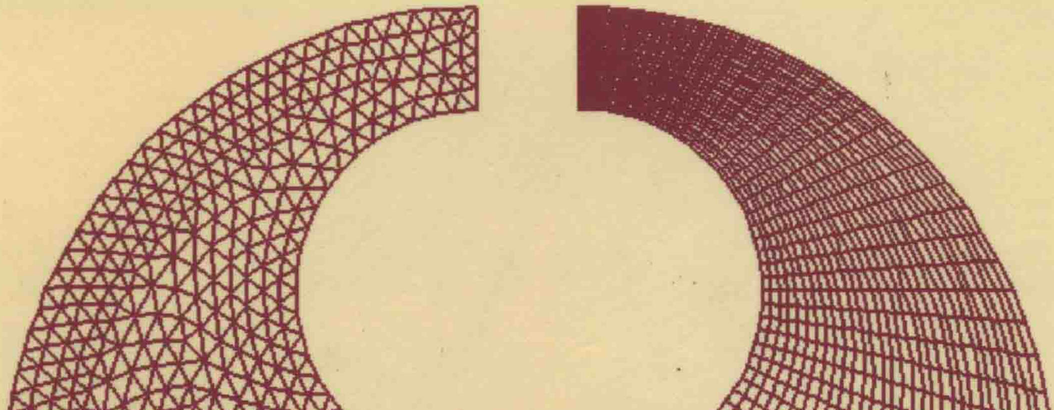
Anil W. Date

Introduction to **Computational Fluid Dynamics**



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Introduction to Computational Fluid Dynamics introduces all the primary components for learning and practicing computational fluid dynamics (CFD). The book is written for final year undergraduates and/or graduate students in mechanical, chemical, and aeronautical engineering who have taken basic courses in thermodynamics, fluid mechanics, and heat and mass transfer. Chapters cover discretisation of equations for transport of mass, momentum, and energy on Cartesian, structured curvilinear, and unstructured meshes; solution of discretised equations; numerical grid generation; and convergence enhancement. The book follows a consistent philosophy of control-volume formulation of the fundamental laws of fluid motion and energy transfer and introduces a novel notion of "smoothing pressure correction" for solution of flow equations on collocated grids within the framework of the well-known SIMPLE algorithm.

There are more than 50 solved problems in the text and 130 end-of-chapter problems. Practicing industry professionals will also find this book useful for continuing education and refresher courses.

PROFESSOR ANIL W. DATE obtained his bachelor's degree in mechanical engineering from Bombay University; his master's degree in thermo-fluids from UMIST Manchester, UK; and his doctorate in heat transfer from Imperial College of Science and Technology, London. He has been a member of the Thermo-Fluids Engineering group of the Mechanical Engineering Department at IIT Bombay since 1973. Over the past thirty years, he has taught courses at both the undergraduate and postgraduate levels in thermodynamics, energy conversion, heat and mass transfer, and combustion. He has been engaged in research and consulting in thermo-fluids engineering and is an active reviewer of research proposals and papers for various national and international bodies and journals. He has been Editor for India of the *Journal of Enhanced Heat Transfer* and has contributed research papers to several international journals in the field. He has been a visiting scientist at Cornell University and a visiting professor at the University of Karlsruhe, Germany. He has delivered seminar lectures at universities in Australia, Hong Kong, Sweden, Germany, the United States, the UK, and India. Professor Date derives great satisfaction from applying thermo-fluid science to rural-technology problems in India and has taught courses in science, technology, and society and in appropriate technology at ITT Bombay. Professor Date is a Fellow of the Indian National Academy of Engineering (FNAE).

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Dedicated to the memory of

Aai, Kaka, and Walmik

Nomenclature

Only major symbols are given in the following lists.

AE, AW, AN, AS, AP, Sp, A_k	Coefficients in Discretised Equations
B	Body Force (N/kg) or Spalding Number
C_p	Constant-Pressure Specific Heat (J/kg-K)
C_v	Constant-Volume Specific Heat (J/kg-K)
D	Mass Diffusivity (m^2/s)
e	Turbulent Kinetic Energy or Internal Energy (J/kg)
f	Fanning Friction Factor Based on Hydraulic Diameter
Gr	Grashof Number
h	Enthalpy (J/kg) or Heat Transfer Coefficient (W/m^2-K)
k	Thermal Conductivity (W/m-K)
M	Molecular Weight or Mach Number
Nu	Nusselt Number
P	Peclet Number
P_c	Cell Peclet Number
Pr	Prandtl Number
p	Pressure (N/m^2)
q	Heat Flux (W/m^2)
q''', Q'''	Internal Heat Generation Rates (W/m^3)
R	Residual or Gas Constant (J/kg-mol-K)
Re	Reynolds Number
S, Su	Source Term
Sc	Schmidt Number
St	Stanton or Stefan Number
T	Temperature (°C or K)
t	Time (s)
u, v, w	x-, y-, z-Direction Velocities (m/s)

u_i	Velocity in $x_i, i = 1, 2, 3$ Direction
V	Volume (m^3)

