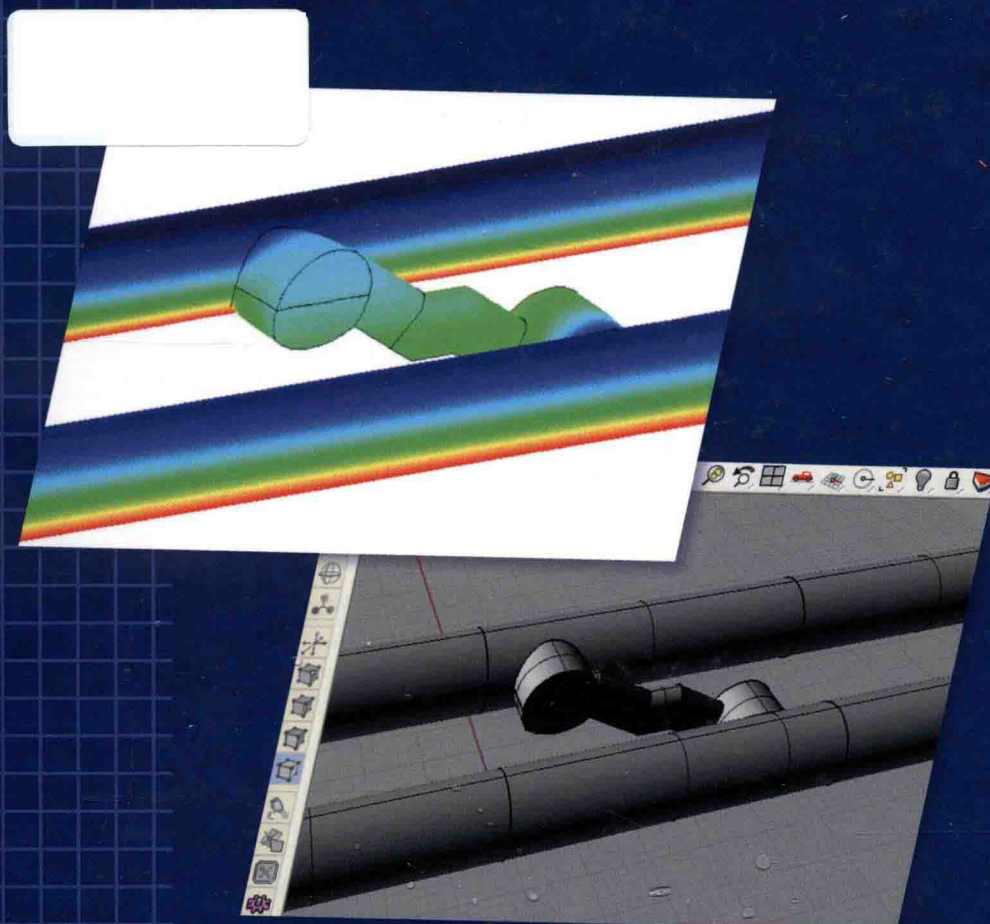


Gernot Beer



Advanced numerical simulation methods

From CAD Data directly to simulation results

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From CAD Data Directly to Simulation Results

Gernot Beer

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The cover is a display of the CAD model and the simulation results (displacement contours) for two tunnels with a cross passage (copyright G. Beer).
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Preface

*The real voyage of discovery consists not in seeking new landscapes,
but in having new eyes.*

M. Proust

In today's age one cannot imagine a world without numerical simulation. It plays an important role in engineering design, weather forecasting and many other aspects of our life. The design of a new type of aircraft such as the A380 for example would not have been possible without it. Simulation methods have become extremely sophisticated and we can now solve large fluid structure interaction problems involving millions or even billions of unknowns. This has been made possible by the tremendous increase in computing power, unthinkable a decade ago and the development of very sophisticated mesh generation programs.

However, as anyone that has been involved in serious simulation can attest, one of the main bottlenecks is still the need to generate a mesh for the analysis. Engineers use Computer Aided Design (CAD) software for the design, be it a new type of aircraft or underground works to create a digital representation of the geometry. The mesh generation process not only repeats this work, but also introduces an unnecessary approximation of the CAD representation. This is a far from ideal situation and came about because CAD and simulation software developed completely independently. The main aim of CAD software is a visual representation of the prototype whereas the aim of the simulation software is the prediction of its behavior, two completely different objectives. Therefore, the digital representation produced by the CAD programs is not *analysis suitable* and hence the need for mesh generation.

