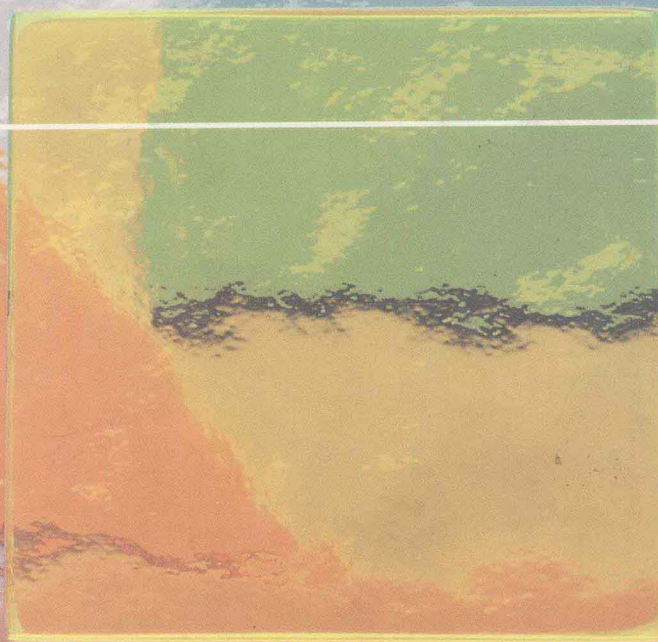


**Larson  
Hostetler  
Edwards**

# Precalculus with Limits

**A GRAPHING APPROACH**



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We have included examples and exercises that use real-life data. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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# PREFACE

*Precalculus with Limits: A Graphing Approach* has two basic goals. The first is to help students develop a good understanding of algebra and trigonometry. The other goal is to show students how algebra can be used as a modeling language for real-life problems.

## FEATURES

The text has several key features designed to help students develop their problem-solving skills, as well as acquire an understanding of mathematical concepts.

## GRAPHICS

The ability to visualize a problem is a critical part of a student's ability to solve the problem. To encourage the development of this skill, the text has many figures in examples and exercise sets and in answers to odd-numbered exercises in the back of the text. Various types of graphics show geometric representations, including graphs of functions, geometric figures, symmetry, displays of statistical information, and numerous screen outputs from graphing technology. All graphs of functions, computer- or calculator-generated for accuracy, are designed to resemble students' actual screen outputs as closely as possible.

## APPLICATIONS

Numerous pertinent applications are integrated throughout every section of the text, both as solved examples and as exercises. This encourages students to use and review their problem-solving skills. The text applications are current and often involve multiple parts. Students learn to apply the process of mathematical modeling to real-world situations in many areas, such as business, economics, biology, engineering, chemistry, and physics. Many applications in the text use real data, and source lines are included to help motivate student interest. We tried to use the data accurately—to give honest and unbiased portrayals of real-life situations. In the cases in which models were fit to data, we used the least-squares method. In all cases the square of the correlation coefficient  $r^2$  was at least 0.95. In most cases it was 0.99 or greater.

## EXPLORING DATA

Modeling real-life problems requires the ability to organize, represent, and interpret real data. The *Exploring Data* sections throughout the text help students learn basic statistical skills to represent data graphically, to find linear and nonlinear models for data, and to describe sets of data. The exercises in the sections allow students to practice these skills using real data.

## EXAMPLES

Each example was carefully chosen to illustrate a particular concept or problem-solving technique. Examples are titled for quick reference, and many include color side comments to justify or explain the solution. We have included problems solved graphically, analytically, numerically, or by a combination of these strategies, and the text helps students choose appropriate approaches to the problems. Several examples also preview ideas from calculus, building an intuitive foundation for future study.

## DISCOVERY

Discovery boxes encourage students to strengthen their intuition and understanding by exploring the relationships between functions and the behaviors of functions. The powerful features of graphing utilities enhance the study of functions because a graph of each step in a solution can be generated quickly and easily for use in the problem-solving process.

## **INTUITIVE FOUNDATION FOR CALCULUS**

Throughout the text, many examples discuss algebraic techniques or graphically illustrate concepts that are used in calculus. These help students develop a natural and intuitive foundation for later work.

## **EXERCISE SETS**

Exercise sets, including warm-up exercises, appear at the end of each text section. Many sets include a group of exercises that provide the graphs of functions involved. Review exercises are included at the end of each chapter, and cumulative tests are included to review what students have learned from the preceding chapters. The opportunity to use calculators is provided with topics that allow students to see patterns, experiment, calculate, or create graphic models.

## **DISCUSSION PROBLEMS**

The discussion problems offer students the opportunity to think, reason, and communicate about mathematics in different ways. Individually or in teams, for in-class discussion, writing assignments, or class presentations, students are encouraged to draw new conclusions about the concepts presented. The problem might ask for further explanation, synthesis, experimentation, or extension of the section concepts. Discussion problems appear at the end of each text section.

## **TECHNOLOGY NOTES**

Technology notes to students appear in the margins throughout the text. These notes offer additional insights, help students avoid common errors, and provide opportunities for problem solving using technology.

## **WARM-UP EXERCISES AND CUMULATIVE TESTS**

We have found that students can benefit greatly from reinforcement of previously learned concepts. Most sections in the text contain a set of ten warm-up exercises that efficiently give students practice using techniques studied earlier in the course that are necessary to master the new ideas presented in the section.

Cumulative tests are included after Chapters 2, 5, 8, and 11. These tests help students assess their level of success and help them maintain the knowledge base they have been building throughout the text—preparing them for other exams and future courses.

These and other features of the text are described in greater detail on the following pages.



# FEATURES OF THE TEXT

## CHAPTER OPENER

Each chapter begins with a list of the topics to be covered. Each section begins with a list of important topics covered in that section.

## DEFINITIONS

All of the important formulas and definitions are boxed for emphasis. Each is also titled for easy reference.

## INTUITIVE FOUNDATION FOR CALCULUS

Special emphasis has been given to skills that are needed in calculus. Many examples include algebraic techniques or graphically show concepts that are used in calculus, providing an intuitive foundation for future work.

