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VOLUME 44

Edited by

Hassan Aref
Erik van der Giessen



ADVANCES IN APPLIED MECHANICS

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UNIVERSITY OF GRONINGEN
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Advances in Applied Mechanics

Volume 44

Preface

Although 60 years ago now, there is no relationship between the publication of the first volume of this series and Irwin's seminal contributions to that branch of applied mechanics called "fracture mechanics." Since then, *Advances in Applied Mechanics* has regularly kept the community up to date about developments in this field, while it became the core of design criteria to ensure reliability and safety. Recent progress in mathematical techniques and approaches to incorporate material specificity in the description of damage and fracture warrants a volume that is fully dedicated to fracture: Volume 44. Each of the three contributions, covering both brittle and ductile materials, and under quasi-static or dynamic loading conditions, defines the present state of the art and is expected to serve as reference point for future developments in their area.

All papers fall in the category of so-called "local approaches to fracture," distinct from the global fracture mechanics approach initiated, together with Griffith, by Irwin. If substructuring within this volume is desired, one possibility would be to cluster the papers by Forquin and Hild and by Silling and Lehoucq as dealing with brittle materials, while Benzerga and Leblond dedicate their chapter entirely to materials that fail by a ductile mechanism. Yet, even a glance at the list of contents will reveal the substantial differences in focus and methodology. Notably, Forquin and Hild propose an approach that is probabilistic, in terms of both material properties and distribution of stress and damage (crack density). Their probabilistic theory of fragmentation departs from the idea of a local weakest link and supplements this with the concept of obscuration. The resulting theory is supported by multiscale simulations and by experimental findings for ceramics and concrete.

By contrast, Silling and Lehoucq present a deterministic theory, and more in particular, one that is nonlocal. Peridynamics is a general framework to describe material behavior, which, contrary to standard continuum mechanics, is able to deal with discontinuities. While it is not strictly limited to elastic materials, peridynamics offers the possibility of modeling fracture in brittle solids in a natural manner. The roots of the theory go back to the turn of the century but it is presented here in a novel manner, with due emphasis on the relation to classical continuum theory and with application to dynamic fracture.

The chapter by Benzerga and Leblond provides a comprehensive review of the latest developments in the modeling of ductile failure. It describes various

refinements and extensions of the theory that was presented 20 years ago in this series by Tvergaard (Volume 27). Special attention is given to modeling of the later stages of void growth and coalescence, and its integration in a top-down approach to ductile fracture. In addition to addressing the materials science background to the models, the paper also provides comparisons with experiments on crack initiation and propagation.

While the likelihood that fracture in engineering systems can be avoided completely has not faded since Volume 1, it is advances like those compiled here that will enable reducing it. We hope that this volume will be a source of inspiration for further work to improving systems reliability for future generations.

Erik Van der Giessen

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A Probabilistic Damage Model of the Dynamic Fragmentation Process in Brittle Materials

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Abstract

Dynamic fragmentation is observed in brittle materials such as ceramics, concrete, glass, or rocks submitted to impact or blast loadings. Under such loadings, high-stress-rate tensile fields develop within the target and produce fragmentations characterized by a high density of oriented cracks. To improve industrial processes such as blast loadings in open quarry or ballistic efficiencies of armors or concrete structures against impact loadings, it is essential to understand the main properties of such damage processes (namely, characteristic time of fragmentation, characteristic density, orientation and extension of cracking, ultimate strength) as functions of the loading rate, the size of the structure (or the examination volume), and the failure properties of the brittle material concerned. In the present contribution, the concept of probability of nonobscuration is developed and extended to predict the crack density for any size, shape of the loaded volume, stress gradients, and stress rates. A closed-form solution is used to show how a brittle and random behavior under quasi-static loading becomes deterministic and stress rate dependent with increasing loading rates. Two definitions of the tensile strength of brittle materials are proposed. As shown by Monte Carlo simulations, for brittle materials, the “ultimate macroscopic strength” applies under high loading rate or in a large domain, whereas the “mean obscuration stress” applies under low stress rate or in a small domain. Next, a multiscale model is presented and used to simulate damage processes observed during edge-on impact tests performed on an ultra-high-strength concrete. Finally, the fragmentation properties predicted by modeling of six brittle materials (dense and porous SiC ceramics, a micro-concrete, an ultra-high-strength concrete, a limestone rock, and a soda-lime silicate glass) are compared.

