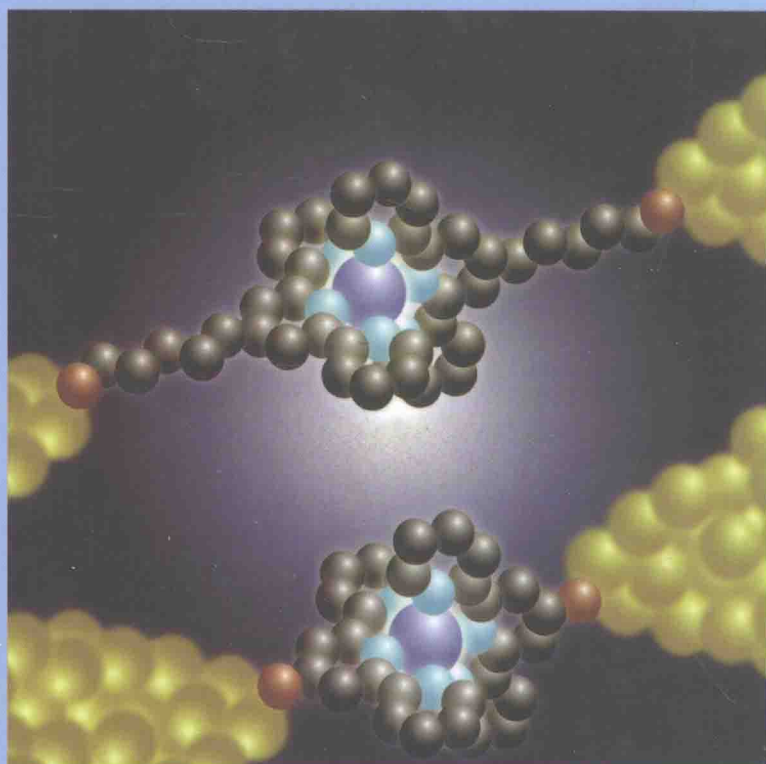


Gerald D. Mahan

Condensed Matter in a Nutshell

凝聚态物质



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Preface

This book is a text for a graduate course in material science, condensed matter physics, or solid state physics. The student is expected to have taken previous courses in quantum mechanics and electromagnetic theory. No prior knowledge or course in condensed matter physics is required.

The earlier chapters introduce basic concepts, such as crystal structures, energy-band theory, phonons, and types of crystal binding. Intermediate chapters discuss the basic features of transport and optical properties of solids. Later chapters discuss current research topics, such as magnetism, superconductivity, and nanoscience. There is an extensive treatment of metals, from the viewpoints of free electrons, tight binding, and strong correlations. There is an extensive discussion of semiconductors, from the viewpoints of both intrinsic, and then extrinsic properties. All chapters except the first have homework problems. These problems have been worked by a generation of students.

I have taught this course many times. The course syllabus in the first semester is fairly standard, and covers the basic material. This course material for the second semester varies from year to year, depending on what topic is “hot” in condensed matter physics. In writing this book, I have included many of the hot topics of the past.

I wish to thank my wife, Sally, for her patience as I wrote still another textbook. I also thank Princeton University Press for encouraging me to finish this project, which was half-done for years. I also thank the Physics Department at Penn State for allowing me to teach this course for several years as I finished the manuscript.

Condensed Matter in a Nutshell

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1

Introduction

The history of material science is closely tied to the availability of materials. Experiments must be done on samples. In the early days of the twentieth century, most of the available materials were found in nature. They were minerals or compounds.

1.1 1900–1910

Scanning the table of contents of the *Physical Review* for the decade 1900–1910, one finds that experiments were done on the following elements and compounds:

- Alkali metals: Na, K, Rb
- Noble metals: Cu, Ag, Au
- Divalent metals: Zn, Cd
- Multivalent metals: Al, Sn, Hg, Bi, Pb
- Transition metals: Ti, Fe, Ni, Mo, Rh, Ta, W, Ir, Pt
- Rare earth metals: Er
- Semiconductors: C, Si, Se, P
- Binary compounds: CaO, MgO, ZnS, HgS, CdS, H₂O, AgCl, AgBr, NaF, NaBr, NaCl, LiCl, KCl, TiCl, TlBr, PbCl₂, PbCl₂, PbI₂
- Oxides: KNO₃, LiNO₃, NaNO₃, AgNO₃, K₂Cr₂O₇, NaClO₃

The binary compounds were identified by their chemical name, such as cadmium sulfide, calcium oxide, or ice.

Table 1.1 A partial list of minerals that were used in experiments reported in the *Physical Review* during the period 1900–1910

Name	Formula
anatase	TiO_x
aragonite	CaCO_3
brookite	TiO_2
calcite	CaCO_3
cinnabar	HgS
eosin	$\text{C}_{20}\text{H}_8\text{Br}_4\text{O}_5$
fluorite	CaF_2
glass	$\alpha\text{-SiO}_2$
pyrites	FeS_2
magnetite	Fe_3O_4
molybdenite	MoS_2
mica	silicates
quartz	SiO_2
sidot blende	ZnS

Among the most interesting materials were minerals. They were usually, and often only, identified by their mineral name. A partial list is given in table 1.1. Several minerals we were unable to identify from their names. The point of this list is that all of these compounds are found in nature as crystals. The samples were not grown in the laboratory, they were found in caves or mines. ZnS was then called sidot blende, but today is called zincblende.

A few materials were actually grown in a laboratory. One was silicon, which was grown in the research laboratory of the General Electric Company. Other artificial materials used in experiments were rubber, brass, asphalt, steel, constantan, and carborundum.