How I became interested in isogeometric analysis

I first became aware of the efforts by Tom Hughes and his group to address this problem, fairly late in 2011, after listening to a keynote lecture in the wonderful Greek island of Kos. I subsequently read the fine and very readable book "Isogeometric Analysis".

I soon became very excited about this development, as in my view it offered a real breakthrough in technology similar to the emergence of the Finite Element Method (FEM) or Boundary Element Method (BEM). A seamless integration of CAD and simulation has the potential of changing the industry.

After becoming emeritus professor and having enough time on my hands, I became fascinated by Non-Uniform Rational B-Splines (NURBS) and the possibilities they offered. I found that the MATLAB programming language (or its freeware counterpart OCTAVE) offered a quick and easy way to try out different things, the main advantage being the ease of generating graphical output.

As most engineers, I rely on a graphical representation to help me understand theoretical concepts. After understanding NURBS and their power, I became convinced that a seamless integration of CAD and simulation was a possibility.

The first task was to study the IGES standard for CAD data exchange, a 650 page document. I then developed a parser that could read and interpret the data from the ASCII file created by CAD programs. It turned out that CAD programs use trimming of NURBS surfaces, so there was the task of converting surface and trimming information into an analysis suitable form. This led to the development of a simple algorithm which was published in the journal *Computer Methods in Applied Mechanics and Engineering* in 2015.

Next, I began to develop Finite Element software to understand the subtle differences in the implementation and the advantages that NURBS offered. It became soon clear that the NURBS functions were superior to Serendipity and Lagrange functions currently in use. I also realized that the isoparametric concept (using the same functions for the description of the geometry as for the approximation of the unknown) was not a very efficient way to proceed. Since the geometrical description is taken directly from the CAD program there is no need to refine it, but in most cases there would be a need to refine the approximation. Therefore, the isoparametric concept was abandoned.

Turning to 3-D solid analysis, it soon became obvious that mesh generation could only be avoided if the Boundary Element Method (BEM) was used since this method only involves a definition of bounding surfaces, the same as for CAD. Therefore, the emphasis in the further development was in the BEM.

In May 2013 Stephane Bordas and I organized a course on “Isogeometric methods for numerical simulation” at the International Center for Mechanical Sciences (CISM) in Udine, which was well attended. A research project funded by the Austrian Science Fund (FWF) enabled a small but very active group to be established at the Graz University of Technology, that made some significant progress especially in the field of the isogeometric BEM. Some of this work is included in this book, but at the time of publication a PhD thesis on “Seamless integration of CAD and simulation” is still in preparation.

Why this book was written

The idea of writing this book came from the desire to make a contribution towards the goal of achieving a seamless integration of CAD and simulation without mesh generation. This includes developing a toolkit that can be used by researchers as a basis from which to start new developments. The main emphasis of the book is on implementation and for each stage programs have been developed so readers can try out and get a feel for the new developments. The main aim is to introduce readers to the novel aspects offered by isogeometric modeling. It is not intended as a comprehensive treatise on simulation, so many advanced topics, such as geometrically nonlinear analysis and fluid structure interaction are left out.

This book is written by an engineer for engineers. It is hoped that mathematicians will excuse my sometimes liberal approach to mathematical theory and that the book will provide an impetus for the development of next generation simulation software that integrates seamlessly with CAD.

For whom this book was written

The book was written for users of simulation software, so they can understand the benefits of this new technology and demand progress from a somewhat conservative industry. It is written for software developers, so they can see that this is a technology with a big future. Finally, it is written for researchers with the hope that it will attract more people to work in this exciting new field.

How to read this book

The book is written like a road book, leading the reader on a journey towards understanding isogeometric analysis and the state of development. I have divided the book into stages and after each stage the reader will have gained knowledge that is required for the next stage. A road map is shown in the Introduction.

Programs available

OCTAVE functions, that have been used in this book, are available for readers. They should run with no or minor modifications in MATLAB. Send an e-mail to gernot.beer@tugraz.at with the subject **Book programs** and stating your name and affiliation to request access to a dropbox account that contains the sources. The NURBS toolbox, used by the software, can be downloaded free of charge from <http://octave.sourceforge.net/nurbs/index.html>.

Acknowledgements

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To my daughter Gisela, for designing the cover and the road map and to my wife Sylvia for carefully proofreading the manuscript. Last but not least thanks to the people at CRC Press for their support and excellent work in publishing.

