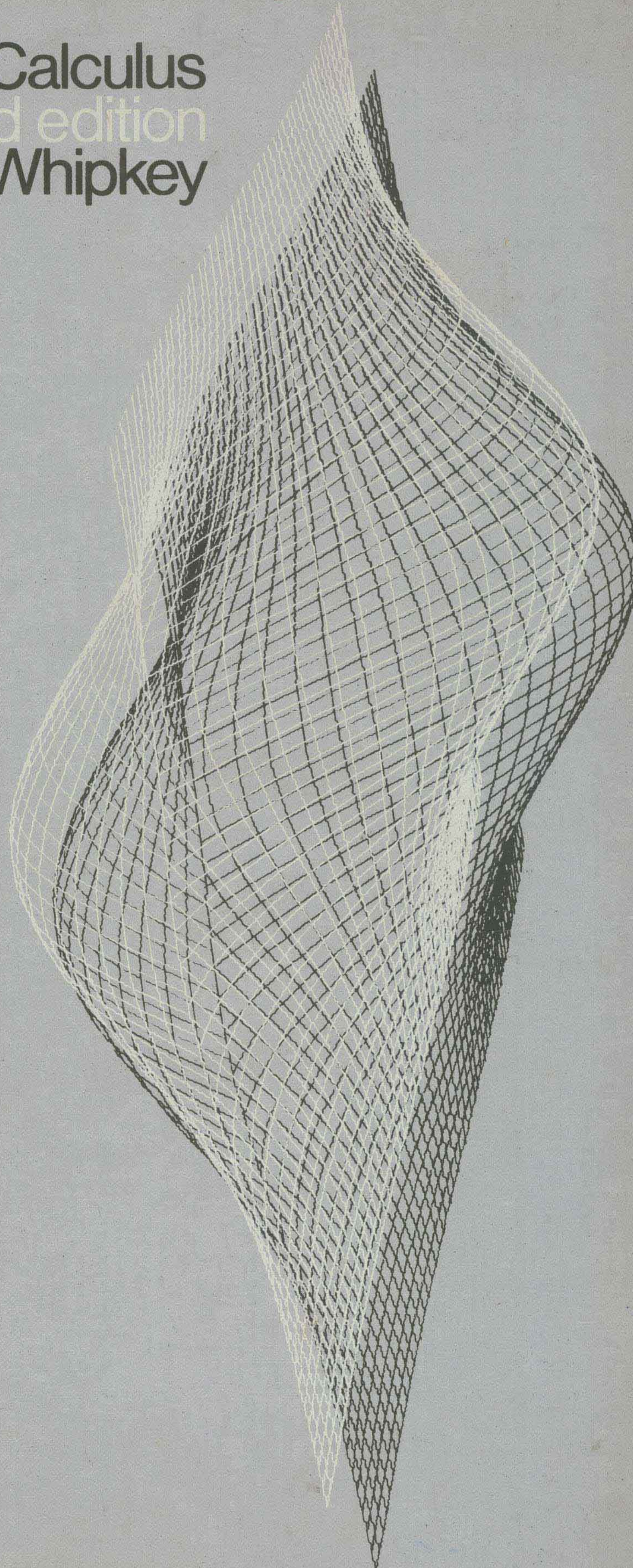
The Power of Calculus
second edition
Whipkey and Whipkey



The Power of Calculus
Second Edition

Kenneth L. Whipkey
Westminster College
Mary Nell Whipkey
Youngstown State University

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To our parents with gratitude

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Preface

This edition, as did the first, evolved from our experience in teaching calculus to social science and business students. Although various textbooks have been written for this purpose, we felt the need for a readable text that will better meet the objectives of a one-term course. Hopefully, the student who reads this text will develop an appreciation of the power, elegance, and efficiency of the calculus.

We are grateful to the thousands of students and to the instructors who used the first edition. We especially are indebted to the instructors who wrote to us and gave us their students' opinions or who permitted the use of the student questionnaire in their classrooms. The present edition covers the topics recommended by The Committee on the Undergraduate Program in Mathematics for students in the biological, management, and social sciences. However, this textbook reflects the opinions and suggestions of colleagues and students and, as such, is a workable model of what the Committee had in mind.