### CHAPTER 1

- 1.1 GRAPHS AND GRAPHING UTILITIES
- 1.2 LINES IN THE PLANE
- 1.3 FUNCTIONS
- 1.4 GRAPHS OF FUNCTIONS
- 1.5 SHIFTING, REFLECTING, AND STRETCHING GRAPHS
- 1.6 COMBINATIONS OF FUNCTIONS
- 1.7 INVERSE FUNCTIONS

## FUNCTIONS AND GRAPHS

### 1.1 GRAPHS AND GRAPHING UTILITIES

The Graph of an Equation / Using a Graphing Utility / Determining a Viewing Rectangle / Applications

#### The Graph of an Equation

News magazines often show graphs comparing the rate of inflation, the federal deficit, wholesale prices, or the unemployment rate to the time of year. Industrial firms and businesses use graphs to report their monthly production and sales statistics. Such graphs provide geometric pictures of the way one quantity changes with respect to another.

Frequently, the relationship between two quantities is expressed as an equation. This section introduces the basic procedure for determining the geometric picture associated with an equation.

For an equation in variables  $x$  and  $y$ , a point  $(a, b)$  is a **solution point** if the substitution of  $x = a$  and  $y = b$  satisfies the equation. Most equations have infinitely many solution points. For example, the equation  $3x + y = 5$  has solution points  $(0, 5)$ ,  $(1, 2)$ ,  $(2, -1)$ ,  $(3, -4)$ , and so on. The set of all solution points of an equation is the **graph** of the equation.

SECTION 1.4 GRAPHS OF FUNCTIONS 97

#### Technology Note

When you use a graphing utility to estimate the  $x$ - and  $y$ -values of a relative minimum or relative maximum, the automatic zoom feature will often produce graphs that are nearly flat. To overcome this problem, you can manually change the vertical setting of the viewing rectangle. You can stretch the graph vertically by making the values of  $Y_{\min}$  and  $Y_{\max}$  closer together.

#### Technology Note

Some graphing utilities, such as a TI-85, can automatically determine the maximum and minimum value of a function defined on a closed interval. If your graphing utility has this feature, use it to graph  $y = -x^3 + x$  on the interval  $-1 \leq x \leq 1$  and verify the results of Example 5. What happens if you use the interval  $-10 \leq x \leq 10$ ?

#### EXAMPLE 4 Approximating a Relative Minimum

Use a graphing utility to approximate the relative minimum of the function  $f(x) = 3x^2 - 4x - 2$ .

#### SOLUTION

The graph of  $f$  is shown in Figure 1.35. By using the zoom and trace features of a graphing utility, you can estimate that the function has a relative minimum at the point

$(0.67, -3.33)$ . Relative minimum

Later, in Section 3.1, you will be able to determine that the exact point at which the relative minimum occurs is  $(\frac{2}{3}, -\frac{10}{3})$ .

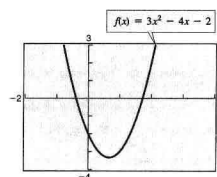


FIGURE 1.35

#### EXAMPLE 5 Approximating Relative Minima and Maxima

Use a graphing utility to approximate the relative minimum and relative maximum of the function  $f(x) = -x^3 + x$ .

#### SOLUTION

A sketch of the graph of  $f$  is shown in Figure 1.36. By using the zoom and trace features of the graphing utility, you can estimate that the function has a relative minimum at the point

$(-0.58, -0.38)$ . Relative minimum

and a relative maximum at the point

$(0.58, 0.38)$ . Relative maximum

If you go on to take a course in calculus, you will learn a technique for finding the exact points at which this function has a relative minimum and a relative maximum.

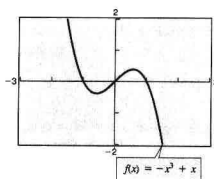


FIGURE 1.36



# CALCULATORS AND COMPUTER GRAPHING UTILITIES

To broaden the range of teaching and learning options, notes for working with calculators occur in many places. Students with access to graphics calculators or graphing utilities can solve exercises both graphically and analytically beginning with Chapter 1. Additionally, many exercises require a graphing utility.

## TECHNOLOGY NOTES

These notes appear in the margins. They provide additional insight and help students avoid common errors.

### Technology Note

Most graphing utilities have two log keys:  $\ln$  is the natural logarithmic function and  $\log$  is the logarithm to base 10. You can graph logarithms to other bases,  $y = \log_a(x)$ , by using the change of base formula.

$$y = \log_a(x) = \frac{\ln x}{\ln a} = \frac{\log x}{\log a}$$

### EXAMPLE 1 Changing Bases

Use common logarithms to evaluate the following.

- A.  $\log_4 30$       B.  $\log_2 14$

#### SOLUTION

- A. Using the change of base formula with  $a = 4$ ,  $b = 10$ , and  $x = 30$ , convert to common logarithms and obtain

$$\log_4 30 = \frac{\log_{10} 30}{\log_{10} 4} = \frac{1.47712}{0.60206} \approx 2.4534.$$

- B. Using the change of base formula with  $a = 2$ ,  $b = 10$ , and  $x = 14$ , convert to common logarithms and obtain

$$\log_2 14 = \frac{\log_{10} 14}{\log_{10} 2} = \frac{1.14613}{0.30103} \approx 3.8074.$$

### EXAMPLE 2 Changing Bases

Use natural logarithms to evaluate the following.

- A.  $\log_4 30$       B.  $\log_2 14$

#### SOLUTION

- A. Using the change of base formula with  $a = 4$ ,  $b = e$ , and  $x = 30$ , convert to natural logarithms and obtain

$$\log_4 30 = \frac{\ln 30}{\ln 4} = \frac{3.40120}{1.38629} \approx 2.4534.$$

- B. Using the change of base formula with  $a = 2$ ,  $b = e$ , and  $x = 14$ , convert to natural logarithms and obtain

$$\log_2 14 = \frac{\ln 14}{\ln 2} = \frac{2.63906}{0.693147} \approx 3.8074.$$

Note that the results agree with those obtained in Example 1, using common logarithms.