Gernot Beer
Nelson Bay, Australia 2015

About the author

The author started numerical simulation as part of his PhD work in 1973 where he developed a simulation method for arc welding of steel plates. Since that time he has been active in modelling, mainly in the area of underground works (mining and tunnelling) and developed a commercial computer package (BEFE) that combines two methods of analysis. He has been involved as a consultant in many interesting projects around the world such as the design of caverns for the Hadron collider at CERN and an underground power station in Iran.

He has written 3 textbooks on the subject, the first one being about 2 methods (Finite Element and Boundary Element Method) and has coordinated many research projects, including the world's largest project on underground construction (EC project TUNCONSTRUCT).

Currently he is emeritus professor of Graz University of Technology, Austria and conjoint professor of the University of Newcastle, Australia.

Table of contents

<i>Preface</i>	xi
<i>About the author</i>	xv
I Introduction	I
1 A brief history of simulation	1
1.1 The world's first simulation	1
1.2 Emergence of mathematics and mechanics	3
1.3 Computer age	4
2 Basic steps in simulation	9
2.1 Geometry description	9
2.2 Approximation of the unknown	11
2.3 Solution	12
2.4 Recovery of the results	12
3 A change of paradigm: towards a more efficient and accurate simulation	12
4 Organization of the text	13
2 Stage 1: Basis functions	17
1 One-dimensional basis functions	17
1.1 Lagrange and Serendipity functions	17
1.2 From B-splines to NURBS	22
2 Two-dimensional basis functions	33
2.1 Lagrange and Serendipity functions	33
2.2 B-splines	38
2.3 NURBS	38
2.4 T-splines	40
3 Programming	44
4 The NURBS toolkit	45
5 Summary and conclusions	51
3 Stage 2: Geometry	55
1 Coordinate systems	55
1.1 Coordinate transformation	55
2 Curves	56
2.1 Mapping with Serendipity/Lagrange basis functions	56
2.2 Mapping with NURBS	57

3	Programming	58
3.1	NURBS toolkit	58
3.2	Geometry functions	59
3.3	Examples	63
3.4	Example 1: Circular arc	63
3.5	Example 2: Horseshoe tunnel	65
3.6	Example 3: Plate with hole	67
4	Surfaces	68
4.1	Mapping with Serendipity/Lagrange basis functions	69
4.2	Mapping with NURBS basis functions	70
4.3	Programming	71
5	Surface of revolution	73
5.1	Example 1: Cylindrical surface	74
5.2	Example 2: Spherical surface	75
5.3	Example 3: Bell shaped surface	77
6	Lofted surfaces	79
7	NURBS surfaces with cutouts	81
7.1	Analysis suitable trimmed NURBS surfaces	82
8	Infinite NURBS patch	87
8.1	Example	92
9	Summary and conclusions	93
4	Stage 3: Computer Aided Design	95
1	Introduction	95
2	IGES data structure	98
3	How CAD programs describe geometry – entity types	100
3.1	Line entity (110)	100
3.2	Surface of revolution entity (120)	100
3.3	Rational B-spline entity (126)	101
3.4	Rational B-spline surface entity (128)	101
3.5	Boundary entity (141)	101
4	NURBS surfaces	102
5	Trimmed NURBS surfaces	104
6	Summary and conclusions	113
5	Stage 4: Introduction to numerical simulation	117
1	One-dimensional simulation	117
1.1	Ritz method	119
1.2	Approximation	121
2	Steps in the simulation	126
2.1	Description of the geometry	126
2.2	Description of known values	126
2.3	Convergence tests	127
2.4	Approximation of unknown	128
2.5	P-refinement or order elevation	128
2.6	H-refinement, the Finite Element Method	128
2.7	Knot insertion, isogeometric method	132

2.8	K-refinement	132
2.9	Summary and conclusions	133
3	2-D simulation, plane stress and plane strain	135
3.1	Boundary Conditions (BC)	138
3.2	Using one NURBS patch	140
3.3	Comparison with classical FEM	141
3.4	Example	142
3.5	Multiple NURBS patches	143
3.6	Bezièr elements	147
3.7	Trimmed NURBS patches	148
3.8	Convergence test	150
4	Summary and conclusions	150
6	Stage 5: Plates and shells	153
1	Kirchhoff plate	153
1.1	Plates	154
1.2	Examples	158
2	Kirchhoff shells	161
2.1	Example 1: Scordelis roof	162
2.2	Example 2: Trimmed Scordelis roof	164
2.3	Example 3: Arched Scordelis roof	166
3	Multiple patches	168
3.1	Assembly	168
3.2	Example	169
4	Summary and conclusions	172
7	Stage 6: Integral equations	175
1	Introduction	175
2	Trefftz method	176
2.1	Example	180
2.2	Conclusions	183
3	Integral equations	183
3.1	Theorem of Betti	187
3.2	Rigid body trick	191
3.3	Conclusions	193
4	Numerical solution of integral equations	193
4.1	Nyström method	193
4.2	Galerkin method	196
4.3	Collocation	196
4.4	Discretisation	197
5	Summary and conclusions	199
8	Stage 7: The boundary element method for plane problems	201
1	Introduction	201
2	Classical isoparametric approach	201
2.1	Numerical evaluation of integrals	203