We have minimized abstract theory and symbol shock. However, we agree with the Committee that students should learn the correct and precise definitions of concepts and statements of theorems. It would be an injustice to teach them only techniques. Their future needs are hard to predict, and they will benefit more from a broad preparation than from knowledge of only specialized mechanical techniques. Techniques cannot be used as a substitute for understanding.

We hope this volume accomplishes our objectives. If errors are found, Mr. Whipkey still holds Mrs. Whipkey responsible and Mrs. Whipkey accordingly blames Mr. Whipkey.

Kenneth L. Whipkey
Mary Nell Whipkey

Organization of the Book

Since some students are reviewing mathematics as they take this course, we have resisted the temptation to deal with certain topics simultaneously. Therefore, we have discussed one topic at a time. The spiral effect is used. Thus concepts build from chapter to chapter, culminating in Chapter 7 where previously discussed techniques are reviewed and reapplied.

Changes from the First Edition. At the students' request, the appendix contains a review of basic algebra including exponents and factoring, the answers to more problems are given, tables of logarithms and exponentials are provided, more graphs are used to illustrate concepts, and chapter reviews have been included. The ϵ–δ limit definition has been moved from Chapter 3 to the appendix. The sections on inequalities, limits, logarithms, and exponentials have been rewritten to make them both clearer and easier to teach. The definite integral in Chapter 6 has been defined in a less formal manner and the pace of the entire chapter slowed. Integration by parts has been moved to the section after logarithms where its application is better motivated. The examples have more steps provided and provide better preparation for working the exercises. The exercises contain more applied problems that relate to business, economics and biology. Finally, the format has been improved to set off definitions and theorems from examples and explanations.

Worked-Out Examples. Extensive examples are given. The concepts are reexplained more frequently than in the usual calculus textbook. Since the steps in each example are complete, there are no gaps in the solutions. The purpose of each example is stated in a separate sentence. This crystalizes the main idea that the example conveys.

Problems. The exercises are thorough and occur throughout the chapters. Also, at the end of each chapter is a set of OPTIONAL review problems. Most instructors will not have the class time to formally assign and go over each chapter review. However, students have requested that we include such a review so that they may, on their own, gain additional practice. (The appendix also contains the brief review of algebra, exponents, and logarithms that many students requested.) A knowledge of trigonometry is not required for the text itself; however, there is a self-contained trigonometry section in the appendix. Applied problems do not require an extensive background in any of the social science or business courses, since all terms from these disciplines are defined.

In a one-term course it is not feasible to teach complicated business or social science theory. We believe that applications requiring complicated theory are better handled in later courses.

Theorems. When a theorem is proved, it is thoroughly and carefully proved. When it is to be justified only by an intuitive argument, we clearly state that this argument does not constitute a proof of the theorem. Instructors may include proofs as part of the course or leave them for interested and inquisitive students to study. Omission of proofs does not disrupt the context.

K.L.W.
M.N.W.

Acknowledgments

We express our appreciation to the many professors and students who used the first edition and related their experiences to us, and to Gary Ostedt and Fred Corey, mathematics editors of Wiley for their help and support.

Also, we are indebted to Thomas Nealeigh and Bernard Yozwiak for permission to use the original manuscript in classes at Westminster College and Youngstown State University. This preliminary version was influenced by our high school teachers, Florence Wall and Mary Craver, and by two college professors, Ralph Anttonen and Kenneth Cummins.

Although final decisions on content and style were made by us and are our responsibility, we had the assistance or opinions of the following persons.

Reviewers (line by line)

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Reviewers

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Production

Joanne Jarocki, Kathy Anderson, and Nancy Cooper, Typists
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We welcome any suggestions and comments that users of this edition may have. We look forward to receiving them!

K.L.W.
M.N.W.

