GEOMETRY

Moise / Downs

Teacher's Edition

TEACHERS' EDITION

GEOMETRY

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PREFACE

The *Teachers' Edition* provides a section-by-section commentary on the student text and includes: reproduction of each page of the student text; a general introduction to each chapter; an outline of the chapter to aid teacher lesson planning; mathematical discussions of most sections of the text; proofs of some theorems which were considered too difficult for the student text; suggestions, based on our own classroom experience, on how to teach particular topics and how to handle particular problem sets; quizzes for individual sections; answers to many of the problems; and suggested test questions (in the front of the book). The pages of the student text have been annotated with answers to short-answer and numerical problems. In addition there is a Solution Section at the end of the *Teachers' Edition* which contains all the solutions not given either in the annotations or in the margins of the *Teachers' Edition.* It also contains expansions of some of the answers and discussions of some of the more difficult, interesting problems.

Most of the remarks in the *Teachers' Edition* are intended to furnish practical help in teaching the course. Some of them give the reasons for various novel features of the text. The intent is to make the *Teachers' Edition* complete, in its essentials. At some points, for further explanations of the mathematical background of the course, references are made to *Elementary Geometry from an Advanced Standpoint,* by Edwin Moise (Addison-Wesley, 1963).

GENERAL INTRODUCTION

As the Preface and the Table of Contents in the text indicate, there is nothing very novel in the choice of material in this book. It is devoted mainly to plane geometry, with shorter introductions to solid geometry, solid mensuration, and plane coordinate geometry.

In its style, however, the book is much less conventional. The reasons for this are explained, as we go along, in the appropriate sections of this *Teachers' Edition.* But the spirit in which the book is written may be clearer if we explain, at the outset, certain general considerations.

A. THE METRIC POSTULATES

Probably the most novel feature of this book, at first glance, is the postulates. In most elementary books on geometry, the postulates are modifications of those of Euclid. ("When equals are added to equals, the sums are equal," and so on.) By contrast, the postulates used here are the *metric* postulates; they involve the idea of measurement, by real numbers, of distances, and angles, and areas.

One reason commonly given for this change is that it permits a higher level of logical rigor. This is true, and it is important; but in the opinion of the authors, it is not the most important virtue of the metric postulates.

An examination of almost any geometry text in common use will show that the idea of measurement plays a central role. Sides of triangles are marked with numbers, indicating their lengths; angles are marked with numbers, indicating their degree measures; and almost every day the student is asked to solve problems whose answers are numerical. This means that in fact and practice the theory that is being taught is a metric theory, in which there really are such things as the length of a segment and the measure of an angle.

To present one set of ideas in postulates and theorems, undertaking at the same time to teach another set of ideas by extended examples, necessarily involves a certain expository awkwardness. And it has a further weakness which may be less obvious. One of the hazards, in teaching young students, is that so often they regard the theory being presented to them merely as a gesture; they take for granted that their real job is to learn to do each day's homework assignment by acquiring, in the previous class session, the appropriate set of behavior patterns. When this happens, the theory is not serving its purpose, either in communicating skills or in preparing the student for later courses which are supposed to build on it.

To avoid this sort of thing is no easy task. It requires, obviously, that the exposition of the theory be intelligible. It requires also that the ideas presented in the theory be the ideas that the student is going to *use* when he solves problems. This, we believe, is the crucial reason for the use of metric postulates.

The metric theory has other important advantages:

(1) In earlier courses (and in everyday life) measurement is commonplace. Usually the concept of the number line is used, so that geometric intuition can contribute to the understanding of arithmetic and algebra. Thus it is natural, in the introduction to geometry, to use the Ruler Postulate (pages 37–38).

We are saying, in effect, that *any* line can be regarded as a number line. Thus we appeal to the student's earlier experience, and we bring arithmetic and algebra to the aid of formal geometry.

(2) Metric geometry is a better preparation for the courses which will be studied in the following few years. Analytic geometry is automatically metric; and when geometry is used as an intuitive model in the study of advanced algebra and calculus, it appears in a metric form. These considerations apply with special force to the ideas of area and volume. (See the commentary on Chapter 11.)

(3) Geometry is such a complicated subject that any teachable presentation of it in high school must leave logical gaps. In a way it is an evasion of this issue to "presuppose" an understanding of the real numbers, because in fact the student does not have, at this stage, a very thorough understanding of them. But under this scheme, the gaps that we leave are smaller, and they are of such a kind that they will be filled when the student, in later studies, learns more about the number system. (The gaps in a synthetic treatment are such that they would be filled only by a thorough course in the foundations of geometry, which would come many years later, if at all.)

B. MATHEMATICAL DESCRIPTIONS AND MODELS

It is possible, of course, to think of geometry as an abstract deductive system. But it is also possible—and, at this stage, it is far more important—to think of it as a *concrete* deductive system; that is, as a mathematical model of the physical world, studied by deductive methods.

The standard procedure, in applications of mathematics, is to examine a physical situation and observe that some of its essential features can be described by mathematical conditions. We then pursue, by mathematical methods, the consequences of these conditions, and eventually compare our results with further observations of the physical situation with which we began. Probably the greatest triumph of this method was in the work of Sir Isaac Newton. Newton began with a simple mathematical assumption, the inverse square law of gravitational attraction. He pursued, by mathematical methods, the consequences of this law, and found that the results agreed with the actual behavior of the solar system, with an error so small that even today it is hard to detect and to measure.

The prototype of this method is the deductive geometry developed by the Greeks. This is a description of physical space; and it should still be so regarded even now that we know that the Euclidean description is not the only possible one, and is not necessarily exact. For this reason, geometry offers an excellent opportunity to teach the relation between definitions and the ideas that they are supposed to describe. Nearly all of the ideas of geometry are highly intuitive, and can be conveyed by pictures. Nearly all of them are conveyed first in this way, throughout this book. We then give a definition and invite the student to compare the pictures with the definitions, to make sure that the definition says what it ought to say.

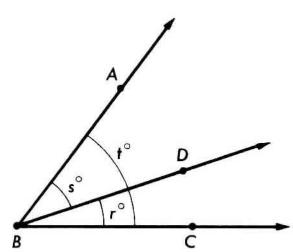
This style of presentation surely makes the definitions easier to understand and to remember, but this is not its sole purpose. The *relation* between intuitive ideas and mathematical definitions is part of the substance of what we are trying to teach. Naturally, the text does not attempt to theorize about this relation; it can only be learned by experience.

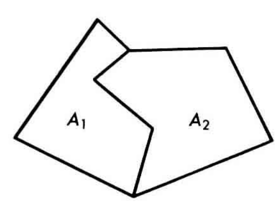
C. CLARITY AND ACCURACY OF LANGUAGE

An extreme attention to accuracy and explicitness in the use of language is often regarded as a burden to the student; and in some cases no doubt it is. But in many cases we believe that an accurate and explicit statement is the easiest to understand and to use. Consider, for example, the familiar statement: *The whole is equal to the sum of its parts.* If this is regarded as a general principle, then we have a fourfold problem, namely, the problem of assigning exact meanings to the words *whole, equal, sum,* and *part.* In practice, the student learns by experience that he is supposed to split the general principle into three specifically geometric statements, as follows:

(1) If B is between A and C, on a line, then $AC = AB + BC$ (where AC, AB, and BC denote distances).

(2) If D is in the interior of $\angle ABC$, as in the figure, then $r + s = t$.





$A = A_1 + A_2$.

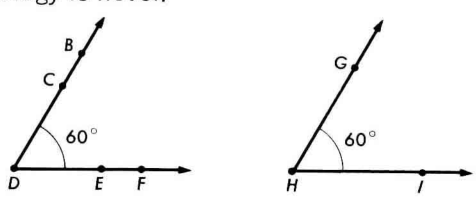
(3) If a plane region is divided into two nonoverlapping regions, then the area of the whole region is the sum of the areas of the parts.