1. Introduction

Fragile, easily breakable, difficult to machine, expansive, weakly tough, “brittle materials” do not have the best reputation in the world of industry—materials science or mechanical engineering. However, under extreme loading situations such as high temperatures, high confining pressures, or dynamic loads, brittle materials may appear as very attractive in comparison with metals, polymers, or some composite materials. Ceramics have been known since antiquity as heat-resistant materials and are used very often under high temperature conditions (Kingerly, Bowen, & Uhlmann, 1976). For example, the tensile or compressive strength of ceramics such as silicon carbide or alumina is virtually constant for

temperatures as high as 1000–1200 K (Lankford, 1981, 1991). Concrete materials are also very popular in civil engineering applications for their low conductivity and residual strength after exposure to high temperatures up to 700–900 K (Arioz, 2007; Ghan, Peng, & Anson, 1999; Saad, Abo-El-Eneini, Hanna, & Kotkakat, 1996; Savva, Manita, & Sideris, 2005; Xiao & König, 2004).

The quasi-static confined strength of brittle materials has been extensively studied during the last three decades. In particular, triaxial compression tests were developed to provide an evaluation of the strength of geomaterials under different confinement pressures. During these tests, a purely hydrostatic pressure is first applied on a cylindrical specimen. Afterwards, an axial compression is added. The strength, in a von Mises sense, is taken as the maximum axial stress on withdrawal of the pressure applied by the confinement fluid. These tests have been carried out for several decades on concretes (Bridgman, 1952; Kotsovos & Newman, 1980; Palaniswamy & Shah, 1974; Zimmerman, 1972), on rocks (Cagnoux & Don, 1994; Hoek & Franklin, 1968) (limestone and quartzite), and on ceramics (Heard & Cline, 1980) (alumina-type ceramics, aluminum nitride, and beryllium or magnesium oxide). All these works show that these materials known for their brittleness or quasi-brittleness under uniaxial compression undergo a change to a more ductile behavior under high pressure with an increase of strength under pressures of about few hundreds of megapascals that may exceed 10 times those under unconfined or nearly unconfined loading.

The increase of strength and ductility with confining pressure is particularly spectacular in brittle materials; the same remark can be made concerning their strain-rate sensitivity. Influence of strain rate on the failure stress of brittle materials has been investigated extensively during the two last decades. For example, quasi-static and dynamic uniaxial compression tests were performed on ordinary or high-strength concretes. A review proposed by Bischoff and Perry (1991) shows that above a strain rate of 10^{-5} s^{-1} , a smooth and linear (in a log–log plot) increase of strength with strain rate is observed up to $20\text{--}30 \text{ s}^{-1}$. This increase of strength is due to the free water within the specimens of concrete since this phenomenon totally disappears with dry concretes (Gary & Klepaczko, 1992; Gary, Klepaczko, Hamelin, & Rossi, 1991). Rossi (1988) showed a significant influence of free water on the toughness of concretes. Therefore, the strain-rate sensitivity is certainly the result of the viscosity of internal fluid that impedes the opening of microcracks (Maugis, 1988; Rossi, 1991). Above the strain-rate level of transition (i.e., $20\text{--}30 \text{ s}^{-1}$ for concretes (Bischoff & Perry, 1991)), the increase of strength in dynamic uniaxial compression is more pronounced. This time free water is not the main reason of the phenomenon since it is also observed in dry concretes (Gary et al., 1991). According to Gorham (1989)

and Weerheijm (1992), this increase of ultimate stress is the consequence of inertia effects that restrain the lateral expansion associated with compression. As reported above, brittle materials are very sensitive to any lateral pressure, and the inertia of the specimen is acting as an artificial confinement that explains the apparent strain-rate sensitivity that is observed in concrete-like materials under such loading rates. A confirmation of this interpretation, namely, no strain-rate sensitivity, was observed in dry concrete in dynamic quasi-oedometric compression tests for which the radial displacement is considerably reduced by a metallic confinement cell that surrounds the concrete specimen (Forquin, 2003; Forquin, Arias, & Zaera, 2007; Forquin, Gary, & Gatuingt, 2008a). Furthermore, in saturated concrete, the apparent strain-rate sensitivity is mainly related to pore pressure and drainage conditions (Forquin, Safa, & Gary, 2009).