Note to the Instructor

Chapter 1 is an optional chapter that may be used at the discretion of the instructor. The Committee on the Undergraduate Program in Mathematics assumes that students are prepared to begin their collegiate mathematics with a calculus-level course. We have found that this is not always true. Therefore, Chapter 1 is included only for the students who need additional background before undertaking calculus.

Although the text was designed for a one-term course of approximately 50 class sessions of 50 minutes each, it is adaptable to other time requirements. A short course of 37 sessions may be devised by limiting the study basically to Chapters 2 to 7. Also a two-term course may be designed by including all optional material and the appendices, and by pacing the course at an optimal rate. By omitting most optional sections and proofs of lesser importance, and by leaving the worked examples for the students to study, the following is a realistic syllabus for approximately a 50-session course of about 50 minutes each.

Chapter 1	4 lectures	Preparation for the Calculus, Sets Order Relationships, the Coordinate Plane, and the Straight Line
2	4 lectures	Functions, Inequalities, and Absolute Values
3	7 lectures	Limits, Differentiation and Continuity of Functions of One Independent Variable
4	6 lectures	Rules of Differentiation
5	10 lectures	Mean Value Theorem, Extremization of Functions and Applications of Differential Calculus
6	11 lectures	The Definite Integral, the Indefinite Integral, the Fundamental Theorem of Integral Calculus, Applications of the Definite Integral (Omit Sections 6.7 and 6.8.)
7	8 lectures	The Logarithmic and Exponential Functions, Differential Equations, Growth and Decay Problems

K.L.W.
M.N.W.

Contents

1
Review and Preparation for the Study of Calculus

1.1 The Arrest of a Seventeenth Century Nonconformist and the Development of Analytic Geometry

You, René Descartes, are hereby ordered to appear before the magistrates to answer charges brought against you, to wit, that you are an atheist, that you lead an unsettled, irresponsible, or disreputable life, moving from place to place without a fixed home, and that you are completely given up to dissipation and licentiousness.

versus

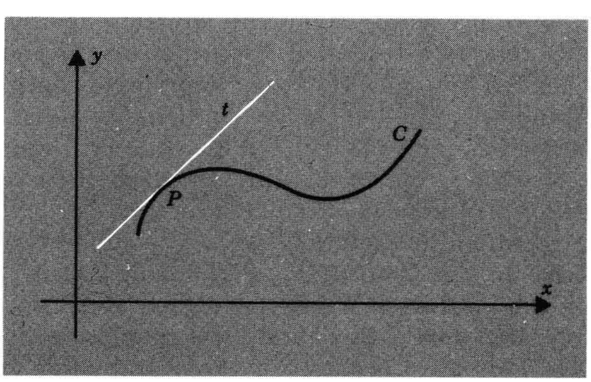
But, there are other men who attain greatness because they embody the potentiality of their own day and magically reflect the future. They express the thoughts which will be everybody's two or three centuries after them. Such a one was Descartes.

Thomas Huxley

Such a one was Descartes—ex-soldier, philosopher, tutor to Queen Christina of Sweden, mathematician, and the person credited with the invention of analytic geometry. However, analytic geometry, which became the foundation for the calculus, was not entirely the brainchild of Descartes. It built on the works of previous seventeenth century mathematicians who were concerned with two basic problems:

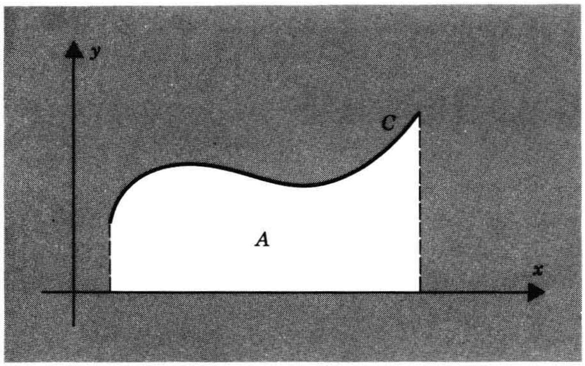
Basic Problem I. Given any curve, find the tangent line drawn to the curve at a point on the curve.