### Properties of Logarithms

You know from the previous section that the logarithmic function with base  $a$  is the *inverse* of the exponential function with base  $a$ . Thus, it makes sense that the properties of exponents should have corresponding properties involving logarithms. For instance, the exponential property  $a^0 = 1$  corresponds to the logarithmic property  $\log_a 1 = 0$ .

### DISCOVERY

Use a graphing utility to graph  $y = \ln x$  and  $y = \ln x / \ln a = \log_a x$  with  $a = 2, 3$ , and  $5$  on the same viewing rectangle. (Use a viewing rectangle in which  $0 \leq x \leq 10$  and  $-4 \leq y \leq 4$ .) On the interval  $(0, 1)$ , which graph is on top? Which is on the bottom? On the interval  $(1, \infty)$ , which graph is on top? Which is on the bottom?

Throughout the text, when solving equations, be sure to check your solutions—either *algebraically*, by substituting in the original equation, or *graphically*. For instance, the graphs shown in Figure 2.22 visually reinforce the solutions obtained in Example 1.

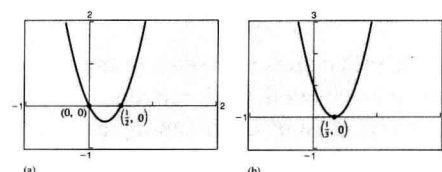


FIGURE 2.22

### EXAMPLE 2 Extracting Square Roots

Solve the quadratic equations.

- A.  $4x^2 = 12$       B.  $(x - 3)^2 = 7$

#### SOLUTION

- A.  $4x^2 = 12$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

- B.  $(x - 3)^2 = 7$

$$x - 3 = \pm\sqrt{7}$$

$$x = 3 \pm \sqrt{7}$$

Original equation  
Divide both sides by 4  
Extract square roots  
Original equation  
Extract square roots  
Add 3 to both sides

The graphs of  $y = 4x^2 - 12$  and  $y = (x - 3)^2 - 7$ , shown in Figure 2.23, reinforce these solutions.

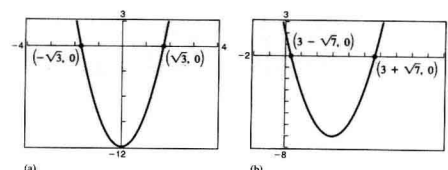


FIGURE 2.23

## DISCOVERY

Throughout the text, the discovery boxes take advantage of the power of graphing utilities to explore and examine the behavior of complicated functions.

## PROBLEM SOLVING

A consistent strategy for solving problems is emphasized throughout: analyze the problem, create a verbal model, construct an algebraic model, solve the problem, and check the answer in the statement of the original problem. This problem-solving process has wide applicability and can be used with analytical, graphical, and numerical approaches to problem solving.

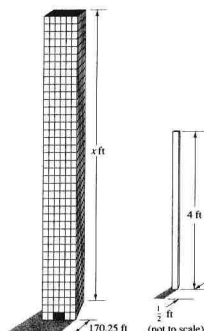


FIGURE 2.3

### EXAMPLE 6 An Application Involving Similar Triangles

To measure the height of the World Trade Center (in New York City), you measure the shadow cast by the building to be 170.25 feet long, as shown in Figure 2.3. Then you measure the shadow cast by a 4-foot post and find that its shadow is 6 inches long. Use this information to find the height of the building.

#### SOLUTION

To solve this problem, use a theorem from geometry that states that the ratios of corresponding sides of similar triangles are equal.

VERBAL MODEL:	$\frac{\text{Height of building}}{\text{Length of shadow}} = \frac{\text{Height of post}}{\text{Length of shadow}}$	
LABELS:	Height of building = $x$	(feet)
	Length of building's shadow = 170.25	(feet)
	Height of post = 4	(feet)
	Length of post's shadow = $\frac{1}{2}$	(feet)
EQUATION:	$\frac{x}{170.25} = \frac{4}{\frac{1}{2}}$	
	$x = 1362 \text{ feet}$	

The World Trade Center is about 1362 feet high.

### EXAMPLE 7 An Inventory Problem

A store has \$30,000 of inventory in 12-inch and 19-inch television sets. The profit on a 12-inch set is 22%. The profit on a 19-inch set is 40%. If the profit for the entire stock is 35%, how much was invested in each type of television set?

#### SOLUTION

VERBAL MODEL:	$\text{Profit from 12-inch sets} + \text{Profit from 19-inch sets} = \text{Total profit}$	
LABELS:	Inventory of 12-inch sets = $x$	(dollars)
	Inventory of 19-inch sets = $30,000 - x$	(dollars)
	Profit from 12-inch sets = $0.22x$	(dollars)
	Profit from 19-inch sets = $0.40(30,000 - x)$	(dollars)
	Total profit = $0.35(30,000) = 10,500$	(dollars)
EQUATION:	$0.22x + 0.40(30,000 - x) = 0.35(30,000)$	
	$0.22x + 12,000 - 0.4x = 10,500$	
	$-0.18x = -1500$	
	$x \approx \$8333.33 \text{ 12-inch sets}$	
	$30,000 - x \approx \$21,666.67 \text{ 19-inch sets}$	

### EXAMPLE 14 The Cost of a New Car

Between 1970 and 1990, the average cost of a new car increased according to the model

$$C = 30.5t^2 + 4192, \quad 0 \leq t \leq 20,$$

as shown in Figure 2.34. In this model the cost is measured in dollars and the time  $t$  represents the year with  $t = 0$  corresponding to 1970. If the average cost of a new car continued to increase according to this model, when would the average cost reach \$20,000? (Source: Commerce Department, American Automobile Association)

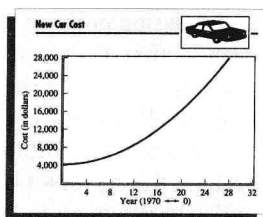


FIGURE 2.34

#### SOLUTION

To solve this problem, let the cost be \$20,000 and solve the equation  $20,000 = 30.5t^2 + 4192$  for  $t$ .

