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Bayesian Methods for Characterizing Geotechnical Model Uncertainty

描述岩土工程模型不确定性特征的贝叶斯方法

Edited by ZHANG Jie
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内 容 提 要

As any model is only an abstraction of the real world, model uncertainty always exists. In geotechnical engineering, the model uncertainty could be large. Lack of knowledge about model uncertainty may lead to unrealistic predictions. When back analysis from observed performances, model uncertainty is often mixed with parameter uncertainty and observational uncertainty. Hence it is generally difficult to isolate and characterize model uncertainty. This book introduces the state-of-the-art theories and methodologies for geotechnical model uncertainty characterization based on the Bayesian theory, including both rigorous solution and approximate but practical solutions, where the effects of parameter uncertainty and observational uncertainty on model uncertainty characterization are appropriately addressed. The theories and methodologies are illustrated in detail with various geotechnical problems. The book will be of general interest to readers in the profession and particularly useful for those specializing in geotechnical inverse analysis and geotechnical reliability.

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Preface

It is widely recognized that the uncertainties involved in geotechnical engineering are much larger than those in other disciplines such as structural engineering. The reliability theory is one of the most effective ways for modeling and assessing the effect of uncertainties in a geotechnical design, which has been the basis for the ongoing revision of many current geotechnical codes in Japan, Europe, Canada, and USA. There are two types of uncertainties in the geotechnical engineering, i.e., the uncertainties associated with input parameters, and the uncertainties associated with calculation models. Fundamental to geotechnical reliability analysis is the knowledge about both parameter and model uncertainties. While the variability of model input parameters have been studied extensively, how to determine the model uncertainty has been considered as difficult for a long time.

Through introducing the state-of-the-art methodologies for geotechnical model uncertainty characterization, this book intends to bridge the gap between the need for model uncertainty characterization and the lack of appropriate methodologies to do so. The key in the methods described in this book is how to consider the effect of parameter uncertainty and observational uncertainty appropriately and realistically during the process of model uncertainty characterization. Both theoretically stringent and computationally efficient methods for model uncertainty characterization are introduced and compared utilizing various types of available observed performance data. The introduced methods are illustrated in detail through a variety of geotechnical examples. It is believed that with the methods and examples introduced in this book characterizing geotechnical model uncertainty in the presence of parameter uncertainty and observational uncertainty is feasible and is no longer difficult.

The intended readers of this book are practicing engineers and researchers dealing risk and reliability issues in geotechnical engineering, planning, management, and decision making.

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Contents

Chapter 1 Introduction

TANG W H, GILBERT R B. Case study of offshore pile system reliability [C]// Proceedings of the 25th Offshore Technology Conference. Houston: Society of Petroleum Engineer, 1993:677 - 686.

1.1 Background

Geotechnical models built on physics and mathematics are widely used in daily geotechnical analysis to predict the behavior of various systems. On one hand, the quality of geotechnical design has been significantly improved by adopting these models. On the other hand, as soil is a type of natural material with highly complicated and varying properties, accurate modeling of soil systems is usually not easy. In developing geotechnical models, idealized assumptions are often made to simplify the underlying physical process. As a result, the predicted geotechnical performance may not be the actual performance. The model uncertainty is defined as the uncertainty in the disparity between model predictions and actual performances excluding the uncertainty in disparity caused by uncertain model input parameters.

Knowledge on uncertainty associated with a geotechnical model is essential for assessing the credibility of geotechnical analyses. If a geotechnical model is unconservative, predictions based on this model could be unconservative even if conservative parameters are adopted in the analysis, and vice versa. Tang and Gilbert (1993) and Lacasse and Nadim (1994) commented that the calculated failure probabilities without considering model uncertainty were not actual failure probabilities of geotechnical systems. Since the existence of model uncertainty could distort the whole decision-making process, it is highly desirable to understand the characteristics of model uncertainty associated with a prediction model (Gilbert and Tang 1995; Whitman 2000; Juang et al. 2004; Nadim 2007).

In theory, one may assess the model uncertainty through a systematic comparison between model predictions and observed system performances. During such comparison, however, the model input parameters such as soil properties may be uncertain due to the spatial variability of soil property, test uncertainty, and insufficient number of tests. Moreover, the observed response may not be the actual system response due to the presence of measurement error. Thus, model uncertainty in geotechnical engineering is often mixed with uncertainty in model input parameter uncertainties and observational errors, making the characterization of model uncertainty generally not easy.