In this text, statements (1), (2), and (3) are made separately, at the places where they are needed: (1) is conveyed by the definition of *between,* on page 37; (2) is the Angle Addition Postulate, on page 86; and (3) is the Area Addition Postulate, on page 332. Logically speaking, this is preferable, because (1), (2), and (3) cannot really be deduced from any general philosophical principle. And our accuracy is surely no burden on the student, because we have stated our three principles at the points where they are needed, *in the forms in which they are going to be used.*

It is in this spirit, and for this sort of purpose, that we have tried to be exact. It appeared to us that if sufficient care was taken, then most of the supposed conflicts between logic and pedagogy could be resolved, to the benefit of the latter. There were some exceptions to this rule, and in the exceptional cases we used a different style. Thus, for example, on page 69 we attempt to convey a topological idea by talking about maps drawn on sheets of rubber. A formal mathematical treatment of this idea seemed out of the question. The treatment of area and volume, in Chapters 16 and 19, has large logical gaps. (Note the absence of a definition of the word *solid.*) Here again we had no choice: the crucial ideas can be made intuitively intelligible, but a formally accurate treatment would be hopeless.

D. THE LANGUAGE OF CONGRUENCE

The terminology of this book is, of course, introduced piecemeal as we go along; but in classroom discussions some topics may come up earlier, and so we should explain now some of the ways in which our terminology is novel.



In the first place, we use the symbol "=" and the word "equals" only in the sense in which these are used in algebra; when we write $A = B$ we mean that the symbols A and B denote exactly the same object. Thus, for the angles shown in the figure above, we can write $\angle BDF = \angle CDE$, because $\angle BDF$ and $\angle CDE$ not only have the same measure, but are *exactly* the same angle. The angles $\angle BDF$ and $\angle GHI$ will be called *congruent* (rather than equal), and we shall write $\angle BDF \cong \angle GHI$.

Similarly for segments. If two segments $\overline{AB}$ and $\overline{CD}$ have the same length, then they will be called *congruent.* We shall never say that $\overline{AB}$ and $\overline{CD}$ are "equal" unless they are exactly the same segment. (Thus, for example, we always have $\overline{AB} = \overline{BA}$, although we rarely have any occasion to say so.)

This terminology may seem strange, if it is new to the reader, but it is a considerable simplification. There are two main ideas involved here. One is the logical identity, the relation *is exactly the same as.* The other is the idea of *congruence,* or *geometric equivalence:* two geometric figures, of any kind whatever, are *congruent* if they have exactly the same size and shape. (Thus two segments are congruent if they have the same length, two circles are congruent if they have the same radius, and two equilateral triangles are congruent if their sides have the same length.)

In this book, each of these two fundamental ideas has a word all to itself, and each of them is always described by the same word. This usage gives a much simpler relation between the ideas and the words that refer to them. And it brings the language of geometry into closer accord with the language of the rest of mathematics.

There is, of course, no need to explain the language of congruence to the student until it comes up in the text. But if the subject does arise, it should not be discussed in a terminology which will be replaced by another in later chapters.

E. PROBLEMS

As everyone knows, problem sequences are the heart of a mathematics course: they are what the student lives with, and any kind of knowledge or skill that is not incorporated in them is unlikely to be learned. Geometry offers a special opportunity—the "originals" that the student is asked to prove are a *part* of the theory, rather than merely an application of it; and this probably is the reason why so many students have gotten for the first time, in a geometry course, some notion of what mathematics means to a mathematician.

The problem sets include a variety of different types of problems of different degrees of difficulty: there are simple exercises in the direct application of definitions and theorems; there are many challenging originals; there are minor theorems which are not included in the text itself; and there are many Honors Problems to challenge the ablest students. Division of problem sets into three levels of difficulty is indicated by single and double rules.

F. COURSE SCHEDULES

This book is much shorter, for teaching purposes, than the number of pages would suggest. It includes many more problems than a teacher would want to assign to any one class. (It is possible to adjust the problem assignments to suit the needs of a great variety of classes.) Moreover, the text proper is designed to be read, not to be deciphered. For this reason, intuitive explanations are frequent, the exposition is full, figures are used liberally, and excessive use of abbreviations is avoided.

Scattered through the text, at points which seem appropriate, are short biographical sketches of eminent mathematicians. The main emphasis is on their mathematical work, because one of the purposes of the biographical notes is to convey to the student that mathematics is the work of human beings: all of the mathematics that is known today was discovered by some person, at some point in time, to solve a problem. If this is understood, then the student will not be surprised on being told that original mathematics is being done today.

Some sections of the text, while not logically necessary, were included because of the high level of student interest that they usually arouse. These include "The Seven Bridges of Königsberg," "How Eratosthenes Measured the Earth," and "The Impossible Construction Problems of Antiquity."

1. General comments on the schedule for an average class

In teaching this text to an average class, you can cover roughly the same topics as you do with a strong class, and you can cover each in approximately the same number of days. There is, however, an important difference in the approach you should take to the subject. The average student is often slow to grasp mathematical ideas; he has difficulty in recalling material covered even a short time ago, he frequently fails to see relationships, and he has difficulty in applying theories to practical situations. Hence, in teaching geometry to the average student, you should concentrate on the rudimentary aspects of the subject. Make sure the student understands the definitions and statements of the theorems, and frequently do prototype problems (especially numerical ones), supplying the student with enough patterns to follow so that he can get off to a good start on his homework.

2. Omission of topics

In order to do justice to those topics which you consider most important for your students, you may find it necessary simply to omit certain chapters or sections of the text. We list here, in rough order of preference, those topics which can be omitted:

(i) Section 6–6 on the proof of the ASA and SSS Postulates and betweenness and separation.

(ii) A considerable block of time can be saved by omitting or covering only informally Chapters 8, 10, and 19 on the geometry of space and Chapters 17 and 18 on trigonometry, transformations, and vectors.

(iii) Section 7–7 on the Hinge Theorem and its converse.

(v) Sections 15–6 through 15–10 on constructions. Logically speaking, the constructions may be omitted, because they will not be needed later; but they arouse so much interest among all students that it would be a pity not to cover them.

[Note: Chapter 13, "Plane Coordinate Geometry," is not included in the list, partly because its place in such a list is controversial, and partly because one of the reasons for omitting other topics is to assure an adequate coverage of coordinate geometry. If you omit Chapter 13, you should also omit Sections 14–8 and 15–2.]

3. Sequence of topics

There are, of course, several ways in which the sequence of topics can be rearranged. A few comments are in order: (a) Chapter 8 is needed for Chapters 10 and 19. (b) Chapter 11 must precede Chapter 12, because area theory is used in the proof of the Basic Proportionality Theorem. (c) Chapter 13, "Coordinate Geometry," must come after Chapter 12, "Similarity," because similarity is used in the discussion of slopes of lines. Chapter 13 can, however, be postponed until the end of the course, since the chapters which follow Chapter 13 are not dependent on it, except in Sections 14–8 and 15–2, as noted above.

Note further that all of the theorems of the first eight chapters, and of Sections 9–1 and 9–2 are proved without use of the Parallel Postulate. We suggest that these chapters be taught in the sequence given in the text.

SUGGESTED TEST QUESTIONS

Note: Answers appear in brackets following each question.

Chapters 1 and 2 Common Sense and Exact Reasoning; Sets, Real Numbers, and Lines

1. G is between H and K if G, H, and K are different points on a line and if ____?____. [$HG + GK = HK$]
2. According to the Ruler Postulate, the distance between any two points is the ____?____ of the corresponding numbers. [absolute value of the difference]
3. (a) The union of $\overrightarrow{RT}$ and $\overrightarrow{TR}$ is ____?____. [$\overleftrightarrow{RT}$]
   (b) The intersection of $\overrightarrow{RT}$ and $\overrightarrow{TR}$ is ____?____. [$\overline{RT}$]
4. (a) If $x < y$, then $x - y$ is ____?____. [negative]
   (b) If $x = y$, then $x - y$ is ____?____. [zero]
   (c) If $x > y$, then $x - y$ is ____?____. [positive]
5. Three points A, B, and C are on a line. $AC = 3$ and $AB = 9$. $BC =$ ____?____, and point ____?____ is not between the other two points. [6 or 12 (*both must be given*); B]
6. If points M, N, and P are on a line and $PM = PN$, then P is called the ____?____ of $\overline{MN}$. [midpoint]

Supply the missing symbol, if any, over each letter pair.

7. C is an endpoint of CB, but B is not. B is an endpoint of CB. [$\overrightarrow{CB}$; $\overline{CB}$]
8. If A is a point such that $CA - BA = BC$, then C is a point of BA, but not of BA nor of BA. [$\overleftrightarrow{BA}$; $\overrightarrow{BA}$; $\overline{BA}$]

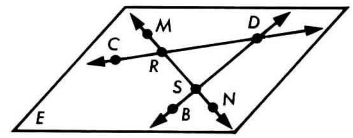
On the number line below show A, B, and C in the order specified by Question 6.