$30.5t^2 + 4192 = 20,000$	Set cost equal to 20,000
$30.5t^2 = 15,808$	Subtract 4192 from both sides
$t^2 \approx 518.295$	Divide both sides by 30.5
$t \approx \sqrt{518.295}$	Extract positive square root

Thus, the solution is  $t \approx 23$ . Because  $t = 0$  represents 1970, you can conclude that the average cost of a new car reached \$20,000 in 1993. You could use a graphing utility to solve this problem. By finding the positive zero of  $30.5t^2 + 4192 - 20,000 = 0$ , you can see that  $t \approx 23$ .

## EXAMPLES

The text contains over 650 examples. They are titled for easy reference, and many include side comments that explain or justify steps in the solution. Students are encouraged to check their solutions.

## EXPLORING DATA

Students work with real data to develop basic statistical skills in these sections, learning to graph, model, and describe sets of data in the examples and exercises.

### AN APPLICATION OF SLOPE

In 1982, a college had an enrollment of 5000 students. By 1992, the enrollment had increased to 7000 students.

- What is the average annual change in enrollment from 1982 to 1992?
- Use the average annual change in enrollment to estimate the enrollment in 1986, 1990, and 1994.

Year	1982	1986	1990	1992	1994
Enrollment	5000			7000	

- Graph the line represented by the data given in the table in part (b). What is the slope of this line?
- Write a short paragraph that compares the concepts of *slope* and *average rate of change*.

### WARM-UP

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, simplify the expression.

1.  $\frac{4 - (-3)}{-3 - (-1)}$

2.  $\frac{-5 - 8}{0 - (-3)}$

3. Find  $-1/m$  for  $m = 4/5$ .

4. Find  $-1/m$  for  $m = -2$ .

In Exercises 5–10, solve for  $y$  in terms of  $x$ .

5.  $2x - 3y = 5$

6.  $4x + 2y = 0$

7.  $y - (-4) = 3(x - (-1))$

8.  $y - 7 = \frac{1}{3}(x - 3)$

9.  $y - (-1) = \frac{3 - (-1)}{2 - 4}(x - 4)$

10.  $y - 5 = \frac{3 - 5}{0 - 2}(x - 2)$

## 2.6 EXPLORING DATA: LINEAR MODELS AND SCATTER PLOTS

Scatter Plots / Fitting a Line to Data

### Technology Note

Most graphing utilities have built-in statistical programs that can create scatter plots. Use your graphing utility to plot the points given in Table 2.2.

### Scatter Plots

Many real-life situations involve finding relationships between two variables, such as the year and the number of people in the labor force. In a typical situation, data are collected and written as a set of ordered pairs. The graph of such a set is called a **scatter plot**.

### EXAMPLE 1 Constructing a Scatter Plot

The data in Table 2.2 show the number of people  $P$  (in millions) in the United States who were part of the labor force from 1980 through 1990. In the table,  $t$  represents the year, with  $t = 0$  corresponding to 1980. Sketch a scatter plot of the data. (Source: U.S. Bureau of Labor Statistics)

TABLE 2.2

$t$	0	1	2	3	4	5	6	7	8	9	10
$P$	109	110	112	113	115	117	120	122	123	126	126

### SOLUTION

Begin by representing the data with a set of ordered pairs.

(0, 109), (1, 110), (2, 112), (3, 113), (4, 115), (5, 117),  
(6, 120), (7, 122), (8, 123), (9, 126), (10, 126)

Then plot each point in a coordinate plane, as shown in Figure 2.48.

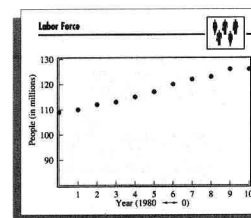


FIGURE 2.48

## DISCUSSION PROBLEMS

A discussion problem appears at the end of each section. Each one encourages students to think, reason, and write about mathematics, individually or in groups. Presenting the mathematics in a different way from in the section, these problems emphasize synthesis and experimentation.

## WARM-UPS

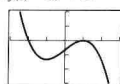
Each section (except those in Prerequisites) contains a set of 10 warm-up exercises for students to review and practice the previously learned skills that are necessary to master the new skills and concepts presented in the section. All warm-up exercises are answered in the back of the text.

## SECTION 3.2 • EXERCISES

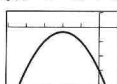
- Compare the graph of  $f$  with the graph of  $y = x^3$ .
  - $f(x) = (x - 2)^3$
  - $f(x) = x^3 - 2$
  - $f(x) = (x - 2)^3 - 2$
  - $f(x) = -\frac{1}{2}x^3$
- Compare the graph of  $f$  with the graph of  $y = x^4$ .
  - $f(x) = (x + 3)^4$
  - $f(x) = x^4 - 3$
  - $f(x) = 4 - x^4$
  - $f(x) = \frac{1}{2}(x - 1)^4$

In Exercises 3–10, match the polynomial function with its graph and describe the viewing rectangle. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

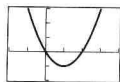
- $f(x) = -3x + 5$
- $f(x) = -2x^2 - 8x - 9$
- $f(x) = -\frac{1}{2}x^3 + x - \frac{3}{2}$
- $f(x) = 3x^2 - 9x + 1$
- $f(x) = -\frac{1}{2}x^4 + 2x^2$
- $f(x) = x^2 - 5x^3 + 4x$



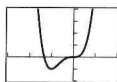
(a)



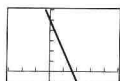
(b)



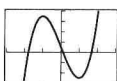
(c)



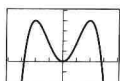
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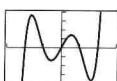
(e)



(f)



(g)



(h)

In Exercises 11–14, use a graphing utility to graph the functions  $f$  and  $g$  in the same viewing rectangle. Zoom out sufficiently far to show that the right-hand and left-hand behavior of  $f$  and  $g$  are identical.

- $f(x) = x^3 - 9x + 1$   $g(x) = x^3$
- $f(x) = -\frac{1}{2}(x^3 - 3x + 2)$   $g(x) = -\frac{1}{2}x^3$
- $f(x) = -(x^4 - 4x^3 + 16x)$   $g(x) = -x^4$
- $f(x) = 3x^4 - 6x^2$   $g(x) = 3x^4$

In Exercises 15–20, determine the right-hand and left-hand behavior of the graph of the polynomial function based on the Leading Coefficient Test. Use a graphing utility to verify your result.