Geotechnical systems can be divided into two broad categories, i.e., scalar prediction models and vector prediction models as defined below. A scalar prediction model refers to a model from which the prediction is a scalar. For instance, the model for predicting the capacity of a shallow foundation (Fig. 1.1) based on plasticity theory of the following form (e.g., Craig 1997) is a scalar prediction model

$$q_u = (N_c c) + (N_q q_o) + \left(N_\gamma \frac{\gamma B}{2}\right) \quad (1.1)$$

where q_u=capacity of a shallow foundation; c=soil cohesion; q_o=surcharge; γ=gravity of the soil; B=width of the strip footing; and N_c, N_q, N_γ=bearing capacity factors for cohesion, surcharge and self weight of the soil, respectively.

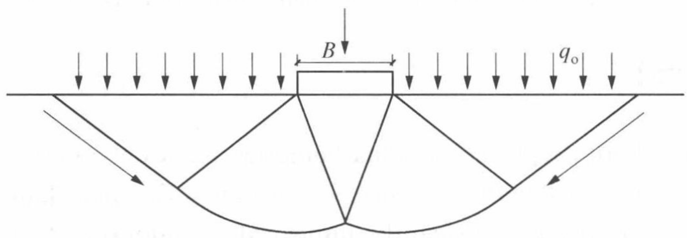

Fig. 1.1 Illustration of scalar prediction model: a shallow foundation capacity model based on plasticity theory

A vector prediction model refers to a model from which the prediction is more than a scalar. An example of the vector prediction model is a finite element model for ground settlement analysis, as shown in Fig. 1.2. This model can predict the settlement at any point within the model domain.

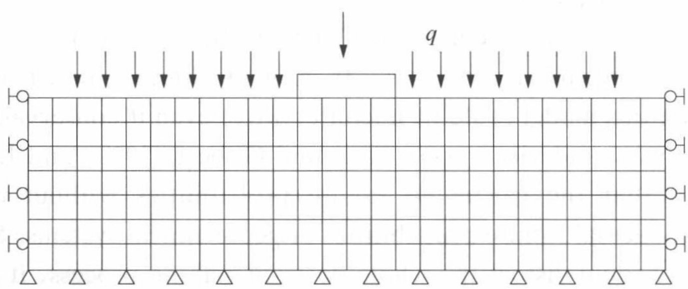

Fig. 1.2 Illustration of vector prediction model: a finite element model for ground deformation analysis

The methods to characterize model uncertainty of the two types of models are different. For a vector prediction model, one can collect a set of observations from the system under study to calibrate the corresponding model. For a scalar prediction model, as the number of observation for each system is only one, the information from a single system is too limited for model uncertainty characterization. In such a case, observed data from many similar systems should be collected such that a systematic comparison between model predictions and observed performances can be made (e.g., Gilbert and Tang 1995; Juang et al. 2004). The characterizations of model uncertainty using data from one system and from multiple systems are called within-system characterization and cross-system characterization, respectively.

The purposes of within-system model uncertainty characterization and cross-system model uncertainty characterization are different. In a within-system model uncertainty characterization problem, the purpose of calibration is to provide better prediction about the current system which provides the calibration data. In a cross-system model uncertainty characterization problem, the purpose is to gain knowledge on the model uncertainty of the prediction model, and to improve prediction for a future system.

Much of the previous research work on model uncertainty characterization in geotechnical engineering belongs to within-system characterization (e. g., Ledesma 1987; Ledesma 1991; Ledesma et al. 1996a, 1996b; Honjo et al. 1994; Honjo and Kudo 1998; Honjo and Kashiwagi 1999; Lee et al. 1997; Lee and Kim 1999; Welker and Gilbert 2003; Cho et al. 2006). A common assumption adopted in such work is that the prediction model is on average unbiased. In geotechnical engineering, however, a prediction model could be biased either towards the conservative side or towards the unsafe side. In the process of within-system model calibration characterization, model uncertainty and model input parameters are usually characterized simultaneously if possible. In such a case, if a prediction model is indeed biased, ignoring this bias in model calibration will essentially lump the model bias into the model input parameters. If the calibrated model uncertainty and model input parameters are used to predict the future performance of the system which provides the calibration data, the possible model bias would have been implicitly considered in the prediction. Therefore, the assumption of no model bias is acceptable in within-system characterization.

