JON ROGAWSKI

CALCULUS

SECOND EDITION

Early Transcendentals

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JON ROGAWSKI

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ALGEBRA

Lines

Slope of the line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

Point-slope equation of line through $P_1 = (x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Point-point equation of line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$y - y_1 = m(x - x_1) \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines of slope m_1 and m_2 are parallel if and only if $m_1 = m_2$.
Lines of slope m_1 and m_2 are perpendicular if and only if $m_1 = -\frac{1}{m_2}$.

Circles

Equation of the circle with center (a, b) and radius r:

$$(x - a)^2 + (y - b)^2 = r^2$$

Distance and Midpoint Formulas

Distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Laws of Exponents

$x^m x^n = x^{m+n}$ $\qquad \frac{x^m}{x^n} = x^{m-n}$ $\qquad (x^m)^n = x^{mn}$

$x^{-n} = \frac{1}{x^n}$ $\qquad (xy)^n = x^n y^n$ $\qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad \sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$ $\qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Special Factorizations

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$
$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot k}$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

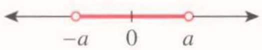
If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

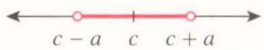
If $a < b$ and $c < 0$, then $ca > cb$.

$|x| = x \quad$ if $x \geq 0$

$|x| = -x \quad$ if $x \leq 0$



$|x| < a$ means $-a < x < a$.



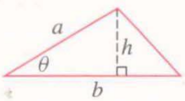
$|x - c| < a$ means $c - a < x < c + a$.

GEOMETRY

Formulas for area A, circumference C, and volume V

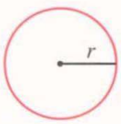
Triangle

$A = \frac{1}{2}bh$
$\quad = \frac{1}{2}ab\sin\theta$



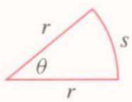
Circle

$A = \pi r^2$
$C = 2\pi r$



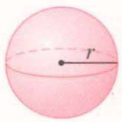
Sector of Circle

$A = \frac{1}{2}r^2\theta$
$s = r\theta$
(θ in radians)



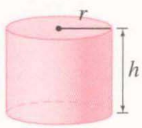
Sphere

$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$



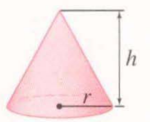
Cylinder

$V = \pi r^2 h$



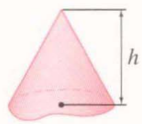
Cone

$V = \frac{1}{3}\pi r^2 h$
$A = \pi r\sqrt{r^2 + h^2}$



Cone with arbitrary base

$V = \frac{1}{3}Ah$
where A is the area of the base



Pythagorean Theorem: For a right triangle with hypotenuse of length c and legs of lengths a and b, $c^2 = a^2 + b^2$.

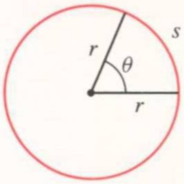
TRIGONOMETRY

Angle Measurement

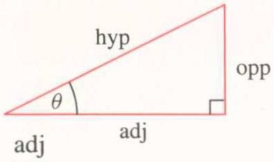
π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta \quad (\theta \text{ in radians})$



Right Triangle Definitions



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\text{adj}}{\text{opp}}$

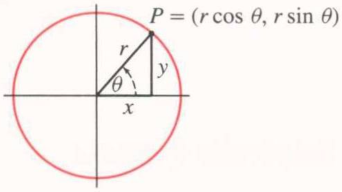
$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$

Trigonometric Functions

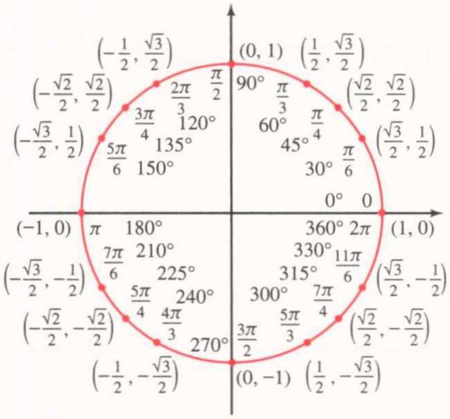
$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