Given a coordinate system on a line, such that the coordinate of T is −3, of W is 9, of A is x, and of B is y. Under a new coordinate system, W corresponds to 4 and T corresponds to −8.

9. The new coordinates of A and B are ____?____ and ____?____, respectively. [$x - 5$; $y - 5$]
10. The new coordinate of the point K such that $TK = WK$ is ____?____. [−2]
11. Show that the distance AB is the same in both coordinate systems. [In the old system, $AB = |x - y|$. In the new system, $AB = |(x - 5) - (y - 5)| = |x - y|$.]

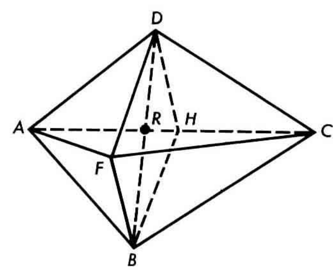
Chapter 3 Lines, Planes, and Separation

In the figure for Questions 1–3, points B, C, D, M, N, R, and S all lie in plane E.

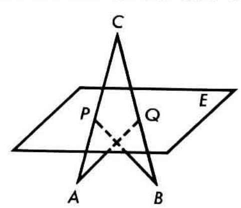


1. Points C and N are in different half-planes formed by ____?____, and C and D are in different half-planes formed by ____?____. [$\overleftrightarrow{BD}$; $\overleftrightarrow{MN}$]
2. According to Postulate 6, if we know only that points R and S lie in plane E, we can conclude that E contains ____?____. [$\overleftrightarrow{RS}$]
3. Since $\overline{CR}$ does not intersect $\overleftrightarrow{BD}$, we know that C and R are ____?____ of $\overleftrightarrow{BD}$. [on the same side]
4. If a line intersects a plane not containing it, the intersection contains ____?____ point(s). [exactly one]
5. If two planes intersect, the intersection contains ____?____ points. [infinitely many]
6. The number of planes determined by four noncoplanar points is ____?____. [4]
7. Three collinear points are contained in ____?____ plane(s). [infinitely many]
8. The number of lines determined by pairs of the six points R, S, T, X, Y, Z, no three of which are collinear, is ____?____. [15]

Given the noncoplanar figure in which B, D, F, and H are coplanar and $\overline{AC}$ and $\overline{BD}$ intersect at R.



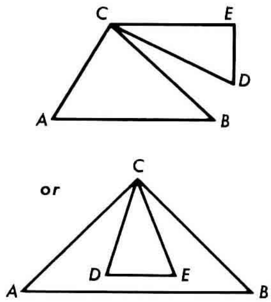
9. Are F, R, and H coplanar? [Yes]
10. Does $\overleftrightarrow{FC}$ intersect $\overleftrightarrow{AR}$? [Yes]
11. Does H lie on $\overline{AC}$? [No]
12. Are A, C, H, and R coplanar? [Yes]
13. Does $\overline{HB}$ intersect $\overline{FC}$? [No]
14. Sketch and label a figure which satisfies the following description: C is a point above a horizontal plane E. A and B are points in the half-space opposite to that which contains C. $\overline{AC}$ and $\overline{BC}$ intersect E in P and Q, respectively. Draw $\overline{AQ}$ and $\overline{BP}$.



Chapter 4 Angles and Triangles

1. In $\triangle ABC$, M is a point of $\overline{BC}$ between B and C. If $m\angle AMC = 78$, what is $m\angle AMB$? [102]
2. What is the measure of the complement of an angle whose measure is $90 - x$? [x]
3. The measure of the supplement of an angle is four times the measure of the complement of the angle. What is the measure of the angle? [60]
4. Sketch a figure which satisfies the description below and then answer Questions 5–9.

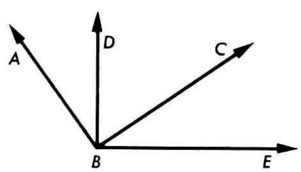
$\triangle ABC$ and $\triangle CDE$ are coplanar and intersect only at C. Points D and E are on the same side of $\overleftrightarrow{AC}$ as B and also on the same side of $\overleftrightarrow{AB}$ as C.



5. Can B be in the interior of $\angle D$? [Yes]
6. D is always in the interior of which angle? [$\angle A$]
7. May E ever be in the interior of $\triangle ABC$? [Yes]
8. Must $\overline{DE}$ intersect $\overline{CB}$? [No]
9. Must $\overleftrightarrow{DE}$ intersect $\overline{CB}$? [No]
10. Complete the proof by filling in the blanks.
    Given: The figure with

$$m\angle ABC = 90 = m\angle DBE.$$

Prove: $\angle ABD \cong \angle CBE$.



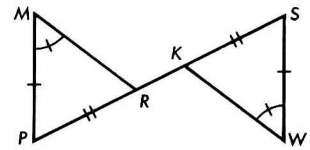
Proof

STATEMENTS	REASONS
1. $m\angle ABC = 90 = m\angle DBE$.	1. Given
2. $\angle ABD$ is complementary to $\angle DBC$.	2. Definition of complementary angles
3. ___?___ [$\angle CBE$ is complementary to $\angle DBC$.]	3. Same as reason 2
4. $\angle DBC \cong \angle DBC$.	4. ___?___ [Every angle is congruent to itself.]
5. ___?___ [$\angle ABD \cong \angle CBE$.]	5. ___?___ [Complements of congruent angles are congruent.]

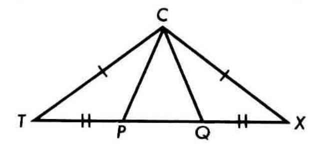
Chapter 5 Congruence

In each pair of triangles below, like markings indicate congruent parts. In each answer blank labeled (a), complete the appropriate congruence, or write *none*. In each answer blank labeled (b), name the congruence postulate which proves the congruence, or write *none*.

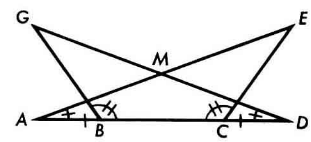
1. (a) $\triangle MPR \cong$ ___?___ [None]
   (b) ___?___ [None]



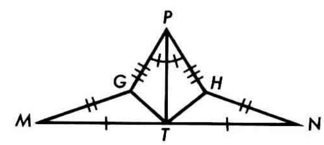
2. (a) $\triangle TCP \cong$ ___?___ [$\triangle XCQ$]
   (b) ___?___ [SAS]



3. (a) $\triangle ACE \cong$ ___?___ [$\triangle DBG$]
   (b) ___?___ [ASA]



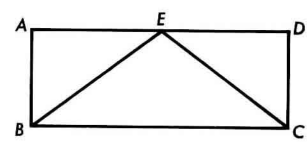
4. (a) $\triangle MTG \cong$ ___?___ [$\triangle NTH$]
   (b) ___?___ [SSS]



5. Point K is called the ___?___ of $\overline{AB}$ if K is between A and B and if $AK = BK$. [midpoint]
6. $\overrightarrow{MK}$ is the bisector of $\angle AMB$ if K is in ___?___ and if ___?___. [the interior of $\angle AMB$; $\angle KMA \cong \angle KMB$]
7. In $\triangle MKH$, $\overline{MK}$ is said to be the side ___?___ by $\angle M$ and $\angle K$. [included]
8. In an ___?___ triangle every median is also an angle bisector. [equilateral]
9. Given: The figure with

$\overline{AB} \perp \overline{AD}$, $\overline{DC} \perp \overline{AD}$, $AB = CD$, and E the midpoint of $\overline{AD}$.

Prove: $\angle EBC \cong \angle ECB$.