The above assumption however cannot be adopted in cross-system model uncertainty characterization problems. The reason is that the purpose of cross-system characterization is to improve the prediction for a new system. During the model characterization process, if one assumes that the prediction model is on average unbiased, the possible bias in the prediction model is lumped into the parameters of the systems which provide calibration data. When the determined model uncertainty is used to predict a future system, the effect of model bias cannot be considered because the parameters of the systems that provide the calibration data are different from those of a new system. The well-established theory for within-system model uncertainty characterization cannot be used for cross-system model uncertainty characterization.

Researchers have conducted studies in cross-system model uncertainty characterization (e.g., Gilbert and Tang 1995; Juang et al. 2004; Zhang 2005; Zhang et al. 2007; Ching et al. 2007). As will be seen later in Chapter 2, although some success was observed, existing methods were usually limited to the characterization of the uncertainty of some specific models and using some specific observational data. Development of general methods for cross-system model uncertainty characterization is still desirable.

1.2 Objective and Scope

The objective of this book is to introduce the state-of-the-art theories, solutions, and examples for cross-system model uncertainty characterization. The problem of within-system model uncertainty characterization is beyond of the scope of this book. The goals set in this book will be achieved through the following tasks:

(1) Critically review previous work on both within and cross-system model uncertainty characterizations as well as the necessary mathematical theory for model uncertainty characterization, so as to identify the advantages and disadvantages of the existing methods, to highlight the characteristics of the cross-system model uncertainty characterization problem, and to provide the background knowledge for cross-system model uncertainty characterization.

(2) Introduce the general framework for cross-system model uncertainty characterization. The framework would be based on the Bayesian theory. The reason to choose a Bayesian approach is that it allows for explicit consideration of uncertainties involved in the model uncertainty characterization process. Besides, the Bayesian method also provides a natural way to incorporate engineering judgment in decision making, which is very important in geotechnical engineering.

(3) Suggest easy-to-apply algorithms that can be used to implement the proposed Bayesian framework efficiently. One practical difficulty to apply Bayesian theory is that the computational work involved is usually large. To apply the proposed Bayesian framework to real problems, computationally effective algorithms are required. Sound approximations may be required to achieve such efficiency.

(4) Check the accuracies of the approximate methods for model uncertainty characterization with rigorous solution. Once approximate methods for model uncertainty characterization are available, an important concern is how accurate these methods are. To answer this question, a rigorous solution for model uncertainty characterization shall be avaible, and the results from the approximate methods shall be checked against those from the rigorous solution.

(5) Illustrate the described Bayesian framework and solution algorithms through a series of application examples in which the model uncertainty is characterized utilizing different types of observed data, and demonstrate how the methods described can help the current geotechnical practice.

1.3 Organization of the Book

There are eight chapters in this book. In the present chapter, the engineering background, the model uncertainty characterization problem, objectives and scope of this book are outlined.

In Chapter 2, a literature review related to model uncertainty characterization is presented. The review consists of three parts, i.e., within-system model uncertainty characterization, cross-system model uncertainty characterization, and computational techniques for implementing the Bayesian method.

In Chapter 3, a Bayesian framework for identifying model uncertainty from parameter and observation uncertainties is presented. In this framework, a model correction factor is used to denote the effect of model uncertainty. The mean and standard deviation of the model uncertainty factor are modeled as random variables, and their distributions are updated simultaneously with other uncertain variables using observed performance data. This Bayesian framework is first introduced for an additive model correction factor with point observations, and is subsequently extended to other cases, including the case of a multiplicative model correction factor, the case of censored observation (censored observation means the actual system response is only known to be larger or smaller than a certain value), and the case of model correction functions. The selection of the use of various model correction factors or model correction functions is also discussed.

In Chapter 4, a first order second moment approximate formulation is introduced to simplify the calculation work involved in the Bayesian formulation presented in Chapter 3. The approximate formulation is first introduced for the additive model correction factor with point observations, and is subsequently extended to more general cases. Two easy-to-apply methods, i.e., a maximum posterior density method and a grid calculation method, are used to solve the approximate formulation. These two methods are illustrated using a slope problem, and the results from the two algorithms are compared.

