

Jorge Sá Silva  
Bhaskar Krishnamachari  
Fernando Boavida (Eds.)

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# Wireless Sensor Networks

7th European Conference, EWSN 2010  
Coimbra, Portugal, February 2010  
Proceedings

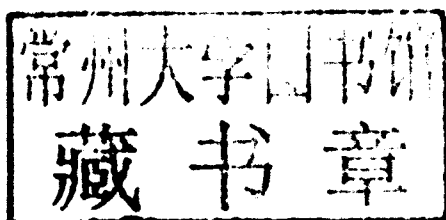


Springer

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# Preface

It is our great pleasure to present the proceedings of the European Conference on Wireless Sensor Networks 2010 (EWSN 2010).

As the field of wireless sensor networks matures, new design concepts, experimental and theoretical findings, and applications have continued to emerge at a rapid pace. As one of the leading international conferences in this area, EWSN has played a substantial role in the dissemination of innovative research ideas from researchers all over the globe.

EWSN 2010 was organized by the University of Coimbra, Portugal, during February 17–19, 2010 and it was the seventh meeting in this series. Previous events were held in Berlin (Germany) in 2004, Istanbul (Turkey) in 2005, Zurich (Switzerland) in 2006, Delft (The Netherlands) in 2007, and Cork (Ireland) in 2009.

A high-quality selection of papers made up EWSN 2010. Based on the reviews and the recommendations from the four live TPC discussions, we selected a total of 21 papers from 109 submissions (19.26% acceptance rate) for EWSN 2010. Topics of interest included hardware design and implementation, operating systems and software, middleware and macroprogramming, communication and network protocols, information and signal processing, fundamental theoretical limits and algorithms, prototypes, field experiments, testbeds, novel applications, including urban sensing, security and fault-tolerance.

Putting together EWSN 2010 was a team effort. We would like to thank the Program Committee members, the reviewers, our sponsors, all authors, and the Organizing Committee for their respective contributions.

We believe the conference program was interesting and that it provided participants with a very valuable opportunity to share ideas with other researchers and practitioners strongly involved in wireless sensor networks.

February 2010

Bhaskar Krishnamachari  
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# Radio Interferometric Angle of Arrival Estimation

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**Abstract.** Several localization algorithms exist for wireless sensor networks that use angle of arrival measurements to estimate node position. However, there are limited options for actually obtaining the angle of arrival using resource-constrained devices. In this paper, we describe a radio interferometric technique for determining bearings from an anchor node to any number of target nodes at unknown positions. The underlying idea is to group three of the four nodes that participate in a typical radio interferometric measurement together to form an antenna array. Two of the nodes transmit pure sinusoids at close frequencies that interfere to generate a low-frequency beat signal. The phase difference of the measured signal between the third array node and the target node constrains the position of the latter to a hyperbola. The bearing of the node can be estimated by the asymptote of the hyperbola. The bearing estimation is carried out by the node itself, hence the method is distributed, scalable and fast. Furthermore, this technique does not require modification of the mote hardware because it relies only on the radio. Experimental results demonstrate that our approach can estimate node bearings with an accuracy of approximately  $3^\circ$  in 0.5 sec.

## 1 Introduction

Spatial coordination in wireless sensor networks (WSNs) has received a lot of attention in recent years. In typical solutions, one or more nodes emit a signal, and some property of that signal (e.g. angle of arrival (AOA), time of arrival (TOA), received signal strength (RSS), etc.) is measured and used to derive bearing or range. Angulation or lateration techniques can then respectively be used to estimate a node's position.

Although several techniques exist for determining node position based on bearing information [1], [2], [3], [4], [5], there are few options for actually measuring signal AOA in WSNs. Currently available methods for bearing estimation require a heavy-weight infrastructure [6], rotating hardware [7], [8], directional antennas [9], and/or expensive and sophisticated sensors [10]. Furthermore, such techniques typically require participating nodes to be stationary for extended periods of time. These constraints are often undesirable for WSN deployments, in

which node size and cost must be kept to a minimum. An AOA approach that does not require additional hardware, runs on the nodes themselves, and is fast enough to support tracking in addition to static localization would be a major step forward.