1.2 Crystal Growth

Today nearly all materials used in experiments are either grown in a laboratory or purchased from a company that grew them in a laboratory. The techniques were discovered one by one during the twentieth century. Some notable landmarks:

1. Jan Czochralski [3] invented a method of pulling crystals from their melt in 1917. His apparatus is shown in fig. 1.1. The crystals are pulled vertically, slowly, starting with a small seed crystal. Today the crystal is rotated to ensure that inhomogenieties in the liquid do not make the crystals inhomogeneous. Two-thirds of crystals are grown using the Czochralski method. Large single crystals are prepared this way. For example, silicon crystals used in the manufacturing of integrated circuits are pulled.

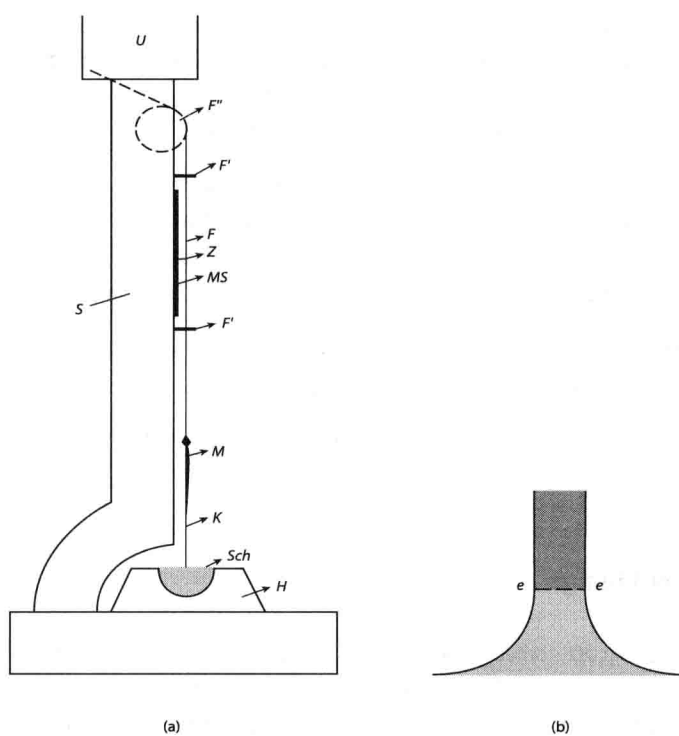


FIGURE I.1. (a) Czochralski apparatus for pulling a crystal from the melt. Melt is at *Sch*, and a tiny seed particle (not shown) is at the end of the silk thread *K*, *F*. (b) Details of region where the crystal grows.

2. Percy Bridgman [2] reported the Bridgman method in 1925. A hollow cylinder is packed with powder or small crystals. It is pulled slowly through a hot region, where the material is melted and recrystallized. Large single crystals can be made this way. The cylinder can be moved vertically or horizontally.

3. William Pfann [4] invented the method of zone refining in 1952, whereby a crystal is pulled through a hot area that locally melts and recrystallizes it. Zone refining generally purifies a crystal, by pushing impurities to the end of the crystal. A crystal may be zone refined several times to obtain a low density of impurities.

4. Large single crystals may be grown from a melt. A superaturated solution of the compound will precipitate the excess material. At the right temperature, it precipitates by growing single crystals. This process happens daily in the author's pantry, as large sugar crystals are grown in the container of maple syrup. This rock candy is a family favorite.

5. Small crystals can be grown in a vapor. The material is inserted into a container, often a glass tube. Then it is heated, so the vapor is supersaturated. At the right temperature, it will grow crystals. This process is slow, but is used for laboratory samples.

The above methods are all traditional, and make three-dimensional, homogeneous samples. Many crystals today are grown using *epitaxy*. Epitaxy is the technique of growing a crystal, layer by layer, on the atomically flat surface of the same, or another, crystal. The atoms are brought to the surface by a variety of methods.

- *Molecular beam epitaxy* (MBE) uses a beam of atoms, or molecules, that are directed toward the surface. John Arthur [1] reported this method in 1968 for growing layers of GaAs. The particle beams originate in a small furnace that creates a vapor of the material, and a hole in the furnace lets atoms out. This process is very slow, but is widely practiced.
- *Chemical vapor deposition* (CVD) uses a vapor of the material in contact with the surface. This method is also called vapor-phase epitaxy (VPE).
- *Liquid phase epitaxy* (LPE) has a liquid of the material in contact with the surface. It is a variation of the solution method mentioned earlier.

1.3 Materials by Design

There are about 92 ~ 100 stable elements in the periodic table. Around 10^4 binary compounds can be formed from pairs of different atoms. Not all pairs form a compound, but many pairs form several different crystals. Putting three elements together has about 10^6 possible compounds, and putting four elements together has about 10^8 possible compounds. The number of new materials that are grown for the first time is thousands each year. Most of these new compounds have rather ordinary properties. However, occasionally one is found that is a high-temperature superconductor, a high-field magnet, or an excellent thermoelectric. Condensed matter physics continues to be an exciting area of research, because new crystals are constantly being discovered. There seems to be no end to this process, since the number of possible new compounds is endless.

An interesting challenge is to try to make this process more efficient. At the moment the scientific community grows thousands of new materials, and a few turn out to be interesting. This process is obviously inefficient. I challenge you, the reader, to find the answer to the following questions:

- What material is the best superconductor? It would have the highest transition temperature T_c to the superconducting phase. Do not tell me the electronic properties or the best density of states. Tell me which atoms are in the crystal, and in what arrangement.
- What material is the best ferromagnet?
- What material has the best magnetoresistance? It is used in computer memories.
- What semiconductor has the highest mobility?
- What material is the best thermoelectric? Typical thermoelectrics have a high figure of merit over a limited ($\Delta T \sim 100^\circ\text{C}$) temperature range. So there are several answers to this question for different temperature regions.