Proof

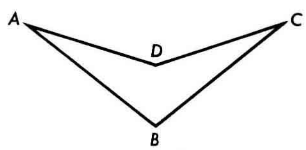
STATEMENTS	REASONS
1. E is the midpoint of $\overline{AD}$.	1. ___?___ [Given]
2. Therefore ___?___ [$AE = DE$.]	2. ___?___ [Definition of midpoint]
3. $\overline{AB} \perp \overline{AD}$ and $\overline{DC} \perp \overline{AD}$.	3. ___?___ [Given]

4. ___?___ [$\angle A$ and $\angle D$ are right angles.]	4. Perpendiculars form right angles.
5. $\angle A \cong \angle D$.	5. ___?___ [All right angles are congruent.]
6. $AB = CD$.	6. Given
7. ___?___ [$\triangle ABE \cong \triangle DCE$.]	7. ___?___ [SAS]
8. ___?___ [$BE = CE$.]	8. Corresponding parts of congruent triangles
9. Therefore $\angle EBC \cong \angle ECB$.	9. ___?___ [The Isosceles Triangle Theorem]

Chapter 6 A Closer Look at Proof

1. In an indirect proof we reason until we reach a ___?___. [contradiction]
2. A point or segment introduced to complete a proof is called an ___?___ [auxiliary set]
3. Which of the following sets are segments? [a, c]
(a) Median (b) Perpendicular bisector (c) Angle bisector of a triangle (d) Bisector of an angle (e) Side of an angle
4. Which of the following are convex sets? [a, c, d, e]
(a) Median (b) Right triangle (c) Bisector of an angle (d) Perpendicular bisector (e) Plane
5. Given: $\square ABCD$ with $\angle DAB \cong \angle DCB$ and $AD = CD$.

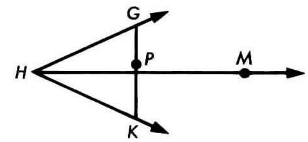
Prove: $\triangle ABD \cong \triangle CBD$.



Proof

STATEMENTS	REASONS
1. Introduce ___?___. [$\overline{AC}$ (and $\overline{DB}$)]	1. ___?___ [The Line Postulate]
2. $AD = CD$.	2. ___?___ [Given]
3. ___?___ [$\angle DAC \cong \angle DCA$.]	3. The Isosceles Triangle Theorem
4. ___?___ [$\angle DAB \cong \angle DCB$.]	4. Given
5. $m\angle DAB + m\angle DAC = m\angle DCB + m\angle DCA$.	5. ___?___ [The Addition Property of Equality]
6. ___?___ [$m\angle BAC = m\angle BCA$.]	6. The Angle Addition Postulate and the meaning of equality
7. $AB = CB$.	7. ___?___ [If two angles of a triangle are congruent, the opposite sides are congruent.]
8. ___?___ [$\triangle ABD \cong \triangle CBD$.]	8. ___?___ [SAS or SSS]

6. Given: $\angle GHK$ with bisector $\overrightarrow{HM}$, $GH = KH$, and P is the midpoint of $\overline{GK}$.



Prove: P lies on $\overrightarrow{HM}$.

Proof

STATEMENTS	REASONS
1. ___?___ [Suppose P is not on $\overrightarrow{HM}$.]	1. ___?___ [Beginning of indirect proof]
2. Introduce ___?___. [$\overline{HP}$]	2. ___?___ [The Line Postulate]
3. ___?___ [$HP = HP$.]	3. Identity
4. P is the midpoint of $\overline{GK}$.	4. Given
5. ___?___ [$PG = PK$.]	5. Definition of midpoint
6. $GH = KH$.	6. Given
7. ___?___ [$\triangle PHG \cong \triangle PHK$.]	7. ___?___ [SSS]
8. $\angle PHG \cong \angle PHK$.	8. Corresponding parts of congruent triangles
9. $\overrightarrow{HP}$ is bisector of $\angle GHK$.	9. ___?___ [Definition of angle bisector]
10. But $\overrightarrow{HM}$ bisects $\angle GHK$.	10. ___?___ [Given]
11. Steps 9 and 10 are contradictory. Thus our assumption is false and the theorem is proved.	11. Every angle has exactly one bisector.

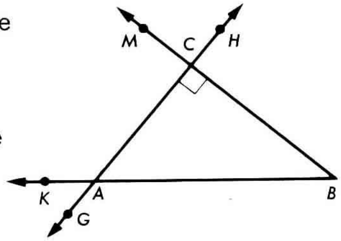
7. Assume you are to prove the following by indirect method. What is the supposition with which you begin?

If a triangle has no two sides congruent, then it is not isosceles. [Suppose the triangle is isosceles.]

Chapter 7 Geometric Inequalities

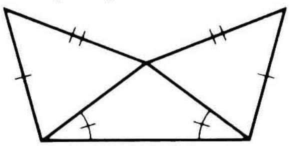
Questions 1–5 refer to the figure below. For each question name the theorem which justifies the statement.

1. $\angle KAC \cong \angle GAB$. [The Vertical Angle Theorem]
2. $AC < AB$. [The First Minimum Theorem]
3. $AB + AC > BC$. [The Triangle Inequality]
4. $\angle KAC > \angle B$. [The Exterior Angle Theorem]

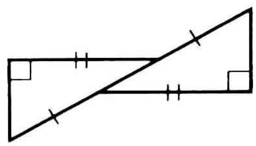


In each pair of triangles below, like markings indicate congruent parts. For each Question 5–10 name the congruence postulate or the congruence theorem which proves the congruence between triangles, or write *none*.

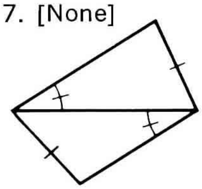
5. [SSS]



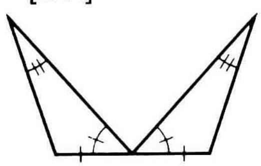
6. [*HL*]



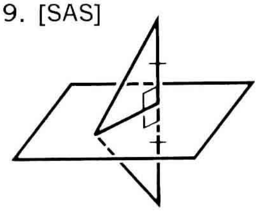
7. [None]



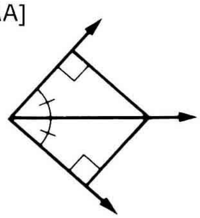
8. [SAA]



9. [SAS]

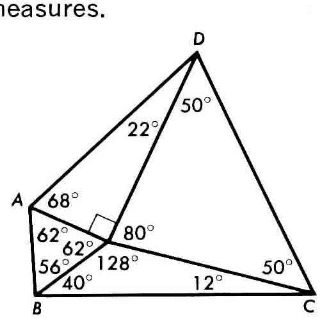


10. [SAA]

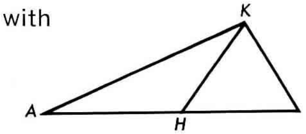


Questions 11–14 refer to the figure below, in which the angles have the indicated measures.

11. Which is greater, *BE* or *EC*? [*EC*]
12. In $\triangle ADE$, what is $\overline{AD}$ called? [hypotenuse]
13. Which is greater, *AE* or *ED*? [*ED*]
14. Which segment of the figure is shortest? Explain. [$\overline{AE}$; $AD > ED > AE$, $DC > EC = ED > AE$, $BC > EC > BE = AB > AE$.]



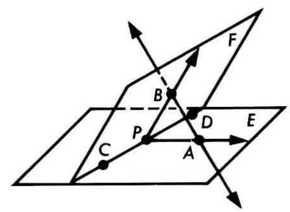
15. Given: The figure with $HK = HB$ and $HB > KB$.
Prove: $AK > KB$.



STATEMENTS	Proof	REASONS
1. $HK = HB > KB$.		1. Given
2. In $\triangle HBK$, $\angle B >$ ___?___. [$\angle KHB$]		2. ___?___ [In a triangle, the larger angle is opposite the longer side.]
3. ___?___ [$\angle KHB > \angle A$.]		3. The Exterior Angle Theorem
4. ___?___ [$\angle B > \angle A$.]		4. Transitivity
5. Therefore, $AK > KB$.		5. ___?___ [In a triangle, the longer side is opposite the larger angle.]