$\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = 0$



Fundamental Identities

$\sin^2\theta + \cos^2\theta = 1 \qquad \sin(-\theta) = -\sin\theta$

$1 + \tan^2\theta = \sec^2\theta \qquad \cos(-\theta) = \cos\theta$

$1 + \cot^2\theta = \csc^2\theta \qquad \tan(-\theta) = -\tan\theta$

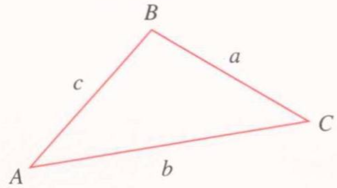
$\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta \qquad \sin(\theta + 2\pi) = \sin\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$

$\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta \qquad \tan(\theta + \pi) = \tan\theta$

The Law of Sines

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-Angle Formulas

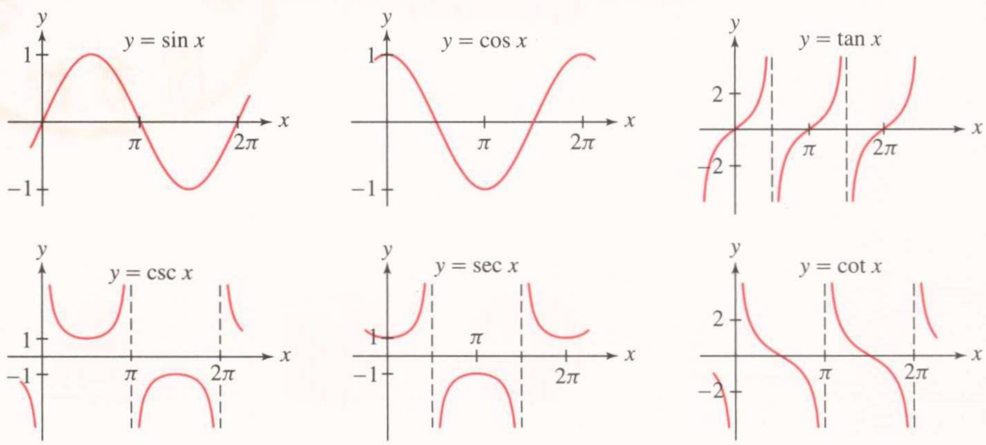
$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin^2 x = \dfrac{1 - \cos 2x}{2} \qquad \cos^2 x = \dfrac{1 + \cos 2x}{2}$

Graphs of Trigonometric Functions



To Julie

ABOUT JON ROGAWSKI

As a successful teacher for more than 30 years, Jon Rogawski has listened to and learned much from his own students. These valuable lessons have made an impact on his thinking, his writing, and his shaping of a calculus text.

Jon Rogawski received his undergraduate and master's degrees in mathematics simultaneously from Yale University, and he earned his PhD in mathematics from Princeton University, where he studied under Robert Langlands. Before joining the Department of Mathematics at UCLA in 1986, where he is currently a full professor, he held teaching and visiting positions at the Institute for Advanced Study, the University of Bonn, and the University of Paris at Jussieu and at Orsay.

Jon's areas of interest are number theory, automorphic forms, and harmonic analysis on semisimple groups. He has published numerous research articles in leading mathematics journals, including the research monograph *Automorphic Representations of Unitary Groups in Three Variables* (Princeton University Press). He is the recipient of a Sloan Fellowship and an editor of the *Pacific Journal of Mathematics* and the *Transactions of the AMS*.

Jon and his wife, Julie, a physician in family practice, have four children. They run a busy household and, whenever possible, enjoy family vacations in the mountains of California. Jon is a passionate classical music lover and plays the violin and classical guitar.

PREFACE

ABOUT *CALCULUS* by Jon Rogawski

On Teaching Mathematics

As a young instructor, I enjoyed teaching but I didn't appreciate how difficult it is to communicate mathematics effectively. Early in my teaching career, I was confronted with a student rebellion when my efforts to explain epsilon-delta proofs were not greeted with the enthusiasm I anticipated. Experiences of this type taught me two basic principles:

1. We should try to teach students as much as possible, but not more.
2. As math teachers, how we say it is as important as what we say.

The formal language of mathematics is intimidating to the uninitiated. By presenting concepts in everyday language, which is more familiar but not less precise, we open the way for students to understand the underlying ideas and integrate them into their way of thinking. Students are then in a better position to appreciate the need for formal definitions and proofs and to grasp their logic.