- $f(x) = 2x^2 - 3x + 1$
- $g(x) = 5 - \frac{2}{3}x - 3x^2$
- $h(t) = -\frac{1}{2}(t^2 - 5t + 3)$
- $f(x) = -\frac{1}{2}(x^3 + 5x^2 - 7x + 1)$
- $f(x) = 6 - 2x + 4x^2 - 5x^3$
- $f(x) = \frac{3x^4 - 2x + 5}{4}$

In Exercises 21–36, find all the real zeros of the polynomial function. In each case, state whether you solved the problem algebraically or graphically and give reasons for your choice.

- $f(x) = x^2 - 25$
- $h(t) = t^2 - 6t + 9$
- $f(x) = x^2 + x - 2$
- $f(x) = 3x^2 - 12x + 3$
- $f(t) = t^3 - 4t^2 + 4t$
- $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$
- $g(t) = 2x^4 - 2x^2 - 40$
- $g(t) = t^3 - 6t^2 + 9t$
- $f(x) = 5x^4 + 15x^2 + 10$
- $f(x) = x^3 - 4x^2 - 25x + 100$
- $f(x) = 49 - x^2$
- $f(x) = x^2 + 10x + 25$
- $g(x) = 2x^2 - 3x + 1$
- $g(x) = 5 - \frac{2}{3}x - 3x^2$
- $h(t) = -\frac{1}{2}(t^2 - 5t + 3)$
- $f(x) = -\frac{1}{2}(x^3 + 5x^2 - 7x + 1)$
- $f(x) = 6 - 2x + 4x^2 - 5x^3$
- $f(x) = \frac{3x^4 - 2x + 5}{4}$

- Navigation** On a certain map, Minneapolis is 6.5 inches due west of Albany, Phoenix is 8.5 inches from Minneapolis, and Phoenix is 14.5 inches from Albany (see figure).
  - Find the bearing of Minneapolis from Phoenix.
  - Find the bearing of Albany from Phoenix.

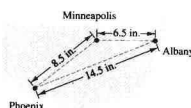


FIGURE FOR 40

- Baseball** In a (square) baseball diamond with 90-foot sides, the pitcher's mound is 60 feet from home plate.
  - How far is it from the pitcher's mound to third base?
  - When a runner is halfway from second to third, how far is the runner from the pitcher's mound?
- Baseball** The baseball player in center field is playing approximately 330 feet from the television camera that is behind home plate. A batter hits a fly ball that goes to the wall 420 feet from the camera (see figure). Approximate the number of feet that the center fielder had to run to make the catch if the camera turned  $9^\circ$  in following the play.

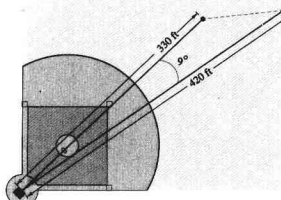


FIGURE FOR 42

## EXERCISES

The approximately 7000 exercises—computational, conceptual, exploratory, and applied problems—are designed to build competence, skill, and understanding. Each exercise set is graded in difficulty to allow students to gain confidence as they progress. Many exercises require the use of a graphing utility. All odd-numbered exercises are solved in detail in the *Study and Solutions Guide*, with answers appearing in the back of the text.

## GEOMETRY

Geometric formulas and concepts are reviewed throughout the text. For easy reference, common formulas are given inside the back cover.

- Awning Design** A retractable awning lowers at an angle of  $50^\circ$  from the top of a patio door that is 7 feet high (see figure). Find the length  $x$  of the awning if no direct sunlight is to enter the door when the angle of elevation of the sun is greater than  $65^\circ$ .

- Circumscribed and Inscribed Circles** Let  $R$  and  $r$  be the radii of the circumscribed and inscribed circles of a triangle  $ABC$ , respectively, and let  $s = (a + b + c)/2$  (see figure). Prove the following.
  - $2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
  - $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$

$$(a) \quad 2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$(b) \quad r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

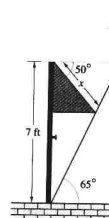


FIGURE FOR 43

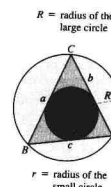


FIGURE FOR 44

**Circumscribed and Inscribed Circles** In Exercises 45 and 46, use the results of Exercise 44.

- Given the triangle with  $a = 25$ ,  $b = 55$ , and  $c = 72$ , find the areas of (a) the triangle, (b) the circumscribed circle, and (c) the inscribed circle.
- Find the length of the largest circular track that can be built on a triangular piece of property whose sides measure 200 feet, 250 feet, and 325 feet.

- Use the Law of Cosines to prove that  $\frac{1}{2}bc(1 + \cos A) = \frac{a+b+c}{2} \cdot \frac{a+b+c}{2}$ .
- Use the Law of Cosines to prove that  $\frac{1}{2}bc(1 - \cos A) = \frac{a-b+c}{2} \cdot \frac{a+b-c}{2}$ .

## APPLICATIONS

Real-world applications are integrated throughout the text in examples and exercises. This offers students insight about the usefulness of algebra and trigonometry, develops strategies for solving problems, and emphasizes the relevance of the mathematics. Titled for reference, many of the applications involve multiple parts, use current real data, and include source lines.

## GRAPHICS

Students must be able to visualize problems in order to solve them. To develop this skill and reinforce concepts, the text has over 1100 figures.

20. **School Enrollment** The table gives the preprimary school enrollments  $y$  (in millions) for the years 1985 through 1991 where  $t = 5$  corresponds to 1985. (Source: U.S. Bureau of the Census)

	5	6	7	8	9	10	11
	10.73	10.87	10.87	11.00	11.04	11.21	11.37

## CHAPTER 2 · REVIEW EXERCISES

In Exercises 1 and 2, determine whether the equation is an identity or a conditional equation.

1.  $6 - (x - 2)^2 = 2 + 4x - x^2$
2.  $3(x - 2) + 2x = 2(x + 3)$

- $$2. \quad 3(x - 2) + 2x = 2(x + 3)$$

In Exercises 3 and 4, determine whether the values of  $x$  are solutions of the equation.