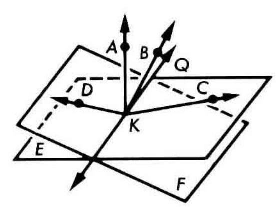
Chapter 8 Perpendicular Lines and Planes in Space

1. In this figure $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are each perpendicular to $\overleftrightarrow{CP}$, the line of intersection of planes *E* and *F*. Which ray and which plane are perpendicular? [c]



(a) $\overrightarrow{PB}$ and plane *E* (b) $\overrightarrow{PA}$ and plane *F*
(c) $\overrightarrow{PC}$ and plane *ABP* (d) $\overrightarrow{BA}$ and plane *F*

2. In this figure, planes *E* and *F* intersect at $\overleftrightarrow{PQ}$, *C* is in *E*, *D* is in *F*, $\overrightarrow{KA} \perp E$, and $\overrightarrow{KB} \perp F$. Which pairs of rays *must be* perpendicular? [a, b, d]



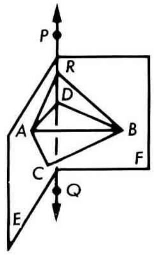
(a) $\overrightarrow{KA}$ and $\overrightarrow{KP}$
(b) $\overrightarrow{KB}$ and $\overrightarrow{KQ}$
(c) $\overrightarrow{KA}$ and $\overrightarrow{KD}$
(d) $\overrightarrow{KA}$ and $\overrightarrow{KC}$

3. The ___?___ of a segment contains all perpendicular bisectors of the segment. [perpendicular bisecting plane]

4. According to the First Minimum Theorem, the shortest segment to a ___?___ from an external point is the ___?___ segment. [line; perpendicular]

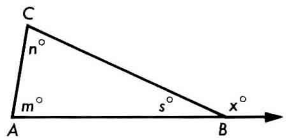
5. In this figure, R and D lie on $\overleftrightarrow{PQ}$, the intersection of planes E and F; A and C lie in E; and B lies in F. $\overline{BA} \perp E$. Which angles listed below *must be* right angles? [c, d]

(a) $\angle ACB$ (b) $\angle ADB$
(c) $\angle DAB$ (d) $\angle RAB$



6. At a given point of a given line there is at most one ___?___ perpendicular to the given line. [plane]

7. Given points A, B, C, D, and E. If $AC = AD$, $CB = BD$, and $ED = CE$, then points ___?___ must be coplanar. [A, B, E]



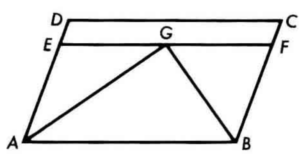
8. If $m = 80$ and $n = 75$, then $x >$ ___?___. [80]
9. If $s = 60$ and $m = 100$, then x ___?___. [$= 120$]
10. If $x = 85$, then n ___?___. [< 85]
11. If $n = 90$, the longest side of $\triangle ABC$ is ___?___. [$\overline{AB}$]
12. If $x = 85$, the longest side of $\triangle ABC$ is ___?___. [$\overline{AC}$]

Chapter 9 Parallel Lines in a Plane

1. If two angles of a triangle have measures $120 - a$ and $60 + b$, respectively, what is the measure of the third angle? [$a - b$]

2. A transversal intersects two parallel lines at points A and B. The bisectors of two interior angles on the same side of the transversal intersect each other at K. What is $m\angle AKB$? [90]

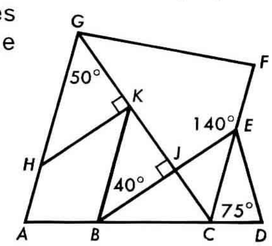
In the figure for Questions 3 and 4, $\overline{DC} \parallel \overline{AB}$, $\overrightarrow{AG}$ bisects $\angle A$, $\overrightarrow{BG}$ bisects $\angle B$, $\overline{EF}$ contains G, and $\overline{EF} \parallel \overline{AB}$.



3. If $m\angle EAB = 80$ and $m\angle FBA = 100$, what is $m\angle FGB$? [50]

4. If $m\angle CDA = 110$, what is $m\angle EGA$? [35]

Supply the angle measures in Questions 5–14 for the figure to the right.



5. $m\angle GHK$ [40]
6. $m\angle BKJ$ [50]
7. $m\angle BKH$ [40]
8. $m\angle AHK$ [140]
9. $m\angle CEJ$ [40]
10. $m\angle EDC$ [75]
11. $m\angle DEF$ [150]
12. $m\angle JCB$ [55]
13. $m\angle JBC$ [35]
14. $m\angle KBA$ [105]

15. In $\triangle ABC$, $\angle A$ is a right angle and $m\angle C = 30$. If $AB = 8$, what is BC? [16]

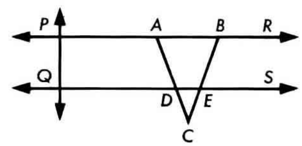
16. In parallelogram $\square ABCD$, $m\angle A$ is eight times $m\angle D$. What is $m\angle A$? [160]

17. In $\triangle PQR$, $\angle Q$ is a right angle and A, B, and C are the midpoints of $\overline{PQ}$, $\overline{PR}$, and $\overline{QR}$, respectively. If $PR = 20$, $QR = 12$, and $PQ = 16$, how long are the diagonals of $\square ABCQ$? [$AC = 10$, $BQ = 10$]

18. In trapezoid $\square ABCD$, $\overline{EF}$ is the median. If $AB = 13$, $BC = 8$, $CD = 9$, and $AD = 5$, what is EF? [11]

19. In parallelogram $\square ABCD$, $m\angle B = 2x + 29$ and $m\angle D = 5x - 7$. What is the numerical measure of $\angle B$? [53]

20. Given: The plane figure with
$\overleftrightarrow{PR} \perp \overleftrightarrow{PQ}$, $\overleftrightarrow{QS} \perp \overleftrightarrow{PQ}$, and $CD = CE$.



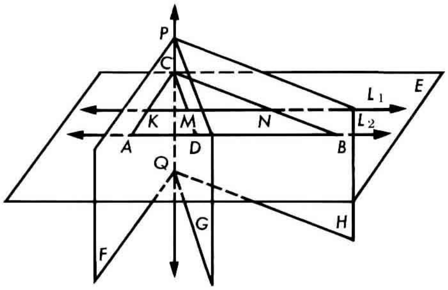
Prove: $\triangle ABC$ is isosceles.

Proof

STATEMENTS	REASONS
1. $\overleftrightarrow{PR} \perp \overleftrightarrow{PQ}$ and $\overleftrightarrow{QS} \perp \overleftrightarrow{PQ}$.	1. ___?___ [Given]
2. Therefore ___?___. [$\overleftrightarrow{PR} \parallel \overleftrightarrow{QS}$]	2. ___?___ [In a plane, two lines perpendicular to a third line are parallel.]
3. $CD = CE$.	3. Given
4. Therefore ___?___. [$m\angle CDE = m\angle CED$]	4. ___?___ [The Isosceles Triangle Theorem]
5. $m\angle CDE =$ ___?___. [$m\angle CAB$] $m\angle CED =$ ___?___. [$m\angle CBA$]	5. If two parallels are cut by a transversal, corresponding angles are congruent.
6. ___?___ [$m\angle CAB = m\angle CBA$.]	6. Steps 4 and 5 and substitution
7. ___?___ [$AC = BC$.]	7. ___?___ [If two angles of a triangle are congruent, the sides opposite the angles are congruent.]
8. $\triangle ABC$ is isosceles.	8. Definition of isosceles triangle

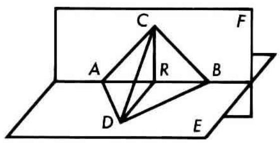
Chapter 10 Parallel Lines and Planes

Plane E, containing parallel lines L_1 and L_2, is perpendicular to $\overleftrightarrow{PQ}$ at C. $\overleftrightarrow{PQ}$ is the edge of right dihedral angle $\angle A\text{-}PQ\text{-}B$. $KC = MC$ and $m\angle A\text{-}PQ\text{-}D = 50$.



1. What is $m\angle B\text{-}PQ\text{-}D$? [40]
2. What is $m\angle CDA$? [65]
3. What is $m\angle ABC$? [25]
4. What is $m\angle CMN$? [115]

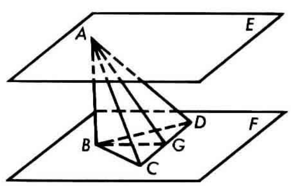
Perpendicular planes E and F intersect in $\overleftrightarrow{AB}$ and plane CDR is the perpendicular bisecting plane of $\overline{AB}$. C is in F and D is in E such that $\overline{AC} \perp \overline{BC}$ and $CR = \frac{1}{2}CD$.