3	NURBS based approach	206
3.1	Boundary conditions	210
4	Assembly of multiple patches	211
4.1	Pure Neumann problem	211
4.2	Mixed Neumann/Dirichlet problem	211
4.3	Symmetry	212
5	Postprocessing	213
5.1	Results on the boundary	213
5.2	Results inside the domain	214
6	Programming	216
7	Examples	227
7.1	Potential problem: Flow past isolator	227
7.2	Elasticity problem: Circular excavation in infinite domain	229
7.3	Practical example: Horseshoe tunnel	231
8	Conclusions	233
9	Stage 8: The boundary element method for three-dimensional problems	235
1	Introduction	235
2	Numerical integration	236
2.1	Regular integration	236
2.2	Determination of the optimal number of Gauss points	237
2.3	Regular integration	238
2.4	Nearly singular integration	239
2.5	Weakly singular integration	243
2.6	Infinite patches	246
3	Symmetry	247
4	Multiple patches	249
5	Postprocessing	249
5.1	Stress recovery	249
5.2	Internal stress computation	251
6	Test examples	252
6.1	Infinite tunnel	252
6.2	Loading on infinite half-space	253
7	Examples	253
7.1	Infinite tunnel in infinite domain near tunnel face	253
7.2	Finite tunnel in a semi-infinite domain	254
7.3	Branched tunnel	255
8	Conclusions	260
10	Stage 9: The boundary element method with volume effects	263
1	Introduction	263
2	Effect of body forces and initial strain	263
2.1	Body forces	264
2.2	Effect of initial strain	265
2.3	Solution	266

3	Implementation for plane problems	266
3.1	Geometry definition	266
3.2	Computation of the volume integral	268
4	Implementation for 3-D problems	268
4.1	Geometry definition	269
4.2	Computation of the volume integral	270
5	Iterative solution algorithm	270
6	Inclusions	271
6.1	Computation of body force	273
6.2	Steps in the analysis	275
7	Inelastic behavior	276
7.1	Yield conditions	277
7.2	Determination of plastic strain increment	278
8	Implementation of plasticity for plane problems	280
8.1	Determination of plastic zone	280
8.2	Computation of the volume term	284
8.3	Numerical integration	286
8.4	Internal stress computation	287
8.5	Extension of plastic zone during iteration	287
9	Implementation for 3-D problems	288
9.1	Determination of the plastic zone	288
9.2	Computation of the volume term	289
9.3	Numerical integration	289
10	Programming	289
11	Example	293
12	Summary and conclusions	295
II	Stage I0: The time domain	297
1	Introduction	297
1.1	Bernoulli beam with mass	297
2	Solutions in the frequency domain	298
2.1	Numerical solution	299
3	Solutions in the time domain	301
3.1	Finite difference method	301
3.2	Newmark method	302
4	Programming	306
5	Summary and conclusions	309
	Appendix: Fundamental solutions	311
1	Stress solution $\Sigma(x,y)$	312
2	Derived solution for displacement $\bar{S}(x,y)$	313
3	Derived solution for traction $\bar{R}(x,y)$	314
4	Derived solution for displacement $S(x,y)$	316
5	Derived solution for traction $R(x,y)$	317
6	Derivatives of kernel $S(x,y)$	319
7	Derivatives of kernel $R(x,y)$	320
	<i>Subject index</i>	325

Introduction

I am a little world made cunningly of elements

Donne

For nearly all physical processes in nature a differential equation can be set up, using the fundamental laws of physics such as equilibrium, the preservation of energy and others. This, combined with laws describing the material behavior, allows setting up equations that can be solved analytically for a very limited class of problems.