Example. Given curve C. Find the tangent line, t, to C at point P.



Basic Problem II. Given any curve C, find the area under that curve.

Example. Given curve C. Find area A.



Both of these problems had to wait for their solution until the calculus was developed. Therefore, the analytic geometry of Descartes was a necessary prelude to the development of the calculus. Descartes' great contribution was that he gave us a way of dealing with curves in general. Thus, instead of handling circles separately, lines separately, etc., as we are forced to do in high school geometry, we can handle whole groups of figures simultaneously.

Incidentally, the previously mentioned charges against Descartes were dropped by the government of Holland. Nevertheless, he was overjoyed to leave Holland, at the invitation of the Queen of Sweden, to become a royal tutor. Little did he know that the Queen required her lessons to be given at 5 a.m. Descartes, always a late riser, had his life-long routine shattered and died of penumonia four months later. (There may be a moral here concerning math classes given before 9 a.m.)

1.2 Rebellion at the University and the Formalization of the Calculus

An honest courage in these matters will secure all, having law on our sides.
Isaac Newton

These were Newton's words about the situation at Cambridge University and his role as a leader of opposition to the monarchy. The government, headed by James II, was attempting to control thinking at the university and to dictate the choice of university personnel. Fortunately, this time, it was James II who left the country. Unfortunately, for mathematics, Newton's role in the university rebellion caused him to devote the rest of his life to governmental service. For it was his prominence in the rebellion that led to Newton's election as representative from Cambridge to the Convention Parliament. During the next 30 years, Newton served first as Warden and then as Master of the Mint. Most of his free time during this period was spent in studying theology and philosophy. This was in contrast to the prerebellion days when he devoted most of his time to the study of scientific and mathematical theory.

During the prerebellion days Newton had formalized the calculus. Concurrently, Leibnitz, in Germany, had also been able to solve the two Basic Problems: I. finding the tangent line to a curve, and II. finding the area under a curve. The "tools" that Newton and Leibnitz independently invented to solve these two basic problems are now called the *derivative* and the *integral.*

Basic Problem	*Tool Invented to Solve Basic Problem*
I. Find the tangent line to a curve.	Derivative
II. Find the area under a curve.	Integral

One of Newton's and Leibnitz's great claims to fame and honor is that they recognized the two basic problems are related. Therefore, the tools invented to solve these problems, the drivative and the integral, are also related.

Moreover, one of the great bonanzas of history is that the derivative and integral, which were invented to solve two particular problems, have applications to a great number of different problems in diverse academic fields.

1.3 The Power of the Calculus and the Rationale for its Study

The power of the calculus is derived from two sources. First, the derivative and the integral can be used to solve a multitude of problems in many different academic disciplines. The second source of power is found in the relevancy of the calculus to the problems facing mankind. Among the present-day applications of the calculus are the building of abstract models for the study of the ecology of populations, cybernetics and its social impact on man, management practices, economics, and medicine. Two examples of basic problems that you will encounter and be able to solve are:

If a country's population is increasing at a continuous rate of 4 percent each year, in how many years will the country's population double?

The owner of an 80-unit deluxe motel can rent all the units nightly at \$20 per night. However, for each dollar he raises the room rate, 2 units will be vacant. How many units should he rent per night and at what rate in order to maximize his daily income?

Indeed, it is the power of the calculus which demands that we become familiar with its fundamental concepts. However, we will not study the subject as it originated but will take advantage of the improvements made in the calculus from Newton's time to the present.

Therefore, as students of the calculus, we are dealing with ideas that have evolved over hundreds of years and that were formalized by some of the greatest geniuses of all time. As we deal with the products of their thinking, we should not be dismayed that certain ideas and concepts may at our first and subsequent readings seem hazy and confusing to us. The key to understanding these concepts is to repeatedly return to each idea until it becomes meaningful. That the task is not impossible is embodied by the basic premise for the existence of this book. For, in writing this book, we assume it is realistic to present the great ideas of the calculus in a meaningful manner that will minimize symbol shock and function fatigue and also meet the recommendations of the Social Science Research Council (SSRC) and the Committee On The Undergraduate Program In Mathematics (CUPM).