In this paper, we propose a novel AOA approach for WSNs that uses radio interferometry [11]. The basic idea is to group together three of the four nodes involved in a typical radio interferometric measurement to form an antenna array, which acts as an anchor node. Two transmitters and one receiver are arranged in such a manner that their antennas are mutually orthogonal to minimize parasitic antenna effects (see Figure 1.) The measured phase difference between the receiver in the array and a target node constrains the location of the latter to a hyperbola. The bearing of the target node can then be estimated by computing the angle of the hyperbola asymptote, assuming the target node is not too close to the array.

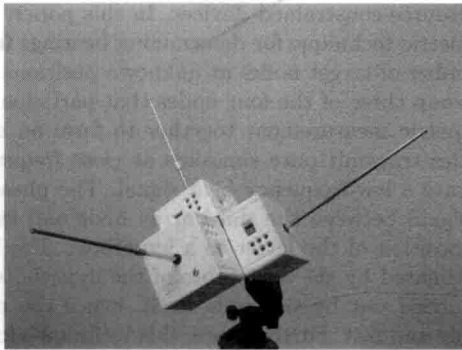


Fig. 1. Antenna array implementation using three XSM motes

We present several new contributions for estimating the angle of arrival in wireless sensor networks.

1. We describe an RF-based technique for determining target-node bearing.
2. We provide a detailed analysis that shows our bearing estimation algorithm is robust to measurement noise and approximation error.
3. We design a real-world implementation using COTS sensor nodes, in which bearing estimation is performed entirely on the resource-constrained motes.
4. We present experimental results that show our approach can rapidly and accurately estimate node bearing.

The remainder of this paper is organized as follows. In Section 2, we discuss other angle of arrival techniques for WSNs. Section 3 describes our proposed system, followed by an error analysis in Section 4. In Section 5, we describe our implementation on a real-world WSN platform. In Section 6, we evaluate our system based on experimental results. Section 7 concludes.

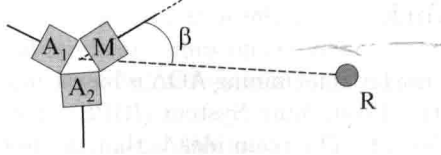
## 2 Related Work

The RF method we use for determining AOA is based on radio interferometry. The Radio Interferometric Positioning System (RIPS) provides accurate RF-based localization in WSNs [11]. The main idea is that the resource-constrained nodes cannot sample a pure RF signal fast enough, but can process the lower-frequency envelope of the beat signal that results from the interference of two high-frequency signals. The difference in signal phase measured by two other nodes is a linear combination of the distances between the transmitters and receivers, modulo the wavelength, and can be used for localizing all participating nodes by solving an optimization problem. Although RIPS has centimeter-accuracy and can support inter-node distances of greater than twice the communication range, it requires centralized processing, suffers from high latency, and involves sampling at several frequencies.

A broad spectrum of acoustic beamforming techniques have been proposed to find the angle of incidence of a signal at an array of sensors. The most common techniques include delay-and-sum beamforming, Capon beamforming [12], MUSIC [13], ESPRIT [14] and min-norm [15] algorithms. Since the time of flight of the signal from the source to sensors in the array varies based on their pairwise distances, sensors receive the signal with different phases. While all of these methods compute the bearing of the source from the data streams sampled at the individual sensors, they differ greatly with respect to their angular resolution as well as their computational requirements. In WSNs, angular resolution is typically within  $10^\circ$  [16].

The Cricket Compass [17] is a device which uses ultrasound to determine orientation with respect to a number of ceiling-mounted beacons. Two receivers are mounted a few centimeters apart on a portable device, and the phase difference of the ultrasonic signal is measured to determine bearing. Although both the Cricket Compass and our approach measure signal phase difference to derive AOA, the two systems use different hardware, signal modalities, phase disambiguation techniques, and bearing derivation algorithms. The Cricket Compass has an accuracy of between  $3^\circ$  and  $5^\circ$ , depending on the orientation of the compass.

Angle of arrival can be used in different ways for spatial coordination. Triangulation, for example, is the process of determining the position of an object from the bearings of known reference positions. Two such reference positions (or three non-collinear ones in degenerate cases) are enough to localize any number of nodes within range. In [2], a method is given to determine position based on the angular separation (the difference in bearings) between beacons. Other angle of arrival positioning approaches have been developed, including multiangulation using subspace methods [4], anchor bearing propagation [1], and semidefinite programming [3]. Bearing estimates can also be useful when anchor positions are unknown. In [18] and [19], mobile robot navigation methods are presented for arriving at a target position by only observing angular separation between two pairs of landmarks.



