



Erdinc Sahin Conkur

PATH PLANNING AND OBSTACLE AVOIDANCE FOR REDUNDANT MANIPULATORS

REAL TIME ALGORITHMS



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NOTATION

${}^R\mathbf{T}_H$	General transformation matrix from the base of the robot to the end-effector
${}^{n-1}\mathbf{A}_n$	Transformation matrix from the frame $n-1$ to n
$\mathbf{Trans}(p_x, p_y, p_z)$	Translation matrix
$\mathbf{Rot}(x, \theta)$	Transformation matrix for the pure rotation about x
$\mathbf{Rot}(y, \theta)$	Transformation matrix for the pure rotation about y
$\mathbf{Rot}(z, \theta)$	Transformation matrix for the pure rotation about z
$\mathbf{RPY}(\phi, \theta, \Psi)$	Roll-pitch-yaw transformation
s_1, s_2, s_3, s_4	$\sin(\theta_1), \sin(\theta_2), \sin(\theta_3), \sin(\theta_4)$
c_1, c_2, c_3, c_4	$\cos(\theta_1), \cos(\theta_2), \cos(\theta_3), \cos(\theta_4)$
\mathbf{r}	Position and orientation matrix of the end-effector
$\dot{\mathbf{r}}$	Linear velocity matrix
$\theta, \dot{\theta}, \Delta\theta$	Joint variables, angular velocity and angular change to the joint variables
ω	Angular velocity matrix, acceleration constant
d_x, d_y, d_z	Cartesian co-ordinates of the end-effector
$\alpha_x, \alpha_y, \alpha_z$	Orientation values of the end-effector
$\mathbf{f}(\theta)$	Function establishing the relationship between joint variables and the position and orientation of the end-effector
$\mathbf{J}, \mathbf{J}(\theta)$	Jacobian matrix
\mathbf{H}	Jacobian matrix of the augmented matrix
\mathbf{J}_i	Jacobian matrix of the i th manipulator variable
$\mathbf{J}_{o1}, \mathbf{J}_{o2}$	Jacobian matrices with respect to assigned velocity vector for the first and second obstacles
$\mathbf{J}^{-1}, \mathbf{J}^{\#}, \mathbf{J}^T$	Inverse, generalised inverse and transpose of the Jacobian matrix
\mathbf{J}^*	Singularity robust pseudoinverse
$(\mathbf{p}_{0n} - \mathbf{p}_{0i})$	Position vector of n th frame
\mathbf{k}_{z_i}	Unit vectors of each frame with respect to the base frame
n	Integer value, surface normal at the boundary and number of the control points

m	Dimension space of the end-effector
t	Task space dimension and parameter of the line equation
$\mathbf{W}, \mathbf{W}_1, \mathbf{W}_2$	Symmetric and positive definite weighting matrices
α	Damping factor and the direction of the current point in the potential field
\mathbf{I}	Unit matrix
\mathbf{z}	Arbitrary vector
$\nabla h(\theta)$	Gradient of the function $h(\theta)$
k	Scalar coefficient, integer value and iteration number
$H_i(\theta)$	Specific definition of the vector \mathbf{z}
$D_{\min i,j}$	Distance between the i th manipulator section and the j th obstacle
D_{stop}	Measure for the thickness of the links and the window
$\hat{\mathbf{K}}_o, \hat{\alpha}_{o1}, \hat{\alpha}_{o2}, \alpha_1, \alpha_2, \alpha_{n1}, \alpha_{n2}$	Weighting constants and functions
$\mathbf{r}_1, \mathbf{r}_2$	First and second manipulator variables
$p(\theta)$	Manipulability measure given as a potential function
\mathbf{U}_{x_d}	Goal presented by an attractive pole
\mathbf{x}	Set of parameters describing the end-effector co-ordinates and orientation
$\mathbf{U}_o(\mathbf{x})$	Obstacles are presented by repulsive surfaces
η	Constant gain
ρ_0	Limit distance of the potential field
ρ	Shortest distance to the obstacle
ϕ, Φ	Harmonic function
$\nabla^2 \phi$	Laplacian of ϕ
$\Omega, \partial\Omega$	Domain, boundary of Ω
$\partial \Gamma$	Closed boundary of an arbitrary region $\Gamma \subset \Omega$
dS	Area element on $(\partial \Gamma)$
$\phi(x), f(x)$	Potential functions representing Dirichlet boundary conditions
$g(x)$	Potential functions representing Neumann boundary conditions
ϕ_d	Solution using the Dirichlet conditions
ϕ_n	Solution using the Neumann condition

h	Grid spacing
T	Virtual torque
a, b, c, d, u, v	Coefficients in line and ellipse equations
$p(x_p, y_p), e(x_e, y_e)$	Start and end point of the link
$s(x_s, y_s), k(x_k, y_k), m(x_m, y_m), n(x_n, y_n)$	Intersections points
pk, ps, pe	Vectors establishing intersection between links and margin circles
hm, hn	Vectors by which it can be decided where the link is to be moved
$\Delta\theta_k$	k th link of the manipulator which is moved a fixed amount $\Delta\theta$
Δs_n	Maximum displacement of the end point of the n th link
l_i	Straight line from the start point of k th link to the end point of
n th l_m	length of each line drawn from each u_k value to the link
$\Delta\theta_n$	n th link angular displacement backwards
l_{link}, l_{max}	Link length and physically possible maximum link length
I	Index value
$f_{b_{x-1}}, f_{b_{x+1}}, f_{b_{y-1}}, f_{b_{y+1}}$	Field values at $b_x-1, b_x+1, b_y-1, b_y+1$ for the point $b(b_x, b_y)$
u	Parameter of B-spline curve segment equation
p_i(u)	B-spline curve segment equation for i curve segment
U_k	Matrix of parameter u for B-spline curve segment equation
M_k	Matrix of coefficients for B-spline curve segment equation
P_k	Matrix of control points for B-spline curve segment equation
d_{min}	Minimum distance from a given point to a curve segment
p^u	Tangent vector at a given point p
$b(b_x, b_y)$	Point at the end of each link used to specify the minimum distance
I_i(t)	i th link vector
u_i	i th point on the curve segment
l_m	Minimum distance from the point $b(b_x, b_y)$ on the link
$c(c_x, c_y), d(d_x, d_y)$	Points on the curve segment used to specify the minimum distance
u_c, u_d	Parameter values between the points $c(c_x, c_y)$ and $d(d_x, d_y)$
u_k	k th parameter value on the curve segment
l_n	Distance between the proximal end of the link and the end of
each	line on the curve
θ_{lim}	Limit value on the already-determined angular change

$s, |ki|, |ai|, |ah|, |jh|, |si|$

Vector magnitudes used to determine the most appropriate line segment

$d(d_x, d_y)$

Point on the curve specifying the direction of angular change

u_δ

Parameter value added to existing value while initialising the

links

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