As students in management, business, education, and the social sciences, it is important to realize, as your professional groups have long recognized, the increased need for greater training in mathematics. The Social Science Research Council has for years made this recommendation. Also, the fields of biology and medicine are experiencing a need for more advanced mathematics. Moreover, most graduate schools in business are requiring more sophisticated courses in mathematics, and often a graduate student may shorten his program by exhibiting adequate preparation in the calculus. Even for those students not contemplating graduate school, the calculus serves as the foundation for many upper-division courses, particularly probability and statistics, which are now required in many curricula.

Before one may reconstruct the greatness, orderliness, and power of the calculus, an awareness of certain fundamental techniques and definations must be attained. Those of you who have made adequate and recent preparation for this excursion in calculus may find the rest of this chapter a review of previous work. However, if your mathematics is weak or rusty through nonuse, it will be an experience that will pay future dividends and, as such, is relevant to our goals.

Also, Appendix II, starting on page 313, includes a more extensive review of the basic algebraic skills needed throughout this book. Some of the topics discussed there are factoring, algebraic simplification, solution of algebraic equations, use of exponents and radicals, and work with logarithms. Many students using this book have requested that these topics be included for easy reference. As you progress through the text you will find it desirable, from time to time, to make use of this Appendix.

1.4 Sets—A Very Brief Introduction

In mathematics, the concept of a set is a convenient way to treat a collection of objects. This review of sets and set operations deals with terminology used in the remainder of the text, reviews techniques for solving equations, and also serves as a basis for subsequent courses in probability and statistics.

Just as point and line are undefined words in geometry, the word "set" will be taken as an undefined word in this course. Roughly speaking, a set is a collection of objects. We define the objects or members of a set as the *elements* of that set.

Example 1

Purpose. To illustrate sets and the mathematical shorthand for sets

(capital letters are used to name the sets; small letters are used to name the elements of the sets):
Problem. Form the set composed of the first two letters of the English alphabet.
Solution. Mathematical shorthand for this set is: $A = \{a,b\}$ or $A = \{b,a\}$.

Example 2

Purpose. To illustrate the concept of elements of a set:
Problem. Given the set $A = \{b,a\}$. a is an element of A. b is an element of A. c is not an element of A. Express each of these statements using mathematical symbols.
Solution. Mathematical shorthand for these statements is:

$$a \in A;\ b \in A;\ c \notin A.$$

DEFINITION

Equal Sets. Two sets A and B are equal, written $A = B$, if and only if A and B have exactly the same elements. That is, each element of A belongs to B, and each element of B belongs to A.

The words "if and only if," abbreviated iff, mean:
If two sets are equal, then they have exactly the same elements and moreover,
If two sets have exactly the same elements, then the two sets are equal.
The logical symbol for if and only if (iff) is $\leftrightarrow$.

Example 3

Purpose. To show a second technique for describing a set and to illustrate the concept of equal sets:
Problem. Let $P(x)$ be an assertion about x. Then $\{x|P(x)\}$ will represent the set of all elements x such that $P(x)$ is true. Find $S = \{x|x$ is a U.S. corporation with more than 1 million shareholders$\}$.
Solution. $S = \{$American Telephone and Telegraph, General Motors$\}$.

Example 4

Purpose. To illustrate the usage of if and only if (iff):
Problem. Which is a correct definition of a square?
(a) A four-sided polygon is a square iff its sides are equal.
(b) A four-side polygon is a square iff its vertex angles are right angles.
(c) A four-sided polygon is a square iff it has equal sides and has vertex angles that are right angles.
Solution. (c) is a correct definition. (a) might be a rhombus, (b) could be any rectangle. However, (c) includes all squares and excludes all other possibilities.

DEFINITION

Empty Set. The empty set or null set is the set that has no elements or members.