**Fig. 2.** Array containing a master node (M) and two assistant nodes ( $A_1, A_2$ ). A target node (R) computes its bearing ( $\beta$ ) from the array.

### 3 System Overview

**Radio Interferometric Measurements.** Our system consists of a stationary antenna array and cooperating wireless sensor nodes at unknown positions. We assume that the position of the midpoint of the array is known, as well as the distance between the antennas in the array. The array contains three nodes, a master (M) and two assistants ( $A_1, A_2$ ), as shown in Figure 2. At a predetermined time, the master,  $M$ , and one of the assistants,  $A_1$ , transmit a pure sinusoidal signal at slightly different frequencies, which interfere to create a low-frequency beat signal whose phase is measured by the other assistant in the array,  $A_2$ , and a receiver node,  $R$ , at an unknown position. Such a measurement is termed a radio interferometric measurement (RIM).

The difference in phase,  $\Delta\varphi = \varphi_R - \varphi_{A_2}$ , measured by receiver nodes  $R$  and  $A_2$  is a linear combination of the distances between the transmitters and receivers,

$$\Delta\varphi = \frac{2\pi}{\lambda} (d_{MA_2} - d_{A_1A_2} + d_{A_1R} - d_{MR}) \pmod{2\pi},$$

where  $\lambda$  is the wavelength of the carrier frequency,  $d_{MR}$  is the distance between the master node and target receiver node,  $d_{A_1R}$  is the distance between the assistant transmitter and the target receiver node, and  $d_{MA_1}$ ,  $d_{MA_2}$ , and  $d_{A_1A_2}$  are the respective distances between all pairs of nodes in the array. Note that the nodes in the array are equidistant from each other, and therefore  $d_{MA_2} - d_{A_1A_2} = 0$ , so the phase difference can be simplified:

$$\Delta\varphi = \frac{2\pi}{\lambda} (d_{A_1R} - d_{MR}) \pmod{2\pi}. \quad (1)$$

We denote the distance difference  $d_{A_1R} - d_{MR}$  by  $d_{A_1MR}$  and refer to it as a *t-range*. From Equation (1), we can see that if  $-\frac{\lambda}{2} < d_{A_1MR} < \frac{\lambda}{2}$ , the phase difference will fall in the interval  $(-\pi, \pi)$ . When this is *not* the case, the possible range of  $\Delta\varphi$  will exceed  $2\pi$ , which results in a modulo  $2\pi$  phase ambiguity. To avoid this, we would like the maximum possible distance difference to be less than  $\frac{\lambda}{2}$ . The maximum distance difference will occur when the receiver node is collinear with the transmitters  $M$  and  $A_1$ .  $d_{A_1MR}$  then corresponds to the distance between the master and assistant. Therefore, to eliminate the modulo  $2\pi$  phase ambiguity, we require the distance between antennas in the array to be less than half the wavelength of the carrier frequency.

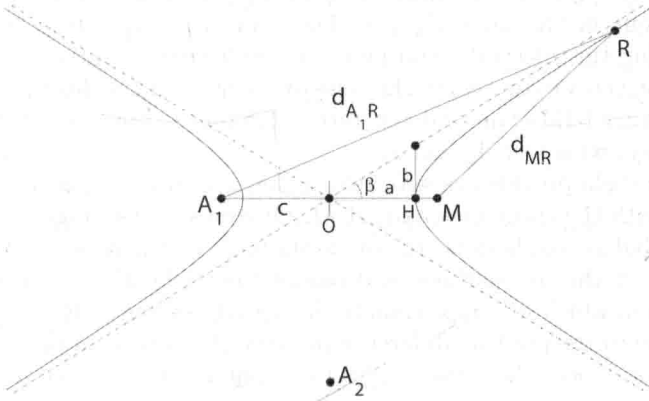
Having removed the modulo operator, we can rearrange Equation (1) so that known values are on the right hand side.

$$d_{A_1MR} = \frac{\Delta\varphi\lambda}{2\pi} \quad (2)$$

The t-range  $d_{A_1MR}$  defines an arm of a hyperbola that intersects the position of node  $R$ , and whose asymptote passes through the midpoint of the line  $\overline{A_1M}$ , connecting the master and assistant nodes. Figure 3 illustrates such a hyperbola with foci  $A_1$  and  $M$ . The absolute value of the distance differences between the foci and any point on a hyperbolic arm is constant, formally defined as

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where  $(x, y)$  are the coordinates of a point on the hyperbola,  $a$  is the distance between the hyperbola center and the intersection  $H$  of the hyperbola with the axis connecting the two foci, and  $b$  is the length of the line segment, perpendicular to the axis connecting the foci, that extends from  $H$  to the asymptote.