Equation	Values
3. $3x^2 + 7x + 5 = x^2 + 9$	(a) $x = 0$ (b) $x = -4$ (c) $x = \frac{1}{2}$ (d) $x = -1$
4. $6 + \frac{3}{x-4} = 5$	(a) $x = 4$ (b) $x = 0$ (c) $x = -2$ (d) $x = 1$

In Exercises 5–28, solve the equation (if possible) and check your answer either algebraically or graphically.

5.  $3x - 2(x + 5) = 10$
6.  $4(x + 3) - 3 = 2(4 - 3x) - 4$
7.  $3\left(1 - \frac{1}{5t}\right) = 0$
8.  $\frac{1}{x-2} = 3$
9.  $6x^2 = 5x + 4$
10.  $15 + x - 2x^2 = 0$
11.  $(x + 4)^2 = 18$
12.  $16x^2 = 25$
13.  $x^2 - 12x + 30 = 0$
14.  $5x^2 - 12x^3 = 0$
15.  $4x^2 - 12x + 8t = 0$
16.  $2 - x^2 = 0$
17.  $\frac{4}{(x-4)^2} = 1$
18.  $\frac{4}{x-3} - \frac{4}{x} = 1$
19.  $\sqrt{x+4} = 3$
20.  $\sqrt{x-2} - \frac{8}{x} = 0$

- Use a computer or calculator to find the least squares regression line. Use the equation to estimate enrollment in 1992.
- Make a scatter plot of the data and sketch the graph of the regression line.
- Use the computer or calculator to determine the correlation coefficient.

In Exercises 29–36, use a graphing utility to solve the equation (if possible).

29.  $x^2 + 6x - 3 = 0$
30.  $12x^3 - 84x^2 + 120x = 0$
31.  $5\sqrt{x} - \sqrt{x-1} = 6$
32.  $\sqrt{3x-2} = 4-x$
33.  $\frac{1}{x} + \frac{1}{x+1} = 2$
34.  $\frac{1}{(t+1)^2} = 1$
35.  $|x^2 - 3| = 2x$
36.  $|2x+3| = 7$

In Exercises 37–40, solve the equation for the indicated variable.

37. Solve for  $r$ :  $V = \frac{1}{3}\pi r^2 h$   
 38. Solve for  $X$ :  $Z = \sqrt{R^2 - X^2}$   
 39. Solve for  $p$ :  $L = \frac{k}{3\pi r^2 p}$   
 40. Solve for  $v$ :  $E = 2k\omega\left(\frac{v}{2}\right)^2$

## REVIEW EXERCISES

A set of review exercises at the end of each chapter gives students an opportunity for additional practice. The review exercises include computational, conceptual, and applied problems covering a wide range of topics.

## CUMULATIVE TESTS

Cumulative tests appear after Chapters 2, 5, 8, and 11. These tests help students judge their mastery of previously covered concepts. They also help students maintain the knowledge base they have been building throughout the text, preparing them for other exams and future courses.

### CUMULATIVE TEST FOR CHAPTERS 3–5

Take this test as you would take a test in class. After you are done, check your work against the answers in the back of the book.

1. Use a graphing utility to graph the quadratic function  $f(x) = \frac{1}{4}(4x^2 - 12x + 17)$ .
2. Find the coordinates of the vertex of the parabola.
3. Find a quadratic function whose graph is a parabola with vertex at  $(0, 6)$  and passes through the point  $(2, 5)$ .
4. Describe the right-hand and left-hand behavior of the polynomial function  $f(x) = -\frac{1}{3}x^4 + 3x^2 - 2x + 1$ .
5. Find a polynomial function with integer coefficients whose zeros are  $-4, \frac{1}{2}$ , and  $2$ .
6. Sketch a graph of the function  $f(x) = \frac{1}{2}x^3 - 2x^2$  without the aid of a graphing utility.
7. Perform the division:  $\frac{6x^2 - 4x^2}{2x^2 + 1}$
8. Use synthetic division to perform the division:  $\frac{3x^3 - 5x^2 + 4}{x - 2}$
9. Find the rational zeros of the function  $f(x) = 6x^3 - 25x^2 - 8x + 48$ .  
(Hint: Use a graphing utility to eliminate some of the possible rational zeros.)
10. Use a graphing utility to approximate (accurate to one decimal place) the zero of the function  $g(x) = x^3 - 5x - 2$  in the interval  $[2, 3]$ .
11. Sketch a graph of each of the following:
  - (a)  $g(x) = \frac{2x}{x-3}$
  - (b)  $g(x) = \frac{2x^2}{x-3}$
12. Sketch a graph of each of the following:
  - (a)  $f(x) = 6i2^{-x}$
  - (b)  $g(x) = \log_2 x$
13. Evaluate without the aid of a calculator: logs 125
14. Use the properties of a logarithm to write the expression  $2 \ln x + \frac{1}{3} \ln(x + 5)$  as the logarithm of a single quantity.
15. Solve each of the following, giving your answers accurate to two decimal places:
  - (a)  $6e^{2x} = 72$
  - (b)  $\log_3 x + \log_2 5 = 6$
16. On the day a grandchild is born, a grandparent deposits \$2500 into a fund earning 7.5%, compounded continuously. Determine the balance in the account at the time of the grandchild's 25th birthday.
17. Express the angle  $\pi/9$  in degree measure and sketch the angle in standard position.