In Chapter 5, an efficient Markov chain is introduced to identify model uncertainty based on the Bayesian framework presented in Chapter 3. This chapter starts with an analysis of the key factors affecting the efficiency of Markov chain Monte Carlo (MCMC) simulation for model uncertainty characterization. A hybrid Markov chain is then describled to characterize model uncertainty based on the Bayesian framework introduced in Chapter 3 integrating the results from the approximate methods for model uncertainty characterization described in Chapter 4. The accuracies of the two approximate methods presented in Chapter 4 are also checked in this chapter.

In Chapter 6, the model uncertainties of limit equilibrium methods for slope stability analysis are further studied using centrifuge model tests, and the role of model uncertainty in a site-specific back analysis of slope failure is investigated. Based on the findings, two convenient procedures for back analysis of slope failure are proposed. The effect of level of uncertainty in pore water pressure on the back analysis of slope strength parameters is also discussed.

In Chapter 7, the model uncertainty of a pile capacity model is studied where the observed data are partially censored. The methods for model uncertainty characterization are further illustrated and compared in this chapter. The effect of data censoring on model uncertainty characterization is discussed. After the model uncertainty is determined, it is used to develop partial factors for reliability based design of pile foundations. This chapter shows how model uncertainty can be incorporated in a daily design once it is calibrated.

In Chapter 8, the Bayesian framework is applied to determine the model uncertainty of a liquefaction model where the calibration data is fully censored. How to consider the choice-based sampling bias in the observed database is discussed. The effect of model uncertainty in reliability analysis is investigated in this chapter.

Chapter 2 Literature Review

2.1 Within-System Characterization

In a broad sense, within-system model uncertainty characterization can be considered as a special case of model calibration where both uncertain model input parameters and model uncertainty parameters are to be characterized using the observed performance data. Model calibration has been widely used in engineering and science. Despite of numerous researches on model calibration, many of them do not consider the existence of model uncertainty, or the model uncertainty is assumed known even it exists. As the purpose of this book is to estimate model uncertainty, the focus of review is on model calibration methods related to model uncertainty characterization. The methods reviewed here include: ① the least square method, ② the maximum likelihood method, ③ Bayesian method, and ④ the extended Bayesian method.

2.1.1 Least Square Method

Let $g(\boldsymbol{\theta})$ denote a vector prediction model, and $\boldsymbol{\theta}$ denote its unknown parameters. The principle of the least square method is to find an estimate for $\boldsymbol{\theta}$ by minimizing the difference between observed system response and the model prediction. This can be done by minimizing the following misfit criterion (e.g., Beck and Arnold 1977)

$$S(\boldsymbol{\theta}) = [g(\boldsymbol{\theta}) - \boldsymbol{d}]^{\mathrm{T}}\boldsymbol{C}^{-1}[g(\boldsymbol{\theta}) - \boldsymbol{d}] \tag{2.1}$$

in which $\boldsymbol{C} = \sigma^2\boldsymbol{V}$, where σ is unknown and $\boldsymbol{V}$ is a known symmetric matrix denoting disparity between the observed data and model prediction. Both the observational uncertainty in $\boldsymbol{d}$ and the model uncertainty associated with $g(\boldsymbol{\theta})$ could contribute to $\boldsymbol{C}$. If the observation uncertainty in $\boldsymbol{d}$ is negligible compared with model uncertainty associated with $g(\boldsymbol{\theta})$, $\boldsymbol{C}$ is dominated by the covariance matrix of the model uncertainty. In such a case, estimating of σ in Eq. (2.1) provides a chance to estimate model uncertainty through least square analysis. σ can be estimated using the following equation (e.g., Beck and Arnold 1977)

$$\hat{\sigma} \approx \frac{[g(\boldsymbol{\theta}^*) - \boldsymbol{d}]^{\mathrm{T}}\boldsymbol{V}^{-1}[g(\boldsymbol{\theta}^*) - \boldsymbol{d}]}{n - k} \tag{2.2}$$

where $\boldsymbol{\theta}^* = \boldsymbol{\theta}$ that minimizes Eq. (2.1); n = number of observations; and k = dimension of $\boldsymbol{\theta}$.

The least square method can be divided into several types. If $\boldsymbol{V}$ is an identity matrix, it is often called ordinary least square analysis; otherwise it is called weighted least square analysis. If $g(\boldsymbol{\theta})$ is a linear model, it is called linear least square analysis; otherwise it is called nonlinear least square analysis (Beck and Arnold 1977).