On Writing a Calculus Text

I began writing *Calculus* with the goal of creating a text in which exposition, graphics, and layout would work together to enhance all facets of a student's calculus experience: mastery of basic skills, conceptual understanding, and an appreciation of the wide range of applications. I also wanted students to be aware, early in the course, of the beauty of the subject and the important role it will play, both in their further studies and in their understanding of the wider world. I paid special attention to the following aspects of the text:

(a) Clear, accessible exposition that anticipates and addresses student difficulties.

(b) Layout and figures that communicate the flow of ideas.

(c) Highlighted features in the text that emphasize concepts and mathematical reasoning: Conceptual Insight, Graphical Insight, Assumptions Matter, Reminder, and Historical Perspective.

(d) A rich collection of examples and exercises of graduated difficulty that teach basic skills, problem-solving techniques, reinforce conceptual understanding, and motivate calculus through interesting applications. Each section also contains exercises that develop additional insights and challenge students to further develop their skills.

Encouraged by the enthusiastic response to the First Edition, I approached the new edition with the aim of further developing these strengths. Every section of text was carefully revised. During the revision process, I paid particular attention to feedback from adopters, reviewers, and students who have used the book. Their insights and creative suggestions brought numerous improvements to the text.

Calculus has a deservedly central role in higher education. It is not only the key to the full range of quantitative disciplines; it is also a crucial component in a student's intellectual development. I hope this new edition will continue to play a role in opening up for students the multifaceted world of calculus.

My textbook follows a largely traditional organization, with a few exceptions. One such exception is the placement of Taylor polynomials in Chapter 8.

Placement of Taylor Polynomials

Taylor polynomials appear in Chapter 8, before infinite series in Chapter 10. My goal is to present Taylor polynomials as a natural extension of the linear approximation. When I teach infinite series, the primary focus is on convergence, a topic that many students find challenging. After studying the basic convergence tests and convergence of power series, students are ready to tackle the issues involved in representing a function by its Taylor series. They can then rely on their previous work with Taylor polynomials and the Error Bound from Chapter 8. However, the section on Taylor polynomials is designed so that you can cover it together with the material on power series and Taylor series in Chapter 10 if you prefer this order.

CAREFUL, PRECISE DEVELOPMENT

W. H. Freeman is committed to high quality and precise textbooks and supplements. From this project's inception and throughout its development and production, quality and precision have been given significant priority. We have in place unparalleled procedures to ensure the accuracy of all facets of the text:

- Exercises and Examples
- Exposition
- Figures
- Editing
- Composition

Together, these procedures far exceed prior industry standards to safeguard the quality and precision of a calculus textbook.

New to the Second Edition

Enhanced Exercise Sets—with Approximately 25% New and Revised Problems: Exercise sets have undergone meticulous reviewing by users and nonusers to refine this very strong feature of Rogawski. Each exercise was worked and evaluated, and carefully revised by the author to further enhance quality and quantity. The Second Edition features thousands of new and updated problems.

New and Larger Variety of Applications: To show how calculus directly relates to the real world, the Second Edition features many fresh and creative examples and exercises centered on innovative, contemporary applications from engineering, the life sciences, physical sciences, business, economics, medicine, and the social sciences.

Updated Art Program: Through the text, there are new and updated figures to enhance the graphics and labeling and link the art with the exposition and student understanding.

Content Changes: Rogawski's Second Edition includes several content changes in response to feedback from users and reviewers. The key changes include:

- Chapter 2 Limits: The topic "Limits at Infinity" has been moved forward from Chapter 4 to Section 2.7 so all types of limits are introduced together.
- Chapter 3 Differentiation: Coverage of differentials has been enhanced.
- Early Transcendentals Chapter 4 Applications of the Derivative: L'Hôpital's Rule (Section 4.5) has been moved up so that it can be used in Section 4.6 on graph sketching.
- The section on "Numerical Integration" has been moved to the end of the Techniques of Integration chapter so that all the techniques of integration appear first.
- A new section on "Probability and Integration," now in the Techniques of Integration chapter, has been added to allow students to explore a new application of integration

which is of importance in the physical sciences, as well as in business and the social sciences.

- Multivariable chapters: Currently recognized as especially strong material in Rogawski's *Calculus*, the multivariable chapters have been refined in minor ways for even greater clarity, accuracy, and precision.
- A new section on "Applications of Multiple Integrals" has been added to the chapter on Multiple Integration to provide an enhanced selection of applied problems from the physical and social sciences.