17. Sketch the angle  $-120^\circ$  in radian measure as a multiple of  $\pi$  and express the angle in standard position.
18. The terminal side of an angle  $\theta$  in standard position passes through the point  $(12, 5)$ . Evaluate the six trigonometric functions of the angle.
19. If  $\cos \theta = r$ , where  $m/2 < \theta < \pi$ , find  $\sin \theta$  and  $\tan \theta$ .
20. Use a calculator to approximate  $\sin(-1.25)$ . Round your answer to four decimal places.
21. Use a calculator to approximate two values of  $\theta$  ( $0^\circ \leq \theta < 360^\circ$ ) such that  $\sec \theta = 2.125$ . Round your answers to two decimal places.
22. Sketch the graph of each of the following functions through two periods.
  - (a)  $y = -3 \sin 2x$
  - (b)  $f(x) = 2 \cos\left(x - \frac{\pi}{2}\right)$
  - (c)  $g(x) = \tan\left(\frac{\pi x}{2}\right)$
  - (d)  $h(x) = \sec t$
23. Write a sentence describing the relationship between the graphs of the functions  $f(x) = \sin x$  and  $g$ .
  - (a)  $g(x) = 10 + \sin x$
  - (b)  $g(x) = \sin \frac{\pi x}{2}$
  - (c)  $g(x) = \sin\left(x + \frac{\pi}{4}\right)$
  - (d)  $g(x) = -\sin x$
24. Find  $a$ ,  $b$ , and  $c$  so that the graph of  $f(x) = a \sin(bx + c)$  matches the graph in the figure.
25. Consider the function  $f(x) = \sin 3x - 2 \cos x$ .
  - (a) Use a graphing utility to graph the function.
  - (b) Determine the period of the function.
  - (c) Approximate (accurate to one decimal place) the zero of the functions in the interval  $[0, \pi]$ .
  - (d) Approximate (accurate to one decimal place) the maximum value of the function in the interval  $[0, 3]$ .
26. Consider the function
$$f(t) = 2 \cdot 10^2 \cos\left(\frac{\pi t}{2}\right)$$
where  $t$  is time in seconds.
  - (a) Use a graphing utility to graph the function.
  - (b) Is the function periodic?
  - (c) Beyond what time  $t$  is the maximum value of the function less than 0.3?
27. Evaluate the expression without the aid of a calculator.
  - (a)  $\arcsin(1)$
  - (b)  $\arctan \sqrt{3}$
28. Write an algebraic expression that is equivalent to  $\sin(\arcsin 2x)$ .
29. From a point on the ground 400 feet from the foot of a cliff, the angle of elevation of the top of the cliff is  $32^\circ 30'$ . How high is the cliff?

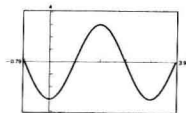


FIGURE FOR 24

# SUPPLEMENTS

This text is accompanied by a comprehensive supplements package for maximum teaching effectiveness and efficiency.

**Instructor's Guide** by Roland E. Larson and Robert P. Hostetler, *The Pennsylvania State University*, and Bruce H. Edwards, *University of Florida*

**Study and Solutions Guide** by Bruce H. Edwards, *University of Florida*, and Dianna L. Zook, *Indiana University—Purdue University at Fort Wayne*

**Test Item File and Resource Guide**

**Graphing Technology Guide** by Benjamin N. Levy

**Transparency Package**

**Precalculus Videotapes** by Dana Mosely

**Test-Generating Software** (IBM, Macintosh)

**The Algebra of Calculus** by Eric J. Braude

**BestGrapher Software** (IBM, Macintosh) by George Best

This complete supplements package offers ancillary materials for students, for instructors, and for classroom resources. Most items are keyed directly to the textbook for easy use. For the convenience of software users, a technical support telephone number is available with all D. C. Heath software products. The components of this comprehensive teaching and learning package are outlined on the following pages.

## PRINTED ANCILLARIES

## SOFTWARE AND VIDEOS

### INSTRUCTORS

#### Instructor's Guide

- Solutions to all even-numbered text exercises, all discussion problems, and all cumulative tests

#### Test Item File and Resource Guide

- Printed test bank
- Over 2000 test items
- Open-ended and multiple-choice test items
- Available as test-generating software
- Sample tests

#### BestGrapher

- Function grapher
- Screen simultaneously displays equation, graph, and table of values
- Some features anticipate calculus
- Includes zoom and print features for use on assignments

#### Computerized Testing Software

- Test-generating software
- Over 2000 test items
- Also available as a printed test item file

#### Derive

- Computer algebra system
- Discount available to adopters

### STUDENTS

#### Study and Solutions Guide

- Solutions to all odd-numbered text exercises
- Solutions match methods of text
- Summaries of key concepts in each text chapter
- Study strategies

#### Graphing Technology Guide

- Keystroke instructions for graphics calculators
- Examples

#### The Algebra of Calculus

- Reviews the algebra, trigonometry, and analytic geometry that students will encounter in calculus
- Over 200 examples
- Pretests and exercise sets

#### BestGrapher

- Function grapher
- Screen simultaneously displays equation, graph, and table of values
- Some features anticipate calculus
- Includes zoom and print features for use on assignments

### CLASSROOM RESOURCES

#### Transparency Package

- 50 color transparencies
- Color-coded by text topic

#### BestGrapher

- Function grapher
- Screen simultaneously displays equation, graph, and table of values
- Some features anticipate calculus
- Includes zoom and print features for use on assignments

#### Videotapes

- Text-specific videos



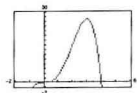
# INTEGRATED LEARNING PACKAGE

## Instructor's Guide

## Study and Solutions Guide

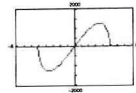
## BestGrapher

40.  $y = 4x^2 - x^4$



Graph intersects x-axis twice, y-axis once.

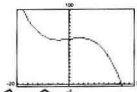
42.  $y = 100x\sqrt{25-x^2}$



Graph intersects x-axis three times, y-axis once.

44.  $2x^3 - 100x - 15,625 + 250y = 0$

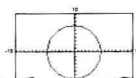
$250y = -2x^3 + 100x + 15,625$   
 $y = -\frac{1}{125}x^3 + \frac{2}{5}x + \frac{15,625}{250}$



46.  $x^2 + y^2 = 49$

$y^2 = 49 - x^2$   
 $y = \pm\sqrt{49 - x^2}$

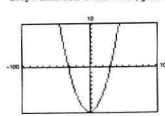
$y_1 = \sqrt{49 - x^2}, y_2 = -\sqrt{49 - x^2}$



43.  $x^3 - 100y - 1000 = 0$

$100y = x^3 - 1000$   
 $y = \frac{1}{100}x^3 - 10$

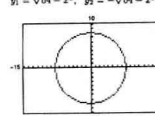
Graph intersects x-axis twice, y-axis once.