Although the least square method was widely used in statistics for regression analysis, calibrating mechanistic models are often much more difficult than performing a regression analysis (e.g., Cooley 1977; Cooley 1979). This is because the mechanistic model $g(\boldsymbol{\theta})$ could be complicated partial differential equations, which may make the minimization process involved computationally challenging. The above statement is also true for other methods reviewed in this chapter for model uncertainty characterization.

Traditionally, the mathematical programming theory was used to optimize Eq. (2.1). Yeh (1986) summarized optimization algorithms used in groundwater flow model calibration and noticed that the modified Gauss-Newton algorithms were most frequently used because they do not require the calculation of Hessian matrix and their convergence rates are superior to gradient based optimization algorithms. In recent years, some newly developed optimization theories were also applied to model calibration problems. For instance, Levasseur et al. (2007) demonstrated the application of genetic algorithm to calibrate a nonlinear finite element model for pit excavation. Meier et al. (2008) applied the particle warm algorithm to calibrate the finite element model for a natural slope.

A problem that can arise in minimizing Eq. (2.1) is that the calibrated results could be unstable (e.g., Shah et al. 1978; Yakowitz and Duckstein 1980). The instability is often associated with the phenomena that a small change in the observed data could result in large change in the model calibration results. If instability indeed occurred, Carrera et al. (2005) summarized four schemes to solve the instability problem:

(1) Regularizing the problem by incorporating prior information in the minimization criteria (e.g., Cooley 1982; Weiss and Smith 1998).

(2) Reducing the number of uncertain variables to calibrate.

(3) Increasing the number and type of data.

(4) Optimizing the observation scheme: observation networks and experiments can be designed to minimize model uncertainty and/or to increase the ability of data to discriminate among alternative models (Knopman and Voss 1989; Usunoff et al. 1992).

Shah et al. (1978) showed that instability was more likely to occur as the number of variables to be calibrated increased while fixing the observed data. This implied that one important cause for instability was that the amount of observed data was not sufficient to calibrate a large number of uncertain variables. The reason for the occurrence of instability in such a case can be explained as follows. In general, both parameter uncertainty and model uncertainty contribute to the prediction uncertainty. As the number of parameters to be calibrated involved in a model increases, while the model has more freedom to fit the data well (i.e., model uncertainty is reduced), the uncertainty involved in the calibrated parameters would also increase. When the number of parameters to be calibrated is too large, the parameter uncertainty would also be very large, thus causing instability to the calibration results. Hence, there shall be an optimum trade-off between model uncertainty and parameter uncertainty, and the number of free parameters involved in a model should be limited. Yeh and Yoon (1976) and Shah et al. (1978) studied the optimum number of free parameters in the framework of least square analysis. Honjo et al. (1994) and Neuman (2003) studied this problem using model selection approaches based on information theory and Bayesian method, respectively. Various model selection approaches will be reviewed in a later section.

A practical difficulty for applying the least square method is that programming work involved in implementing least square method is often extremely heavy (Poeter and Hill 1997) when $g(\boldsymbol{\theta})$ is a complicated numerical model. Some general purpose optimization software has been developed, such as VisualDOC (Vanderplaats 1998), iSIGHT (Engineous 1999), OPTIMUS (LMS 1999), and ModelCenter (Phoenix 1999). By viewing the application of least square method as an optimization problem, such software may potentially used to calibrate geotechnical models. In the groundwater flow modeling field, some inverse software for automated model calibration based on non-linear least square method has also been developed, including MODINV (Doherty 1990), MODFLOWP (Hill 1992), PEST (Doherty 1994) and UCODE (Poeter and Hill 1998). It is likely that such software can also be used to calibrate geotechnical models. In fact, some pioneering work in this direction has been performed (e.g., Calvello 2002; Calvello and Finno 2004; Finno and Calvello 2005; Calvello et al. 2008; Rechea et al. 2008; Hashash and Finno 2008). As an example, Calvello and Finno (2004) calibrated a non-linear finite element model for pit excavation implemented in Plaxis 7.11 using UCODE. Note at present UCODE is free and can be downloaded from http://typhoon.mines.edu/freeware/ucode/. Although such software is developed for calibrating models using the least square method, it is very likely that other methods such as Bayesian methods can also be implemented in such software, thus greatly facilitating the application of these methods.

