastic moment. If it is disregarded, the stress-strain relation becomes that of Fig. 1.3(b), which is often termed the *ideal plastic relation.*

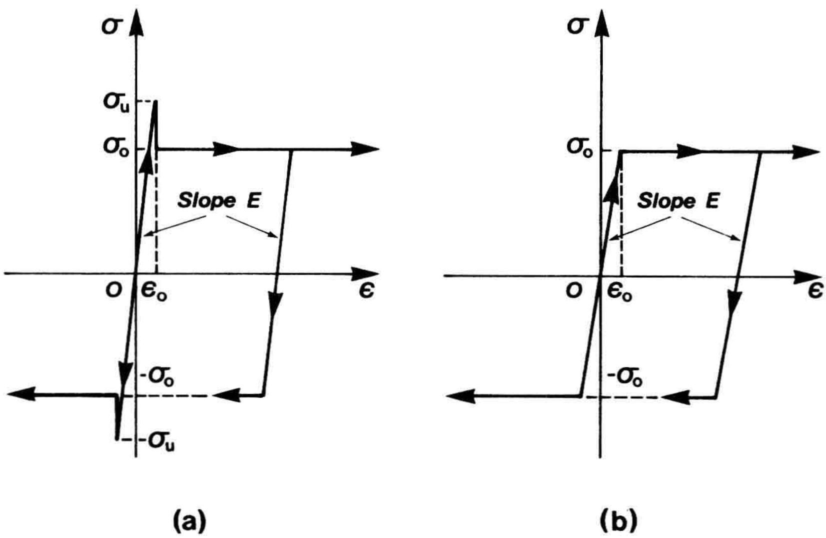

Fig. 1.3 *Stress-strain relations neglecting strain-hardening*

(a) With upper yield stress
(b) Without upper yield stress (ideal plastic)

The neglect of strain-hardening in these idealized relations may seem difficult to justify in view of the fact that the strains will certainly enter the strain-hardening range in many members in actual structures. However, by neglecting the increase of stress during strain-hardening, errors will be introduced which are on the safe side, and it will be seen in Chapter 5 that these errors are usually very small.

1.3 Elastic-plastic bending

For a homogeneous beam of given cross section, the relationship between bending moment and curvature beyond the elastic limit can be derived from the stress-strain relation provided that the usual assumptions of the Bernoulli-Euler theory of bending are made. These are:

(a) The beam is bent by pure terminal couples, so that shear and axial forces are not present.

(b) The deformations are small, so that stresses other than longitudinal normal stresses are negligible.

(c) The relation between longitudinal stress and strain is the same in flexure as in simple tension or compression.

(d) Originally plane cross sections remain plane.

In addition it will be assumed that the stress-strain relation is of the ideal plastic type shown in Fig. 1.3(b), with no upper yield stress. It is further assumed that this relation is obeyed by each individual longitudinal fibre of the beam. In view of the discontinuous nature of the yielding process, this assumption requires experimental verification; several investigators, notably Roderick and Phillipps (1949) have provided evidence in its favour. Finally, it is assumed that there are no residual stresses in the beam. The analysis is simplified considerably if the cross section is symmetrical with respect to an axis which lies in the plane of bending, as happens in many practical cases.

Suppose that the beam is initially straight, and is then bent into an arc of a circle of radius R by pure terminal couples M, say. It is shown in elementary texts on the Strength of Materials that the longitudinal strain ϵ at a distance y from a neutral axis is given by

$$\epsilon = \kappa y \tag{1.1}$$

where $\kappa = 1/R$ is the curvature of the beam. This relation is derived from purely geometrical considerations, and is independent of the properties of the material. If the beam is initially curved, Equation (1.1) is still true provided that κ denotes the change of curvature produced by M.

1.3.1 *Rectangular cross section*

Consider the rectangular cross section of breadth B and depth D which is shown in Fig. 1.4(a), with the bending moment M acting about an axis Ox parallel to the sides of breadth B. In this case the neutral axis will bisect the cross section, because of its double symmetry.

The linear variation of strain across the section implied by Equation (1.1) is shown in Fig. 1.4(b). Here it is supposed that the strain in the outermost fibres exceeds the strain ϵ_0 which corresponds to the yield stress σ_0 (Fig. 1.3(b)). The yield strain ϵ_0 is attained at distances $\pm z$ from the neutral axis. The corresponding distribution of normal stress is shown in Fig. 1.4(c). There is an elastic core of depth $2z$ outside which there are two yielded zones in which the normal stress is of magnitude σ_0.

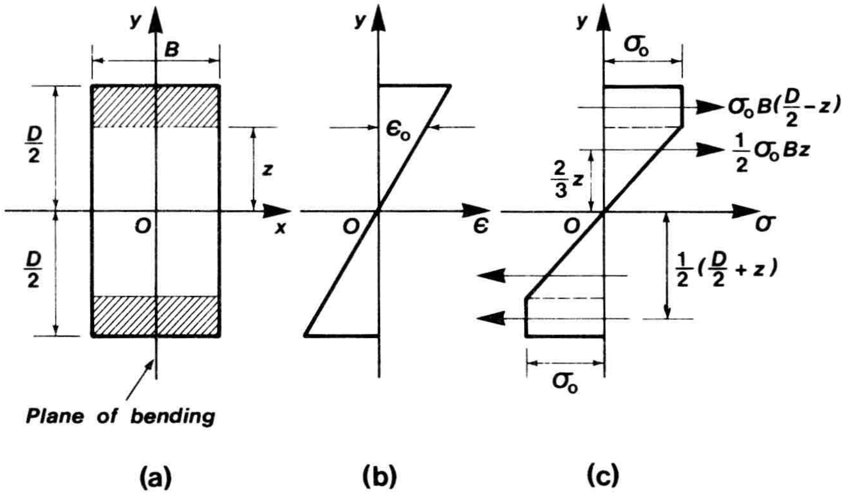

Fig. 1.4 *Elastic-plastic flexure of beam of rectangular cross section*
(a) Cross section
(b) Distribution of strain
(c) Distribution of stress

The bending moment M corresponding to this distribution of stress is readily evaluated. Fig. 1.4(c) shows the resultant normal forces in the two halves of the elastic core and in the yielded zones, and also defines their lines of action. It follows that

$$M = \left(\frac{1}{2}\sigma_0 Bz\right)\frac{4}{3}z + \sigma_0 B\left(\frac{D}{2} - z\right)\left(\frac{D}{2} + z\right) = B\left[\frac{D^2}{4} - \frac{1}{3}z^2\right]\sigma_0. \tag{1.2}$$

The corresponding curvature is obtained from Equation (1.1) by noting that $\epsilon = \epsilon_0$ when $y = z$, so that

$$\kappa = \epsilon_0/z. \tag{1.3}$$

When $z = D/2$, the yielded zones vanish and the stress only just attains the yield value σ_0 in the outermost fibres. The corresponding bending moment M_y is the greatest moment that the section can withstand before yielding. It is termed the *yield moment*; its value is found from Equation (1.2), with $z = D/2$, to be