5. What is $m\angle ACR$? [45]
6. What is $m\angle DCR$? [60]

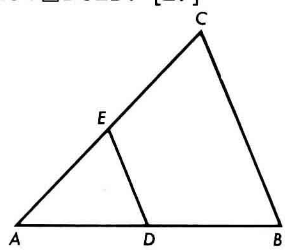
*7. What is $m\angle ADR$? [30]

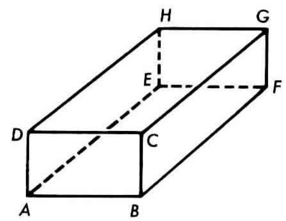
8. The measure of a dihedral angle is 60. A point C in one face of the dihedral angle is 18 inches from its edge. The projection of C into the other face of the dihedral angle is point H. How far is H from the edge of the dihedral angle? [9 in.]
9. One face of a right dihedral angle contains a square. What is the projection of the square into the other face? [A segment]
10. In the figure, E and F are planes, $E \parallel F$, $\overleftrightarrow{AB} \perp F$, and $\overleftrightarrow{AG}$ is the perpendicular bisector of $\overline{CD}$. If $m\angle C\text{-}AB\text{-}D = 30$, what is $m\angle BDC$? [75]



Chapter 11 Polygonal Regions and Their Areas

1. The hypotenuse of a right triangle is 10 in. long and one leg has length 6 in. What is the area of the triangle? [24 sq. in.]
2. In rhombus $\square ABCD$, $AD = 13$ and the altitude to $\overline{BC}$ is 7. What is $a\square ABCD$? [91]
3. The perimeter of a square is 8 in. How long is a diagonal of the square? [$2\sqrt{2}$ in.]
4. A side of an equilateral triangle is 8 in. long. What is the area of the triangle? [$16\sqrt{3}$ sq. in.]
5. The sides of a triangle have lengths 10, 24, and 26. What is the area of the triangle? [120]
6. A parallelogram has sides of lengths 8 and 18. The altitude to the shorter side is 15. What is the altitude to the longer side? [$6\frac{2}{3}$]
7. The area of an isosceles right triangle is 16. How long is its hypotenuse? [8]
8. In parallelogram $\square ABCD$, $AD = 15$, $DC = 22$, and $m\angle A = 60$. What is $a\square ABCD$? [$165\sqrt{3}$]
9. In the figure on the left below, D and E are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. If $a\triangle ABC = 36$, what is $a\square BCED$? [27]





10. In the rectangular solid shown, $BC = 4$, $CG = 10$, and $GH = 4\sqrt{3}$. How long is the diagonal $\overline{CE}$? [$2\sqrt{41}$]

Chapter 12 Similarity

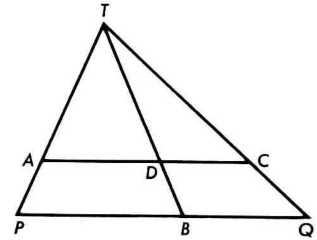
1. Solve for x.

(a) $\dfrac{x+3}{4x} = \dfrac{14}{35}$ [5] (b) $\dfrac{15ma}{4xy} = \dfrac{9ab}{2my}$ $\left[\dfrac{5m^2}{6b}\right]$

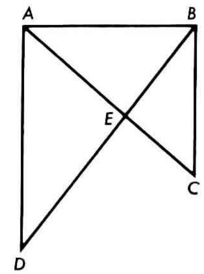
2. Two similar triangles have a pair of corresponding sides of lengths $3\frac{1}{2}$ and $8\frac{3}{4}$, respectively. What is the ratio of their perimeters? [$\frac{2}{5}$]
3. The hypotenuses of two similar right triangles have lengths 6 and 16. What is the ratio of the areas of the two triangles? [$\frac{9}{64}$]

Given the figure with $\overline{AC} \parallel \overline{PQ}$.

4. If $AT = 5$, $AP = 2$, and $CQ = 3$, what is TC? [$7\frac{1}{2}$]
5. If $PT = 33$, $AP = 12$, and $DB = 8$, what is TD? [14]



In this plane figure, $\overline{AD} \perp \overline{AB}$ and $\overline{BC} \perp \overline{AB}$. $AB = 3$, $BE = 2$, and $DE = 3$.

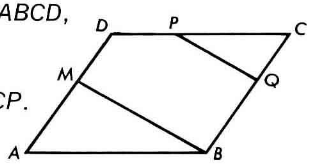


6. What is the numerical value of $\frac{AD}{BC}$? [$\frac{3}{2}$]
7. What is the numerical value of $\frac{a\triangle BCE}{a\triangle DAE}$? [$\frac{4}{9}$]
8. What is the numerical value of $\frac{a\triangle ABC}{a\triangle ABD}$? [$\frac{2}{3}$]

In $\triangle ABC$, $\angle A$ is a right angle and $\overline{AD} \perp \overline{BC}$ with D on $\overline{BC}$. $BD = 8$ and $CD = 10$.

9. What is AD? [$4\sqrt{5}$] 10. What is AB? [12]
11. What is AC? [$6\sqrt{5}$]
12. Given: Parallelogram $\square ABCD$, with $\overline{PQ} \parallel \overline{MB}$.

Prove: $AB \cdot CQ = AM \cdot CP$.

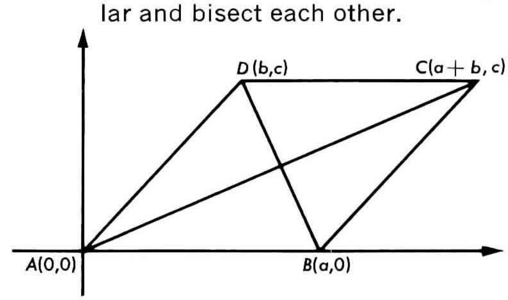


STATEMENTS	**Proof** REASONS
1. $\square ABCD$ is a parallelogram.	1. Given
2. $m\angle A = m\angle C$.	2. __?__ [Opposite angles of a parallelogram are congruent.]
3. $\overline{PQ} \parallel \overline{MB}$.	3. __?__ [Given]
4. __?__ [$m\angle PQC = m\angle MBQ$.]	4. If two lines are parallel, corresponding angles are congruent.
5. $\overline{AD} \parallel \overline{BC}$.	5. __?__ [Definition of a parallelogram]
6. __?__ [$m\angle MBQ = m\angle AMB$.]	6. __?__ [PAI Theorem]
7. Therefore $m\angle PQC = m\angle AMB$.	7. Steps 4 and 6 and transitivity.
8. __?__ [$\triangle ABM \sim \triangle CPQ$.]	8. __?__ [AA Corollary]
9. __?__ $\left[\frac{AB}{CP} = \frac{AM}{CQ}.\right]$	9. __?__ [Definition of similar triangles]
10. Therefore, $AB \cdot CQ = AM \cdot CP$.	10. Multiplying both sides of the equation in step 9 by $AM \cdot CQ$

Chapter 13 Plane Coordinate Geometry

1. If two non-vertical lines are perpendicular, the product of their slopes is __?__. [−1]
2. The midpoint of the segment whose end points are (3, 14) and (13, −4) is __?__. [8, 5]
3. Two segments that have equal slopes are either __?__ or __?__. [parallel; collinear]
4. Any line which has no slope must be a __?__ line. [vertical]
5. The vertices of a quadrilateral are the points (9, 7), (8, −4), (−3, −2), and (0, 11). What is the sum of the lengths of its diagonals? [32]
6. A triangle has vertices at (6, 8), (2, −4), and (−3, 8). What is its area? [54]
7. The vertices of a triangle are the points (0, 0), (6, 6), and (−8, 8). How long is the median to the longest side? [$5\sqrt{2}$]
8. Complete the proof by filling in the blanks.

Prove: The diagonals of a rhombus are perpendicular and bisect each other.



Proof. Let the coordinates of A, B, C, and D be as in the figure. We find the coordinates of the midpoints of $\overline{AC}$ and $\overline{BD}$:

Midpoint of $\overline{AC}$: __?__. $\left[\left(\frac{a+b}{2}, \frac{c}{2}\right)\right]$

Midpoint of $\overline{BD}$: __?__. $\left[\left(\frac{a+b}{2}, \frac{c}{2}\right)\right]$

Since __?__, the diagonals bisect each other. [the midpoints of both segments are the same]

Next find the slopes of the diagonals.

Slope of $\overline{AC}$ = __?__. $\left[\frac{c}{a+b}\right]$

Slope of $\overline{BD}$ = __?__. $\left[\frac{c}{b-a}\right]$

Since $\square ABCD$ is a rhombus, $AB = AD$, or $a =$ __?__. [$\sqrt{b^2 + c^2}$.]