As discussed above, the increase of failure stress in concrete samples under dynamic compression is mainly due to free water in the material or inertia effects. However, strain-rate sensitivity of brittle materials is also observed in dynamic tensile loadings for which no inertia confinement occurs. A review of results obtained with concretes was proposed by Klepaczko and Brara (2001). As in uniaxial compression, a low increase of strength with loading rate is observed up to about 1 s^{-1} . Above this transition, a sharp increase of strength is reported. Free water within concrete samples may explain the sensitivity at low strain rates. Toutlemonde (1994) performed direct tensile tests for strain rates in the range of 2.5×10^{-6} – 2.5 s^{-1} . Significant strain-rate sensitivity was observed in wet concretes for a maximum aggregate size ranging from 2 to 10 mm, and for water-to-cement ratios varying from 0.3 to 0.7. Conversely, a very limited influence of loading rate was detected in dry specimens. Spall tests performed by Klepaczko and Brara (2001) on a dried or water-saturated microconcrete revealed a spectacular increase of dynamic tensile strength in the strain-rate range of 20 – 120 s^{-1} . Other results for the “dynamic increase factor” (i.e., ratio between the dynamic and quasi-static strengths) were obtained in the strain-rate range of 10 – 100 s^{-1} by Weerheijm (1992); Wu, Zhang, Huang, and Jin (2005); Schuler, Mayrhofer, and Thoma (2006); Weerheijm and Van Doormaal (2007); and Erzar and Forquin (2009) on ordinary and high-strength concretes.

To summarize, it is puzzling to observe that most brittle materials show attractive properties under extreme conditions such as high temperatures, high pressures, or high loading rates but ironically these materials are not popular in the field of engineering mechanics. The present study is focused on the modeling of the fragmentation process of brittle materials under dynamic tensile loading. In particular, it is shown that when the stress rate is “sufficiently” increased, the well-known brittle and probabilistic behavior of these materials becomes deterministic and is characterized by an increase of ultimate failure stress and strain

with loading rate. Moreover, the ubiquitous weakest link hypothesis (Freudenthal, 1968) does no longer apply and the failure is not controlled by a unique initiation site but is the consequence of a large number of oriented cracks.

In this study, six brittle materials are selected to compare their fragmentation behavior under dynamic tensile loading. The first two materials are a dense SiC ceramic and a porous SiC ceramic (infiltrated or not by an aluminum alloy). The third material is a soda-lime silicate glass. As geomaterials, two concretes are chosen, namely, an ultra-high-strength concrete (Ductal®) and a microconcrete (MB50), and a limestone rock is selected as the last material. The first part gives examples of the use of such brittle materials under impact or blast loadings. Their damage pattern is illustrated by means of macrographs and micrographs. In the second part, based on the weakest link hypothesis, the Weibull model is used to describe single fragmentation in brittle materials. The Weibull parameters and the basic properties of the six reference materials are presented. Results of edge-on impact (EOI) tests performed with each material are shown in the third part. Open or sarcophagus configurations are used to analyze the main properties of fragmentation in the targets (density, orientation, extension of cracking). In the fourth part, based on a concept of “local” weakest link hypothesis, a description of the obscuration phenomenon is proposed for any fragmentation process (single or multiple). The meaning of the probability of obscuration is discussed depending on the applied loading rate and the size of the examination domain. In the last part, the multiscale fragmentation model is used to simulate the damage process during EOI tests. An analytical solution is also used to deduce the fragmentation properties near the surface of targets for the six brittle materials.

2. Damage of brittle materials under impact or blast loadings

Fragmentation processes are commonly observed in targets made of concrete, rock, ceramic, or glass when they are submitted to impact or blast loadings. In this part, four examples of brittle materials under such conditions are detailed. A silicon carbide ceramic is utilized as a front face of multilayered armor. Next, the fragmentation of a limestone rock under blast loading is shown. Ballistic results obtained with an ultra-high-strength concrete (Ductal®) are shown afterwards. This type of concrete combines low porosity levels and high compressive strengths that makes it interesting as a material for protective structures against impact loads. The last brittle material loaded under a projectile impact is a soda-lime silicate glass used in transparent armored windshields.