SUPPLEMENTS

For Instructors

- Instructor's Solutions Manual
  Brian Bradie, Christopher Newport University; and Greg Dresden, Washington and Lee University
  Single Variable ISBN: 1-4292-5502-1
  Multivariable ISBN: 1-4292-5501-3
  Contains worked-out solutions to all exercises in the text.

- Test Bank
  Printed, ISBN: 1-4292-5507-2
  CD-ROM, ISBN: 1-4292-5505-6
  Includes multiple-choice and short-answer test items.

- Instructor's Resource Manual
  ISBN: 1-4292-5504-8
  Provides suggested class time, key points, lecture material, discussion topics, class activities, worksheets, and group projects corresponding to each section of the text.

- Instructor's Resource CD-ROM
  ISBN: 1-4292-5503-X
  Search and export all resources by key term or chapter. Includes text images, Instructor's Solutions Manual, Instructor's Resource Manual, and Test Bank.

For Students

- Student Solutions Manual
  Brian Bradie, Christopher Newport University; and Greg Dresden, Washington and Lee University
  Single Variable ISBN: 1-4292-5500-5
  Multivariable ISBN: 1-4292-5508-0
  Offers worked-out solutions to all odd-numbered exercises in the text.

- Software Manuals
  Software manuals covering Maple and Mathematica are offered within CalcPortal. These manuals are available in printed versions through custom publishing. They serve as basic introductions to popular mathematical software options and guides for their use with *Calculus*, Second Edition.

- Companion website at **www.whfreeman.com/rogawski2e**

MEDIA

Online Homework Options

WebAssign Premium

http://www.webassign.net/whfreeman

W. H. Freeman has partnered with WebAssign to provide a powerful, convenient online homework option, making it easy to assign algorithmically generated homework and quizzes for Rogawski's *Calculus*, Second Edition. WebAssign Premium for the new edition of *Calculus* offers thousands of exercises, plus tutorial videos. It will also be available with a full eBook option.

CALCPORTAL

www.yourcalcportal.com

CalcPortal combines a fully customizable eBook with exceptional student and instructor resources, including precalculus diagnostic quizzes, interactive applets, student solutions, review questions, and homework management tools, all in one affordable, easy-to-use, and fully customizable learning space. This new iteration of CalcPortal for *Calculus*, Second Edition, represents a dramatic step forward for online teaching and learning, with innovations that make it both more powerful and easier to use. It will include a turnkey solution with a prebuilt complete course, featuring ready-made assignments for you to use as is or modify.

WeBWorK

http://webwork.maa.org

Developed by the University of Rochester, this open-source homework system is available to students free of charge. For adopters of *Calculus*, Second Edition, W. H. Freeman will increase the current first edition offering to include approximately 2400 algorithmically generated questions with full solutions from the text, plus access to a shared national library test bank with thousands of additional questions, including 1500 problem sets correlated to the table of contents.

ADDITIONAL MEDIA

Freeman SolutionMaster

SolutionMaster

SolutionMaster is an innovative new digital tool to help instructors provide selected, secure solutions to their students. With SolutionMaster, instructors can easily create solutions for any assignment from the textbook

Interactive eBook

The Interactive eBook integrates a complete and customizable online version of the text with its media resources. Students can quickly search the text, and they can personalize the eBook just as they would the print version, with highlighting, bookmarking, and note-taking features. Instructors can add, hide, and reorder content, integrate their own material, and highlight key text.

dynamicbooks.

Dynamic Book

Rogawski's *Calculus*, Second Edition, is available as an innovative, customizable, and editable DynamicBook eBook. In DynamicBooks an instructor can easily customize the text presentation by adding, hiding, and modifying content to meet their specific teaching approach to calculus. In addition to highlighting and adding notes, students can link to interactive graphical applets, videos, and other digital assets. Rogawski's DynamicBook can be viewed online, downloaded to a local computer, and downloaded to an iPhone or iPad. Students also have the option to purchase a printed, bound version with the instructor's changes included.

FEATURES

Conceptual Insights encourage students to develop a conceptual understanding of calculus by explaining important ideas clearly but informally.