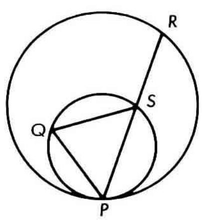
Hence, $a^2 =$ __?__, and $c^2 =$ __?__. [$b^2 + c^2$; $a^2 - b^2$]

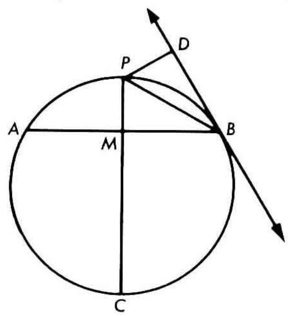
Thus, the product of the slopes of $\overline{AC}$ and $\overline{BD}$ is __?__. [−1]

Therefore __?__. [$\overline{AC} \perp \overline{BD}$]

Chapter 14 Circles and Spheres

1. In a circle whose diameter is 28, a chord of length 14 is drawn. How far is the chord from the center of the circle? [$7\sqrt{3}$]
2. From a point P in the exterior of a circle, secants $\overleftrightarrow{PB}$ and $\overleftrightarrow{PD}$ intersect the circle at A, B and C, D, respectively. If $m\angle P = 30$ and $m\widehat{BD} = 5m\widehat{AC}$, what are the measures of the two intercepted arcs? [15; 75]
3. In a circle, $\overline{AB}$ is a diameter and $\overline{AQ}$ is a tangent segment. $\overline{QB}$ intersects the circle at R. If $QB = 8$ and $QR = 6$, how long is the tangent segment and what is the radius of the circle? [$4\sqrt{3}$; 2]
4. In the figure on the left below, the two circles are internally tangent at P, and $QS = QP$. If $m\widehat{PR} = 140$, what is $m\angle QPS$? [55]





5. Given: A circle with diameter $\overline{CP}$ and chord $\overline{AB}$ such that $\overline{AB} \perp \overline{CP}$ at M. $\overline{PD}$ is perpendicular to the tangent at B.
Prove: $PD = PM$.

Proof

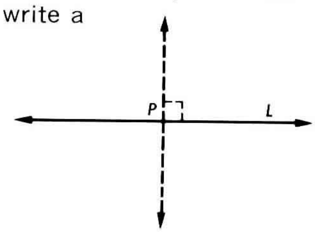
STATEMENTS	REASONS
1. Diameter $\overline{CP} \perp$ chord $\overline{AB}$.	1. Given
2. ___?___. [$m\widehat{AP} = m\widehat{BP}$.]	2. If a diameter is perpendicular to a chord not a diameter, it bisects both arcs of the chord.
3. ___?___. [$m\angle ABP = \frac{1}{2}m\widehat{AP}$.]	3. ___?___. [The measure of an inscribed angle is $\frac{1}{2}$ the measure of the intercepted arc.]
4. ___?___. [$m\angle DBP = \frac{1}{2}m\widehat{BP}$.]	4. The measure of an angle formed by a tangent ray and a secant ray is one-half the measure of the intercepted arc.
5. Therefore $m\angle PBD = m\angle PBM$.	5. Steps 2, 3, and 4
6. ___?___. [$PB = PB$.]	6. ___?___. [Identity]
7. ___?___. [$\angle PMP$ is a right angle.]	7. Perpendiculars form right angles.
8. $\overline{PD} \perp \overleftrightarrow{DB}$.	8. Given
9. ___?___. [$\angle PDB$ is a right angle.]	9. Same as reason 7
10. $\triangle PBD \cong \triangle PBM$.	10. ___?___. [SAA]
11. ___?___. [$PD = PM$.]	11. Corresponding parts of congruent triangles are congruent.

Chapter 15 Characterizations and Constructions

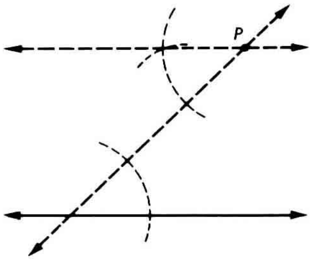
1. In a plane, the set of all points 4 inches from each of the endpoints of a 3-inch segment is ___?___. [d]
   (a) a line (b) a circle (c) two circles
   (d) two points (e) the empty set
2. The set of all points at a given distance from a given point is ___?___. [c]
   (a) a line (b) a circle (c) a sphere
   (d) a plane (e) two lines
3. Two circles in the same plane intersect at two points. If their diameters are 8 and 22, the distance between their centers could be ___?___. [b]
   (a) 4 (b) 12 (c) 16 (d) 20 (e) 24
4. Two altitudes of a triangle intersect at a vertex of the triangle. The triangle must be ___?___. [a]
   (a) right (b) isosceles (c) equilateral
   (d) obtuse (e) acute
5. In $\triangle PQR$, $\angle Q$ is a right angle and the medians intersect at S. If $PR = 12$ and $PQ = 7$, how long is $\overline{QS}$? [b]
   (a) $3\frac{1}{2}$ (b) 4 (c) 6 (d) 7 (e) 8
6. $\{(x, y) \mid x^2 - 4 = 0 \text{ and } x + y = 5\}$ equals ___?___. [b]
   (a) $\{(3, 2)\}$ (b) $\{(2, 3), (-2, 7)\}$
   (c) $\{(2, 3)\}$ (d) $\{(3, 2), (7, -2)\}$
   (e) $\{(2, 3), (-2, -3)\}$

In Questions 7 and 8, sketch the set of all points characterized in each item and write a description of the set.

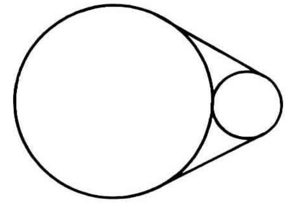
7. In a plane, the set of all points which are centers of circles tangent to line L at point P on L.
(A line perpendicular to L at P minus point P)



8. Construct a line parallel to a line L through a point P not on L. (Use a compass and straightedge only. Show all necessary construction marks.)

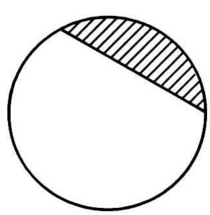


14. Two pipes are bound tightly together with a very thin wire, as in the diagram. The radii of the pipes are 2 in. and 6 in. Neglecting any practical allowances for securing the wire, etc., determine the minimal length of the wire. $\left[\frac{28\pi}{3} + 8\sqrt{3}\right]$



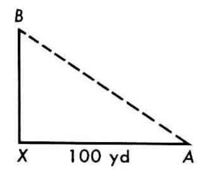
Chapter 16 Areas of Circles and Sectors

1. A circular region is the __?__ of a circle and its interior. [union]
2. The sum of the measures of the angles of a convex n-gon is __?__. [$(n - 2)$ 180]
3. A polygon is regular if it is __?__, __?__, and equilateral. [convex; equilateral]
4. If the perimeter of a regular polygon is three times the diameter of the circumscribed circle, the polygon is called a regular __?__. [hexagon]
5. An annulus is the region bounded by two __?__. [concentric circles]
6. A region bounded by a chord of a circle and an arc of the chord is a __?__ of the circle. [segment]
7. What is the measure of each exterior angle of a regular 12-gon? [30]
8. How many sides has a convex polygon if the sum of the measures of its angles is 2340? [15]
9. Two circles have circumferences of 4π and 20π, respectively. What is the ratio of their areas? [$\frac{1}{25}$]
10. What is the area of the annulus bounded by the inscribed and circumscribed circles of a square whose perimeter is 24? [9π]
11. The radius of a sector is 12 and the measure of its arc is 130°. What is the length of the arc and the area of the sector? $\left[\frac{26\pi}{3}; 52\pi\right]$
12. What is the degree-measure of the arc intercepted by each angle of an inscribed regular pentagon? [216]
13. A chord at a distance 5 from the center of a circle with a diameter 20 forms a segment (shaded) with its minor arc. What is the area of this segment? $\left[\frac{100\pi}{3} - 25\sqrt{3}\right]$