**Fig. 3.** The t-range defines a hyperbola that intersects node  $R$ , and whose asymptote passes through the midpoint of the two transmitters in the array.

**Bearing Approximation.** The hyperbola in Figure 3 is centered at  $O$ , and the distance between  $O$  and either focus is denoted by  $c$ . Furthermore, it can be shown that  $c^2 = a^2 + b^2$  [20]. From the figure, we see that the bearing of the asymptote is  $\beta = \tan^{-1}(\frac{b}{a})$ . Therefore, in order to solve for  $\beta$ , we must determine the values of  $b$  and  $a$ .

We can solve for  $a$  by observing that

$$d_{A_1R} - d_{MR} = d_{A_1H} - d_{MH}$$

because, by definition, the distance differences between the foci and all points on the hyperbola are constant. From Figure 3, we see that we can substitute  $(c + a)$  for  $d_{A_1H}$  and  $(c - a)$  for  $d_{MH}$ , and therefore,

$$d_{A_1R} - d_{MR} = (c + a) - (c - a) = 2a.$$

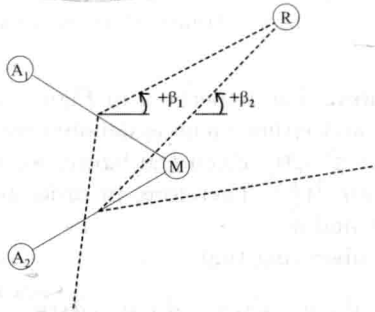
From Equation (2), we know the value of  $d_{A_1R} - d_{MR}$ , which is the t-range, and therefore  $a = \frac{d_{A_1MR}}{2}$ . We can then solve for  $b$ , using  $b = \sqrt{c^2 - a^2}$ . In terms of known distances, the bearing of the asymptote is then defined as

$$\beta = \tan^{-1} \left( \frac{\sqrt{\left(\frac{d_{A_1M}}{2}\right)^2 - \left(\frac{d_{A_1MR}}{2}\right)^2}}{\left(\frac{d_{A_1MR}}{2}\right)} \right). \quad (3)$$

In Figure 3, we see the case where  $d_{A_1MR} > 0$ , and the position of  $R$  lies on the right arm of the hyperbola. If the phase difference is negative (i.e.,  $\varphi_R < \varphi_{A_2}$ ) then the position of  $R$  will lie on the left arm of the hyperbola. When this is the case,  $\beta$  is taken clockwise, and we must adjust it by subtracting it from  $\pi$ .

The line  $\overline{A_1M}$  connecting the two foci is called the transverse axis of the hyperbola, and is a line of symmetry. This implies that although we know  $b$ , we do not know its sign, because mirrored positions on either side of the transverse axis will result in the same  $d_{A_1MR}$ . Therefore, the asymptote bearing  $\beta$  we obtained using this method could be either positive or negative. To find which bearing is correct, we can switch the roles of the assistant nodes in the array and perform another RIM. This will generate a different t-range, and hence another hyperbolic arm with foci  $A_2$  and  $M$ .

Each hyperbola provides us with two angles  $\pm\beta_i$ , where  $\beta_i$  is the angle of the asymptote with the transverse axis,  $\overline{A_iM}$ . Of course, these angles will be offset from the global  $x$ -axis, because the orientation of  $\overline{A_iM}$  may not be 0. Adjusting for this, one of the  $\beta_1$  bearings, and one of the  $\beta_2$  bearings will point in the same direction, which will approximate the actual bearing of  $R$ , as illustrated in Figure 4. Due to the position difference between the centers of the two hyperbolas, we do not expect these two angles to be equal, therefore we define a small



**Fig. 4.** Determining the true bearing of  $R$  is accomplished by selecting  $+\beta$  or  $-\beta$  from each master-assistant pair, such that the difference between the two angles is below the threshold  $\epsilon_\beta$