45.  $x^2 + y^2 = 64$

$y^2 = 64 - x^2$   
 $y = \pm\sqrt{64 - x^2}$

$y_1 = \sqrt{64 - x^2}, y_2 = -\sqrt{64 - x^2}$



47.  $6x^2 + y^2 = 72$

$y^2 = 72 - 6x^2$   
 $y = \pm\sqrt{72 - 6x^2}$

$y_1 = \sqrt{72 - 6x^2}, y_2 = -\sqrt{72 - 6x^2}$



## CHAPTER 1

- 1.1 GRAPHS AND GRAPHING UTILITIES
- 1.2 LINES IN THE PLANE
- 1.3 FUNCTIONS
- 1.4 GRAPHS OF FUNCTIONS
- 1.5 SHIFTING, REFLECTING, AND STRETCHING GRAPHS
- 1.6 COMBINATIONS OF FUNCTIONS
- 1.7 INVERSE FUNCTIONS

# FUNCTIONS AND GRAPHS

## 1.1 GRAPHS AND GRAPHING UTILITIES

The Graph of an Equation / Using a Graphing Utility / Determining a Viewing Rectangle / Applications

### The Graph of an Equation

Newspapers often show graphs comparing the rate of inflation, the federal deficit, wholesale prices, or the unemployment rate to the time of year. Industrial firms and businesses use graphs to report their monthly production and sales statistics. Such graphs provide geometric pictures of the way one quantity changes with respect to another.

Frequently, the relationship between two quantities is expressed as an equation. This section introduces the basic procedure for determining the geometric picture associated with an equation.

For an equation in variables  $x$  and  $y$ , a point  $(a, b)$  is a **solution point** if the substitution of  $x = a$  and  $y = b$  satisfies the equation. Most equations have infinitely many solution points. For example, the equation  $3x + y = 5$  has solution points  $(0, 5)$ ,  $(1, 2)$ ,  $(2, -1)$ ,  $(3, -4)$ , and so on. The set of all solution points of an equation is the **graph** of the equation.

### THE POINT-PLOTTING METHOD OF GRAPHING

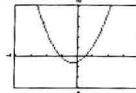
To sketch the graph of an equation by point plotting, use the following steps.

1. If possible, rewrite the equation so that one of the variables is isolated on the left side of the equation.
2. Make up a table of several solution points.
3. Plot these points in the coordinate plane.
4. Connect the points with a smooth curve.

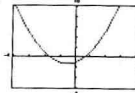
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18. Use a graphing utility to graph  $y = 2x^2 + x - 1$ .

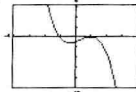
(a)



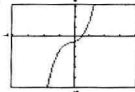
(b)



(c)

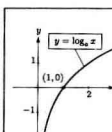


(d)



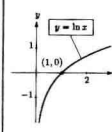
(e) None of these

1-M—Answer: d



Graph of  $y = \log_a x, a > 1$

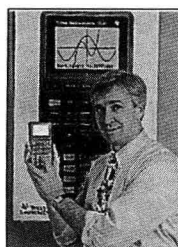
- Domain:  $(0, \infty)$
- Range:  $(-\infty, \infty)$
- Intercept:  $(1, 0)$
- Increasing
- y-axis is a vertical asymptote ( $\log_a x \rightarrow -\infty$  as  $x \rightarrow 0^+$ )
- Continuous
- Reflection of graph of  $y = a^x$  about the line  $y = x$



Graph of  $y = \ln x$

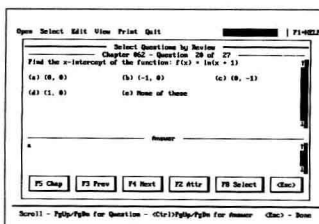
- Domain:  $(0, \infty)$
- Range:  $(-\infty, \infty)$
- Intercept:  $(1, 0)$
- Increasing
- y-axis is a vertical asymptote ( $\ln x \rightarrow -\infty$  as  $x \rightarrow 0^+$ )
- Continuous
- Reflection of graph of  $y = e^x$  about the line  $y = x$

## Transparency Package



## Videotapes

## Computerized Testing Software



## 8

### Functions II: Combinations of Functions, Difficult Graphs

#### REVIEW OF FUNDAMENTALS

1. The sum, product, and quotient of functions  $f$  and  $g$  are defined by the following equations:
  - (a)  $(f+g)(x) = f(x) + g(x)$
  - (b)  $(fg)(x) = f(x)g(x)$
  - (c)  $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ ,  $g(x) \neq 0$
2. The domain of a function  $f$ , denoted by  $D_f$ , is the set of all  $x$  for which  $f(x)$  is defined.
3. The range of a function  $f$ , denoted by  $R_f$ , is the set of all  $y$  for which  $y = f(x)$  for some  $x$  in  $D_f$ .
4. The image of  $f$  is the set of all  $y$  for which  $y = f(x)$  for some  $x$  in  $D_f$ .
5. The inverse of  $f$  is the function  $f^{-1}$  such that  $f^{-1}(f(x)) = x$  and  $f(f^{-1}(y)) = y$ .
6. The graph of  $f$  is the set of all points  $(x, y)$  such that  $y = f(x)$ .
7. The graph of  $f$  is the set of all points  $(x, y)$  such that  $y = f(x)$ .

#### EXAMPLE 1

Name, Problem, and Solution of Functions

Let  $f(x) = x^2 + 1$ ,  $g(x) = x + 1$ , and  $h(x) = x - 1$ .

Find  $(f+g)(x)$ ,  $(fg)(x)$ , and  $(\frac{f}{g})(x)$ .

SOLUTION

(a)  $(f+g)(x) = (x^2 + 1) + (x + 1) = x^2 + x + 2$

(b)  $(fg)(x) = (x^2 + 1)(x + 1) = x^3 + x^2 + x + 1$

(c)  $(\frac{f}{g})(x) = \frac{x^2 + 1}{x + 1}$

## The Algebra of Calculus with Trigonometry and Analytic Geometry

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