Numerical simulation evolved from the need to solve real life problems, where exact solutions are not possible. Such solutions were required, for example, for the safe design or for prediction of behavior. Without numerical simulation we would be unable to design tall buildings or the next generation of aircraft. Numerical simulation always involves an approximation of the real world since most problems are too complex to be analyzed and need to be abstracted. For example, the A380 aircraft has millions of parts and it would be impossible to model it in all detail. Abstraction or simplification of the problem is one of the challenges of numerical simulation, that can not be taken over by a computer, at least in the foreseeable future. However, as we will see next many serious mistakes can be made here. Even if the abstraction is handled correctly, there is still another aspect where errors can be made, namely in the approximation of the geometry and the known and unknown values. We will discuss the emergence of numerical simulation and the milestones associated with it next, before introducing the contents of the book.

I A BRIEF HISTORY OF SIMULATION

I.1 The world's first simulation

Numerical simulation actually started quite early and was driven by the need to understand and predict behavior. An early example of simulation dates back to 1745 (see [4]) when Pope Benedict XVII was worried about the stability of the cupola of the St. Peter's dome since it was observed that it had developed cracks. The cupola had circumferential iron rings installed to ensure stability, but it was questioned if these rings were adequate. The dome was built by artists with knowledge handed down by generations, but no design in the modern sense was done.

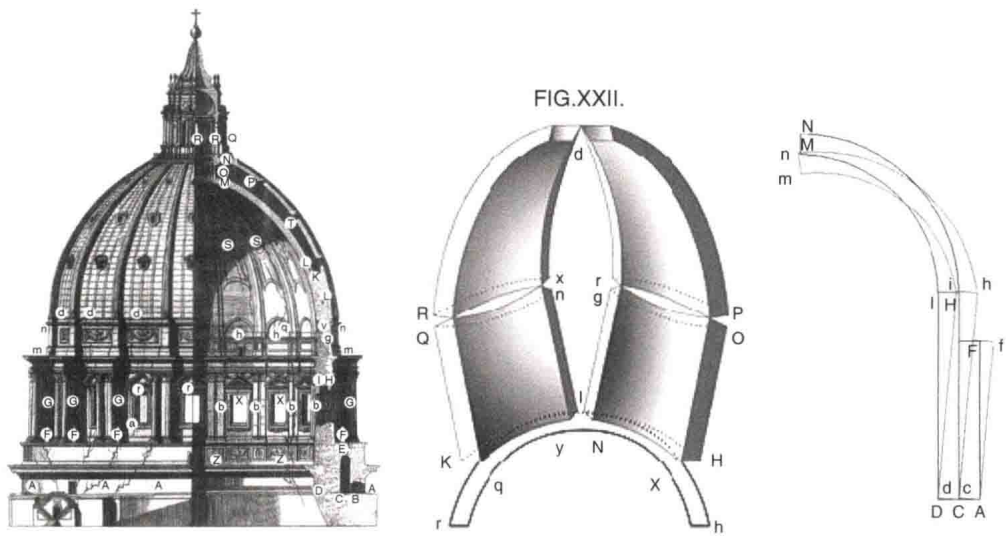


Figure 1 Cupola of St. Peter's dome and model abstractions.

The pope employed 3 mathematicians to solve the problem. The first task for them was to simplify the problem. This involved the following steps:

- Understand the basic mechanics of the problem
- Based on this understanding, identify the important mechanisms
- Develop a mechanical model

The mathematicians realized, that the main driving force for the development of the cracks was the horizontal thrust generated by the weight of the cupola. If they could determine the horizontal thrust, then they could also determine the circumferential force and find out if the iron rings were adequate. However, the mathematical tools available at the time were very limited. For example, they could not deal yet with deformable bodies of arbitrary shape and therefore the mechanical model they devised, involved rigid bodies. To be able to compute the horizontal thrust, hinges had to be assumed as shown in Figure 1. As simulating a 3-D problem was also beyond their capabilities, they simplified it to a plane problem as shown on the right of Figure 1.

However, there was at this stage a good understanding of equilibrium and methods for determining it, published by Johann Bernoulli 27 years earlier: The principle of virtual work or, in particular, the principle of virtual displacements, which states that for a system in equilibrium the work done by the forces times virtual displacements should be zero. It is interesting that this principle is the one used in modern numerical methods, as will be demonstrated later.

The idea of the mathematicians was to apply a virtual displacement to the hinged structure so the horizontal thrust does virtual work. The other virtual work done is due to the derived displacements of parts of the structure times the gravitational force due to self weight. The equation for the equilibrium was quite simple:

$$H \cdot \delta u + \sum \delta W = 0 \tag{1}$$