Greek Symbols

α	Under relaxation Factor or Thermal Diffusivity (m^2/s)
β	Under relaxation Factor for Pressure or Coefficient of Volume Expansion (K^{-1})
δ	Boundary Layer Thickness (m)
Δ	Incremental Value
ϵ	Turbulent Energy Dissipation Rate (m^2/s^3)
Ψ	Stream Function or Weighting Factor
Φ	General Variable or Dimensionless Enthalpy
Γ	General Exchange Coefficient $= \mu, \rho D,$ or k/Cp
κ	Constant in the Logarithmic Law of the Wall
μ	Dynamic viscosity ($N\text{-}s/m^2$)
ν	Kinematic Viscosity (m^2/s)
ω	Species Mass Fraction or Dimensionless Coordinate
ρ	Density (kg/m^3)
λ	Second Viscosity Coefficient or Latent Heat (J/kg)
λ_1	Multiplier of $p - \overline{p}$
σ	Normal Stress (N/m^2)
θ	Dimensionless Temperature
τ	Shear Stress (N/m^2) or Dimensionless Time

Subscripts

P, N, S, E, W	Refers to Grid Nodes
n, s, e, w	Refers to Cell Faces
eff	Refers to Effective Value
f	Refers to Cell Face
l	Liquid or Liquidus
m	Refers to Mass Conservation, Mixture, or Melting Point
s	Solid or Solidus
sm	Refers to Smoothing
sup	Superheated
T	Transferred Substance State
x_i	Refers to $x_i, i = 1, 2, 3$ directions

Superscripts

l	Iteration Counter
o	Old Time
u, v	Refers to Momentum Equations

$\bar{}$	Multidimensional Average
$'$	Correction

Acronyms

1D	One-Dimensional
2D	Two-Dimensional
3D	Three-Dimensional
ADI	Alternating Direction Implicit
CDS	Central Difference Scheme
CFD	Computational Fluid Dynamics
CG	Conjugate Gradient Method
CONDIF	Controlled Numerical Diffusion with Internal Feedback
DNS	Direct Numerical Simulation
GMRES	Generalised Minimal Residual Method
GS	Gauss–Seidel Method
HDS	Hybrid Difference Scheme
HRE	High Reynolds Number Model
IOCV	Integration over a Control Volume Method
LHS	Left-Hand Side
LRE	Low Reynolds Number Model
LU	Lower-Upper Decomposition
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
POWER	Power-Law Scheme
RHS	Right-Hand Side
SIMPLE	Semi-Implicit Method for Pressure Linked Equations
TDMA	Tridiagonal Matrix Algorithm
TSE	Taylor Series Expansion Method
TVD	Total Variation Diminishing
UDS	Upwind Difference Scheme

Preface

During the last three decades, computational fluid dynamics (CFD) has emerged as an important element in professional engineering practice, cutting across several branches of engineering disciplines. This may be viewed as a logical outcome of the recognition in the 1950s that undergraduate curricula in engineering must increasingly be based on *engineering science*. Thus, in mechanical engineering curricula, for example, the subjects of fluid mechanics, thermodynamics, and heat transfer assumed prominence.

I began my teaching career in the early 1970s, having just completed a Ph.D. degree that involved solution of partial differential equations governing fluid motion and energy transfer in a particular situation (an activity not called CFD back then!). After a few years of teaching undergraduate courses on heat transfer and postgraduate courses on convective heat and mass transfer, I increasingly shared the feeling with the students that, although the excellent textbooks in these subjects emphasised application of fundamental laws of motion and energy, the problem-solving part required largely varied mathematical tricks that changed from one situation to another. I felt that teachers and students needed a chance to study relatively more *real* situations and an opportunity to concentrate on the physics of the subject. In my reckoning, the subject of CFD embodies precisely this scope and more.

The introduction of a five-year dual degree (B.Tech. and M.Tech.) program at IIT Bombay in 1996 provided an opportunity to bring new elements into the curriculum. I took this opportunity to introduce a course on computational fluid dynamics and heat transfer (CFDHT) in our department as a compulsory course in the fourth year for students of the thermal and fluids engineering stream. The course, with an associated CFDHT laboratory, has emphasised *relearning* fluid mechanics and heat and mass transfer through obtaining *numerical* solutions. This, of course, contrasts with the *analytical* solutions learnt in earlier years of the program. Through teaching of this CFDHT course, I discovered that this relearning required attitudinal change on the part of the student. Thus, for example, the idea that *all* 1D conduction problems (steady or unsteady, in Cartesian, cylindrical, or spherical coordinates, with constant or variable properties, with or without area change, with or without internal heat generation, and with linear or nonlinear boundary conditions) in a typical undergraduate textbook can be solved by a *single* computer program based on a *single* method is found by the students to be new. Similarly, the idea that a numerical instability in an unsteady conduction problem essentially represents violation of the second law of thermodynamics is found to be new because no book on *numerical analysis* treats it as such. Nothing encourages a teacher to write a book more than the discomfort expressed by the students. At the same time, it must be mentioned that when a student succeeds in writing a generalised computer program for 1D conduction in the laboratory part of the course through struggles of *where and how do I begin*, of debugging, of comparing numerical results with analytical results, of studying effects of parametric variations, and of plotting of results, the computational activity is found to be both enlightening and entertaining.