2.1.2 Maximum Likelihood Method

The basic idea in the maximum likelihood method is to first derive the probability of observing the data in terms of the unknown variables, which is called likelihood function, and then estimating the unknown variables by maximizing the likelihood function (e.g., Azzalini 1996, Ang and Tang 2007). For instance, suppose the model uncertainty can be described by a multivariate normal distribution with a mean of zero and a covariance matrix of $\boldsymbol{C}=\sigma\boldsymbol{V}$, where σ is unknown and $\boldsymbol{V}$ is known. Assuming the observational uncertainty is negligible, the probability to observe $\boldsymbol{d}$ is (e.g., Beck and Arnold 1977)

$$l(\boldsymbol{\theta}, \sigma \mid \boldsymbol{d}) = \frac{1}{(2\pi)^{n/2}|\boldsymbol{C}|^{n/2}}\exp\{[g(\boldsymbol{\theta}) - \boldsymbol{d}]^{\mathrm{T}}\boldsymbol{C}^{-1}[g(\boldsymbol{\theta}) - \boldsymbol{d}]\} \tag{2.3}$$

where n=dimension of $\boldsymbol{d}$.

The logarithm of the likelihood function is

$$\ln l(\boldsymbol{\theta}, \sigma \mid \boldsymbol{d}) = -\frac{1}{2}[g(\boldsymbol{\theta}) - \boldsymbol{d}]^{\mathrm{T}}\boldsymbol{C}^{-1}[g(\boldsymbol{\theta}) - \boldsymbol{d}] - \frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln|\boldsymbol{C}| \tag{2.4}$$

Both $\boldsymbol{\theta}$ and σ can be estimated by maximizing the chance to observe $\boldsymbol{d}$, i.e. Eq. (2.3) or Eq. (2.4). Note the third and forth terms at the right side of Eq. (2.4) do not depend

on $\boldsymbol{\theta}$. Therefore, the maximization of Eq. (2.4) with respect to $\boldsymbol{\theta}$ is equivalent to the minimization of Eq. (2.1) with respect to $\boldsymbol{\theta}$. It can also be shown that the best estimate of σ is the same as Eq. (2.2) (e.g., Beck and Arnold 1977). Therefore, under the assumptions made above, the best estimates for $\boldsymbol{\theta}$ and σ from the maximum likelihood method are the same as that from the least square analysis. If the model uncertainty is not multivariate normal, this conclusion may not be valid. In this sense, the least square method can be viewed as a special case of the maximum likelihood method.

The maximum likelihood method has been widely used for model calibration. In structural engineering, Beck and his coworkers did active research in calibrating structural models using the maximum likelihood method, including the calibration of models of buildings (Beck and Jennings 1980; Werner et al. 1987), a bridge (McVerry and Beck 1983), and an off-shore platform (Mason et al. 1989). The models in the above papers were all linear and the uncertainties in the calibrated parameters were not characterized. The maximum likelihood method was later extended to calibrate nonlinear models (e.g., Jayakumar and Beck 1988), and to consider the uncertainties associated with the calibrated model parameters (e.g., Beck and Katafygiotis 1998a, 1998b). The maximum likelihood method is also suggested to calibrate geotechnical models (e.g., Ledesman et al. 1996a, 1996b; Gens et al. 1996). As an example, Gens et al. (1996) calibrated a tunnel excavation model using the maximum likelihood method.

As the maximum likelihood method depends on the minimizing similar misfit criteria as that in the least square method, problems associated the least square method such as instability also challenge the application of maximum likelihood method. Compared with least square method, however, the maximum likelihood method has several possible advantages. First, the least square method usually can only use uncensored data for model calibration, while the maximum likelihood method can use both censored and uncensored data (Wang 2002). Second, as will be seen later in this review, the results from the maximum likelihood method is closely related to model comparison criteria such that potential models can be ranked easily with respect to their prediction accuracy if the maximum likelihood method is used. Thirdly, as will be seen later, results from the maximum likelihood method can also be used to calculate model weights and model probabilities such that predictions from various models could be combined together for an improved prediction.

2.1.3 Bayesian Method

Bayesian method is a probabilistic approach that can be used to update prior knowledge with observed data in a formal way. In Bayesian method, uncertain variables are modeled as random variables, and their distributions are updated using the observed data. The general Bayesian theory and its computational methods are reviewed in a later section in this chapter. The focus of this section is to review its application in model calibration.

Gavalas et al. (1976) matched the history of a reservoir using the Bayesian method. In Gavalas et al. (1976), the model uncertainty is assumed negligible compared with observational uncertainty. The prior distribution about model input parameters is assumed