Chapter 17 Trigonometry

1. In trapezoid $\square ABCD$, $\overline{DC} \parallel \overline{AB}$, $AD = DC = CB = 2$, and $\cos \angle A = .5$. Find $a\square ABCD$. $\left[\frac{3\sqrt{3}}{4}\right]$
2. In $\triangle ABC$, $m\angle C = 120°$, $m\angle A = m\angle B$, $AC = 2$. Find $a\triangle ABC$. [$\sqrt{3}$]
3. In a race, instead of following county roads, a crosscountry runner cuts across a field, as shown. If $m\angle A$ is 40° and $AX = 100$ yards, how long, to the nearest yard, is his path across the field? How much shorter is this path than the prescribed route from A to B? [131 yds; 53 yds]



4. Convert each of the following degree measures to radian measures.
   (a) 300 $\left[\frac{5\pi}{3}\right]$ (b) $67\frac{1}{2}$ $\left[\frac{3\pi}{8}\right]$ (c) 96 $\left[\frac{8\pi}{15}\right]$
5. Convert each of the following radian measures to degree measures.
   (a) $\frac{16\pi}{9}$ [320] (b) $\frac{11\pi}{30}$ [66] (c) $\frac{59\pi}{60}$ [177]
6. $W(\theta)$ is a point on a unit circle. In which quadrant is $W(\theta)$ for the following values of θ?
   (a) $\frac{9\pi}{8}$ [III] (b) $-\frac{5\pi}{12}$ [IV] (c) $\frac{11\pi}{12}$ [II]
7. For the values of θ in question 6, in which quadrant is $W(\theta + \pi/2)$? [IV; I; III]
8. For a point $W(\theta)$ of a unit circle, $\sin\theta = -\frac{7}{25}$ and $\frac{3\pi}{2} < \theta < 2\pi$. Find the following.
   (a) $\cos\theta$ [$\frac{24}{25}$]
   (b) $\cos(-\theta)$ [$\frac{24}{25}$]
   (c) $\sin(-\theta)$ [$\frac{7}{25}$]
   (d) $\tan\theta$ [$-\frac{7}{24}$]
   (e) $\tan(-\theta)$ [$\frac{7}{24}$]

9. Given that $\sin\theta = \frac{3}{4}$ and $\cos\theta > 0$, find the following.

(a) $\cos\theta$ $\left[\frac{\sqrt{7}}{4}\right]$ (b) $\tan(-\theta)$ $\left[-\frac{3\sqrt{7}}{7}\right]$

(c) $\sin(-\theta)$ $[-\frac{3}{4}]$ (d) $\cos(\theta + \pi/2)$ $[-\frac{3}{4}]$

(e) $\sin(\theta + \pi)$ $[-\frac{3}{4}]$ (f) $\sin(\theta + \pi/2)$ $\left[\frac{\sqrt{7}}{4}\right]$

10. For $\sin\theta_1 = .5$, $\sin\theta_2 = .2$, $\cos\theta_1 > 0$, and $\cos\theta_2 > 0$, find the following.

(a) $\sin(\theta_1 + \theta_2)$ $\left[\frac{\sqrt{6}}{5} + \frac{\sqrt{3}}{10}\right]$

(b) $\sin(\theta_1 - \theta_2)$ $\left[\frac{\sqrt{6}}{5} - \frac{\sqrt{3}}{10}\right]$

(c) $\cos(\theta_1 - \theta_2)$ $\left[\frac{3\sqrt{2}}{5} + \frac{1}{10}\right]$

11. If $\sin\theta = .25$ and $\cos\theta > 0$, find the following.

(a) $\sin 2\theta$ $\left[\frac{\sqrt{15}}{8}\right]$ (b) $\cos 2\theta$ $[\frac{7}{8}]$

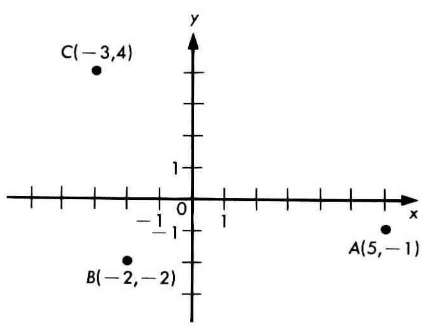
12. If $\sin\theta_1 = \frac{\sqrt{2}}{2}$, $\sin\theta_2 = \frac{\sqrt{5}}{3}$, $\cos\theta > 0$, and $\cos\theta_2 > 0$, find the following.

(a) $\tan(\theta_1 + \theta_2)$ $[-9 - 4\sqrt{5}]$

(b) $\tan(\theta_1 - \theta_2)$ $[4\sqrt{5} - 9]$

Chapter 18 Symmetry, Transformations, and Vectors

1. Given the figure below, give the reflection of the points A, B, and C across



(a) the origin. $[(-5, 1); (2, 2); (3, -4)]$

(b) the x-axis. $[(5, 1); (-2, 2); (-3, -4)]$

(c) the y-axis. $[(-5, -12); (2, -2); (3, 4)]$

(d) the line $x = 2$. $[(-1, -1); (6, -2); (7, 4)]$

(e) the line $y = -3$. $[(5, -5); (-2, -4); (-3, -10)]$

(f) the line $x + y = 1$. $[(2, -4); (3, 3); (-3, 4)]$

2. Find the image of the points A, B, and C in question 1 under the translation by

(a) $(3, -2)$ $[(8, -3); (1, -4); (0, 2)]$

(b) $(-3, -4)$ $[(2, -5); (-5, -6); (-6, 0)]$

3. Find polar coordinates of each of the following points.

(a) $(-2, 0)$ $[2, \pi]$ (b) $(3, -3)$ $\left[3\sqrt{2}, \frac{7\pi}{4}\right]$

(c) $(-5, -5\sqrt{3})$ $\left[10, \frac{4\pi}{3}\right]$

4. Give two other sets of polar coordinates for each of the points in question 3. $[2, -\pi]$, $[2, 3\pi]$; $\left[3\sqrt{2}, -\frac{\pi}{4}\right]$, $\left[3\sqrt{2}, \frac{23\pi}{4}\right]$; $\left[10, -\frac{2\pi}{3}\right]$, $\left[10, \frac{10\pi}{3}\right]$; other answers possible.

5. Let T be a rotation about the origin under which $[r, \theta] \leftrightarrow [r, \theta - \pi/3]$. Find polar coordinates of the image under T of the points in question 3. $\left[2, \frac{2\pi}{3}\right]$; $\left[3\sqrt{2}, \frac{17\pi}{12}\right]$; $[10, \pi]$

6. Let Q be a rotation about the origin under which $[r, \theta] \leftrightarrow [r, \theta + \pi/6]$. Find polar coordinates of the pre-image under Q of each of the following points.

(a) $(-3, 0)$ $\left[3, \frac{5\pi}{6}\right]$ (b) $(-4\sqrt{3}, 4)$ $\left[4, \frac{2\pi}{3}\right]$

(c) $(2, -2)$ $\left[2\sqrt{2}, \frac{19\pi}{12}\right]$

7. Find each of the following vector sums.

(a) $(5, 4) + (4, -3)$ $[(9, 1)]$

(b) $(3, 6) + (1, 4) + (-3, 7)$ $[(1, 17)]$

(c) $(5, 9) - (6, 3) + (1, -2)$ $[(0, 4)]$

(d) $(4, -2) - (3, -1) - (2, -4)$ $[(-1, 3)]$

8. Find each of the following scalar products.

(a) $3(-5, 0)$ $[(-15, 0)]$

(b) $-.5(\sqrt{3}, 1)$ $[(-.5\sqrt{3}, -.5)]$

(c) $-3(-2, 5)$ $[(6, -15)]$

9. Given $\triangle ABC$ with $A(-2, -1)$, $B(-6, 1)$, and $C(-4, -1)$. Under a dilation T from the origin, $A' = T(A) = (-10, -5)$.

(a) If $B' = T(B)$ and $C' = T(C)$, what are the coordinates of B' and C'? $[B'(-30, 5); C'(-20, -5)]$

(b) What is the ratio of a$\triangle ABC$ to a$\triangle A'B'C'$? $[\frac{1}{25}]$

10. Given $\square A'B'C'D'$, the result of a dilation from the origin upon $\square ABCD$. If $A'(-5, -1)$, $B'(-2, -1)$, $C'(-1, 1)$, $D'(-4, 1)$ and $A(-15, -3)$,

(a) What are the pre-images of B', C', and D'? $[B(-6, -3); C(-3, 3); D(-12, 3)]$

(b) What is the ratio of a$\square ABCD$ to a$\square A'B'C'D'$? $[\frac{1}{9}]$