2.1. FRAGMENTATION OF CERAMIC TILES USED AS FRONT FACE OF MULTILAYERED ARMOR

Bilayered armors using hard materials such as ceramics (e.g., alumina, silicon carbide, quartz) as front plate and ductile materials (e.g., steel, aluminum alloy, composite, or polycarbonate) as backing face have been studied for several years to improve the efficiency of light or medium armors against small-to-medium piercing calibers (e.g., armor piercing (AP) 7.62 mm or AP 12.7 mm (den Reijer, 1991)). The high hardness of ceramic materials favors projectile blunting and/or failure and spreads the kinetic energy on a large surface of the ductile backing. The weight of the armor is then reduced in comparison to those made of steel only (Forquin, Tran, Louvigné, Rota, & Hild, 2003b). Fig. 2.1 shows a block of armor made of three tiles of infiltrated ceramic as front face and of 15 mm-thick aluminum layer as backing face. A steel casing that maintains the three tiles in position during squeeze casting and constrains the ceramic during impact surrounds the ceramic tiles. An AP 12.7 mm projectile traveling at 880 m/s impacted the block (Fig. 2.1). One notes no penetration even though a bending deformation of the target is observed. An intense fragmentation made of dense and oriented microcracks occurred mainly in the first layer (see micrographs of Fig. 2.1). This anisotropic damage is due to high tensile stresses that spread out in the target following the incident compressive wave (Denoual, Cottenot, & Hild, 1996; Forquin et al., 2003a). The projectile was found totally pulverized after impact.

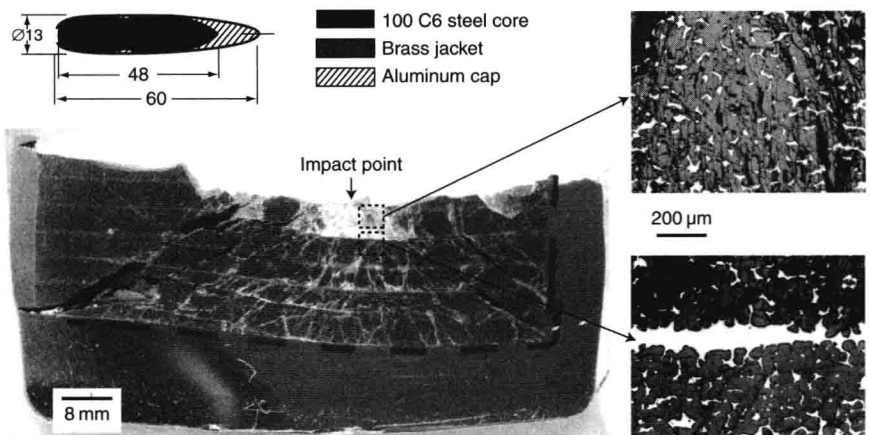


FIG. 2.1 Impact of an armor-piercing projectile AP 12.7 mm traveling at 880 m/s against a multilayered armor made of three infiltrated ceramic tiles (R-SiC-Al) as front face and aluminum alloy as backing. (Forquin, 2003).

During impact, damage in the target and of the projectile develops simultaneously during few microseconds that follow impact. They reduce respectively the strength of the target to be penetrated and the penetrating capability of the projectile. This is why the fragmentation properties of the target (i.e., the characteristic time to damage, the density and orientation of cracking, or the dynamic strength of the material) need to be predicted and modeled, the role of the microstructure of the ceramic tile being an important point to be understood.

2.2. FRAGMENTATION OF A LIMESTONE ROCK SUBMITTED TO BLAST LOADING

Detonating explosives are currently used in geomechanics to blast rocks to build roads, tunnels, bridges, or dams. They are also extensively used in open quarries to produce blocks, aggregates, and gravel. In such situations, fragmentation of rocks needs to be controlled to limit the extension of cracking, to obtain the required size of blocks, and to adjust as well as possible the quantity of explosives, their power, and the number and position of the charges. The distinct zones resulting from rock blasting were identified by Kutter and Fairhurst (1971), namely, a comminuted area (or crushing zone) in the vicinity of the explosive followed by a damaged zone in which dense microcracking is observed and finally a zone where few long cracks develop. Later, it was recognized that inherent flaws are activated, grow, and eventually coalesce to form macrocracks (Shockey, Curran, Seaman, Rosenberg, & Petersen, 1974). For example, the picture of Fig. 2.2 (left) shows the synchronized detonation of eight charges in