CONCEPTUAL INSIGHT Leibniz notation is widely used for several reasons. First, it reminds us that the derivative df/dx, although not itself a ratio, is in fact a *limit* of ratios $\Delta f/\Delta x$. Second, the notation specifies the independent variable. This is useful when variables other than x are used. For example, if the independent variable is t, we write df/dt. Third, we often think of d/dx as an "operator" that performs differentiation on functions. In other words, we apply the operator d/dx to f to obtain the derivative df/dx. We will see other advantages of Leibniz notation when we discuss the Chain Rule in Section 3.7.

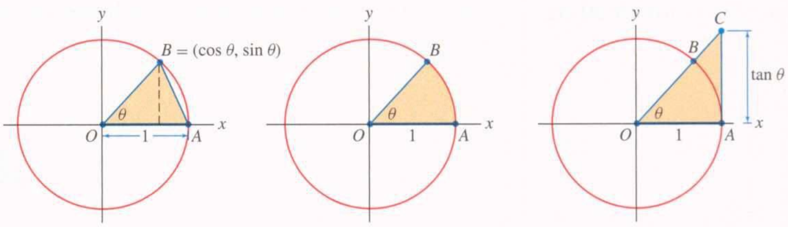
Ch. 3, p. 13

Graphical Insights enhance students' visual understanding by making the crucial connections between graphical properties and the underlying concepts.

GRAPHICAL INSIGHT Keep the graphical interpretation of limits in mind. In Figure 4(A), $f(x)$ approaches L as $x \to c$ because for any $\epsilon > 0$, we can make the gap less than ϵ by taking δ sufficiently small. By contrast, the function in Figure 4(B) has a jump discontinuity at $x = c$. The gap cannot be made small, no matter how small δ is taken. Therefore, the limit does not exist.

Ch. 2, p. 11

Reminders are margin notes that link the current discussion to important concepts introduced earlier in the text to give students a quick review and make connections with related ideas.



Area of triangle $= \frac{1}{2}\sin\theta$ Area of sector $= \frac{1}{2}\theta$ Area of triangle $= \frac{1}{2}\tan\theta$

FIGURE 5

Proof Assume first that $0 < \theta < \frac{\pi}{2}$. Our proof is based on the following relation between the areas in Figure 5:

$$\text{Area of } \triangle OAB < \text{area of sector } BOA < \text{area of } \triangle OAC \qquad (2)$$

Let's compute these three areas. First, $\triangle OAB$ has base 1 and height $\sin\theta$, so its area is $\frac{1}{2}\sin\theta$. Next, recall that a sector of angle θ has area $\frac{1}{2}\theta$. Finally, to compute the area of $\triangle OAC$, we observe that

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AC}{OA} = \frac{AC}{1} = AC$$

Thus, $\triangle OAC$ has base 1, height $\tan\theta$, and area $\frac{1}{2}\tan\theta$. We have shown, therefore, that

$$\underbrace{\frac{1}{2}\sin\theta}_{\text{Area } \triangle OAB} \le \underbrace{\frac{1}{2}\theta}_{\text{Area of sector}} \le \underbrace{\frac{1}{2}\frac{\sin\theta}{\cos\theta}}_{\text{Area } \triangle OAC} \qquad (3)$$

The first inequality yields $\sin\theta \le \theta$, and because $\theta > 0$, we obtain

$$\frac{\sin\theta}{\theta} \le 1 \qquad (4)$$

← **REMINDER** *Let's recall why a sector of angle θ in a circle of radius r has area $\frac{1}{2}r^2\theta$. A sector of angle θ represents a fraction $\frac{\theta}{2\pi}$ of the entire circle. The circle has area πr^2, so the sector has area $\left(\frac{\theta}{2\pi}\right)\pi r^2 = \frac{1}{2}r^2\theta$. In the unit circle ($r = 1$), the sector has area $\frac{1}{2}\theta$.*

Note: Our proof of Theorem 3 uses the formula $\frac{1}{2}\theta$ for the area of a sector, but this formula is based on the formula πr^2 for the area of a circle, a complete proof of which requires integral calculus.

Ch. 2, p. 9

Caution Notes warn students of common pitfalls they may encounter in understanding the material.