I specifically mention these observations because, although there are a number of books bearing the words *Computational Fluid Dynamics* in their titles, most emphasise numerical analysis (a branch of applied mathematics). Also, most books, it would appear, are written for researchers and cover a rather extended ground but are usually devoid of exercises for student learning. In my reckoning, the most notable exception to such a state of affairs is the pioneering book *Numerical Heat Transfer and Fluid Flow* written by Professor Suhas V. Patankar. The book emphasises *control-volume* discretisation (the main early step to obtaining numerical solutions) based on physical principles and strives to help the reader to *write* his or her own computer programs.

It is my pleasure and duty to acknowledge that writing of this book has been influenced by the works of two individuals: Professor D. B. Spalding (FRS, formerly at Imperial College of Science and Technology, London), who unified the fields of heat, mass, and momentum transfer, and Professor S. V. Patankar (formerly at University of Minnesota, USA), who, through his book, has made CFD so lucid and SIMPLE.[1] If the readers of this book find that I have mimicked writings of these two pioneers from which several individuals (teachers, academic researchers, and consultants) and organisations have benefited, I would welcome the compliment.

I have titled this book as *Introduction to Computational Fluid Dynamics* for two reasons. Firstly, the book is intended to serve as a textbook for a student uninitiated in CFD but who has had exposure to the three courses mentioned in the first paragraph of this preface at undergraduate and postgraduate levels. In this respect, the book will also be found useful by teachers and practicing engineers who are increasingly attracted to take refresher courses in CFD. Secondly, CFD, since its inception, has remained an ever expanding field, expanding in its fundamental scope as well as in ever new application areas. Thus, turbulent flows, which are treated in this book through *modelling*, are already being investigated through direct numerical simulation (DNS). Similarly, more appropriate constitutive relations for multiphase

[1] The reader will appreciate the significance of capital letters in the text.

flow or for a reacting flow are being explored through CFD. Newer application areas such as heat and mass transfer in biocells are also beginning to be explored through CFD. Such areas are likely to remain more at the research level than to be part of regular practice and, therefore, a student, over the next few years at least, may encounter them in research at a Ph.D. level. It is my belief that the approach adopted in this book will provide adequate grounding for such pursuits.

Although this is an introductory book, there are some departures and basic novelties to which it is important to draw the reader's attention. The first of these concerns the manner in which the fundamental equations of motion (the Navier–Stokes equations) are written. Whereas most textbooks derive or write these equations for a continuum fluid, it is shown in the first chapter of this book that since numerical solutions are obtained in discretised space, the equations must be written in such a way that they are applicable to both the continuum as well as the discretised space. Attention is also drawn to use of special symbols that the reader may find not in common with other books on CFD. Thus, a *mass-conserving* pressure correction is given the symbol p'_{m} to contrast with the two other pressure corrections, namely, the *total* pressure correction p' and the *smoothing* pressure correction p'_{sm}. Similarly, the velocities appearing at the control-volume faces are given the symbol $u_{\mathrm{f},i}$ to contrast with those that appear at the nodal locations, which are referred to as u_i. Again, in a continuum, the two velocity fields must coincide but, in a discretised space, distinction between them preserves clarity of the physics involved. Novelty will also be found in the discussion of physical principles behind seemingly mathematical activity governing the topics of numerical grid generation and convergence enhancement. It is not my claim that the entire material of the book can be covered in a single course on CFD. It is for this reason that 1D formulations are emphasised through dedicated chapters. These formulations convey most of the essential ingredients required in CFD practice.

The ambience of academic freedom, the variety of facilities and the friendly atmosphere on the campus of IIT Bombay has contributed in no small measure to this solo effort at book writing. I am grateful to my colleagues for their cooperation in many matters. I am particularly grateful for having had the association of a senior colleague like Professor S. P. Sukhatme (FNA, FNAE, former Director, IIT Bombay). It has been a learning experience for me to observe him carry out a variety of roles (including as writer of two well-received textbooks on heat transfer and solar energy) in our institute with meticulous care. Hopefully, some rub-off is evident in this book. I have also gained considerably from my Ph.D. and M.Tech. students who through their dissertations have helped validate the computer programs I wrote.

I would like to express my special gratitude to Mr. Peter Gordon, Senior Editor (Aeronautical, Biomedical, Chemical, and Mechanical Engineering), Cambridge University Press, New York, for his considerable advice and guidance during preparation of the manuscript for this book.

Finally, I would like to record my appreciation of my wife Suranga, son Kartikeya, and daughter Pankaja (Pinky) for bearing my absence on several weekends while writing this book.

Mumbai
June 2004

Anil W. Date