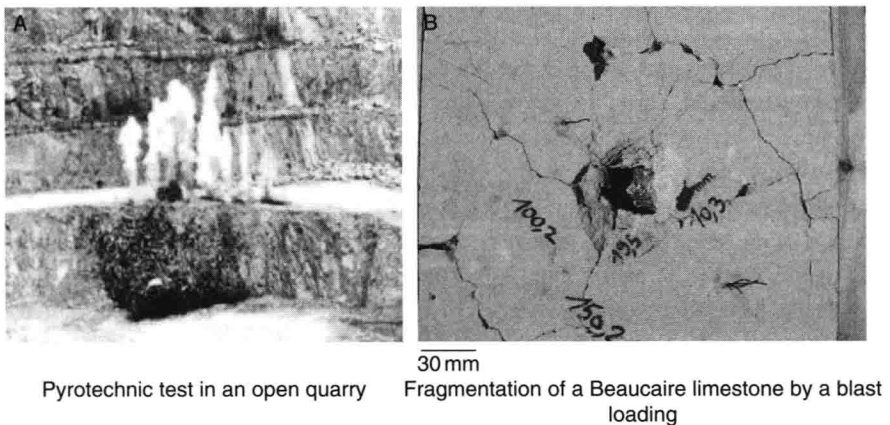


FIG. 2.2 Fragmentation of rock in open quarry (A) and of a slab by blast loading (B) (Hild et al., 2003).

an open quarry. The pressure of gas, the spacing of the charges, and the chronology of the pyrotechnic system directly influence the fragmentation process of the blasted rock. Fig. 2.2 (right) shows the result of a unique blast loading of a Beaucaire limestone slab. The material is reduced in debris in an area close to the explosive. Furthermore, numerous radial and hoop cracks have propagated in the whole block. Compressive damage and compaction in the first zone probably limited the crack density in the volume.

2.3. DAMAGE OF ULTRA-HIGH-STRENGTH CONCRETE DUE TO IMPACT BY A KINETIC STRIKER

During the two last decades, performances of concretes have seen a spectacular and continuous evolution. First, superplasticizers were used in the 1980s to reduce the proportion of water in the mix, keeping at the same time a sufficient malleability. This has led to high-performance concretes with compressive strengths ranging from -40 to -60 MPa, instead of -20 to -30 MPa generally obtained with ordinary concretes. Addition of silica fume in the mix led in the 1990s to very-high-performance concretes (failure stress under simple compression down to -120 MPa (Malier, 1992)). Next, ultra-high-strength concretes (UHSC) also called reactive powder concretes (RPC) were developed during the last decade with failure stresses that may reach at least -200 MPa in simple compression. These concretes benefit from a reduced grain size (maximum size $< 600\ \mu\text{m}$) and an optimal granular skeleton that enables one to homogenize the elastic properties of the grains and the matrix and to decrease internal stresses (Cheyrezy, Maret, & Frouin, 1995; Richard & Cheyrezy, 1995). For example, a micrograph of an UHSC (Ductal®) with no small fibers is shown in Fig. 2.3. Its microstructure is made of fine sand grains whose size is a few hundreds of micrometers, followed by cement grains and crushable quartz grains whose size is in the tens of micrometers. A compact arrangement is obtained by using silica fumes (few micrometer sizes). The amount of water used in the mix [water/(cement + silica fumes) = 0.17 in weight] enables one to minimize the porosity (few percent). In its commercial composition, the Ductal® matrix is reinforced by steel fibers (length: 13 mm, diameter: 0.2 mm, strength: 2400 MPa (Bayard, 2003)).

At the same time, military laboratories have investigated whether the ballistic performances of concretes had grown in proportion to their quasi-static strength. This is why laboratory-scale tests of projectile impact were carried out on ordinary, high-performance concretes (Forrestal, Altman, Cargile, & Hanchak, 1994; Forrestal, Frew, Hanchak, & Brar, 1996; Frew, Hanchak, Green, & Forrestal, 1998; Gomez & Shukla, 2001) and on ultra-high-performance concretes