*CAUTION The Power Rule applies only to the power functions $y = x^n$. It does not apply to exponential functions such as $y = 2^x$. The derivative of $y = 2^x$ **is not** $x2^{x-1}$. We will study the derivatives of exponential functions later in this section.*

We make a few remarks before proceeding:

- It may be helpful to remember the Power Rule in words: To differentiate x^n, "bring down the exponent and subtract one (from the exponent)."

$$\frac{d}{dx}x^{\text{exponent}} = (\text{exponent})\, x^{\text{exponent}-1}$$

- The Power Rule is valid for all exponents, whether negative, fractional, or irrational:

$$\frac{d}{dx}x^{-3/5} = -\frac{3}{5}x^{-8/5}, \qquad \frac{d}{dx}x^{\sqrt{2}} = \sqrt{2}\,x^{\sqrt{2}-1}$$

Ch. 3, p. 131

Historical Perspectives are brief vignettes that place key discoveries and conceptual advances in their historical context. They give students a glimpse into some of the accomplishments of great mathematicians and an appreciation for their significance.

This statue of Isaac Newton in Cambridge University was described in *The Prelude*, a poem by William Wordsworth (1770–1850):

"Newton with his prism and silent face,
The marble index of a mind for ever
Voyaging through strange seas of Thought, alone."

HISTORICAL PERSPECTIVE

Philosophy is written in this grand book—I mean the universe—which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language … in which it is written. It is written in the language of mathematics …

—GALILEO GALILEI, 1623

The scientific revolution of the sixteenth and seventeenth centuries reached its high point in the work of Isaac Newton (1643–1727), who was the first scientist to show that the physical world, despite its complexity and diversity, is governed by a small number of universal laws. One of Newton's great insights was that the universal laws are dynamical, describing how the world changes over time in response to forces, rather than how the world actually is at any given moment in time. These laws are expressed best in the language of calculus, which is the mathematics of change.

More than 50 years before the work of Newton, the astronomer Johannes Kepler (1571–1630) discovered his three laws of planetary motion, the most famous of which states that the path of a planet around the sun is an ellipse. Kepler arrived at these laws through a painstaking analysis of astronomical data, but he could not explain why they were true. According to Newton, the motion of any object—planet or pebble—is determined by the forces acting on it. The planets, if left undisturbed, would travel in straight lines. Since their paths are elliptical, some force—in this case, the gravitational force of the sun—must be acting to make them change direction continuously. In his magnum opus *Principia Mathematica*, published in 1687, Newton proved that Kepler's laws follow from Newton's own universal laws of motion and gravity.

For these discoveries, Newton gained widespread fame in his lifetime. His fame continued to increase after his death, assuming a nearly mythic dimension, and his ideas had a profound influence, not only in science but also in the arts and literature, as expressed in the epitaph by British poet Alexander Pope: "Nature and Nature's Laws lay hid in Night. God said, *Let Newton be!* and all was Light."

Ch. 2, p. 60

Assumptions Matter uses short explanations and well-chosen counterexamples to help students appreciate why hypotheses are needed in theorems.

Section Summaries summarize a section's key points in a concise and useful way and emphasize for students what is most important in each section.

Section Exercise Sets offer a comprehensive set of exercises closely coordinated with the text. These exercises vary in difficulty from routine, to moderate, to more challenging. Also included are icons indicating problems that require the student to give a written response or require the use of technology T.

Chapter Review Exercises offer a comprehensive set of exercises closely coordinated with the chapter material to provide additional problems for self-study or assignments.

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Kyle Pula, *University of Denver*; Jon Von Stroh, *University of Denver*; Sharon Butz, *University of Denver*; Daniel Daly, *University of Denver*; Tracy Lawrence, *Arapahoe Community College*; Shawna Mahan, *University of Colorado Denver*; Adam Norris, *University of Colorado at Boulder*; Anca Radulescu, *University of Colorado at Boulder*; Mike Kawai, *University of Colorado Denver*; Janet Barnett, *Colorado State University–Pueblo*; Byron Hurley, *Colorado State University–Pueblo*; Jonathan Portiz, *Colorado State University–Pueblo*; Bill Emerson, *Metropolitan State College of Denver*; Suzanne Caulk, *Regis University*; Anton Dzhamay, *University of Northern Colorado* **CONNECTICUT** Jeffrey McGowan, *Central Connecticut State University*; Ivan Gotchev, *Central Connecticut State University*; Charles Waiveris, *Central Connecticut State University*; Christopher Hammond, *Connecticut College*; Kim Ward, *Eastern Connecticut State University*; Joan W. 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