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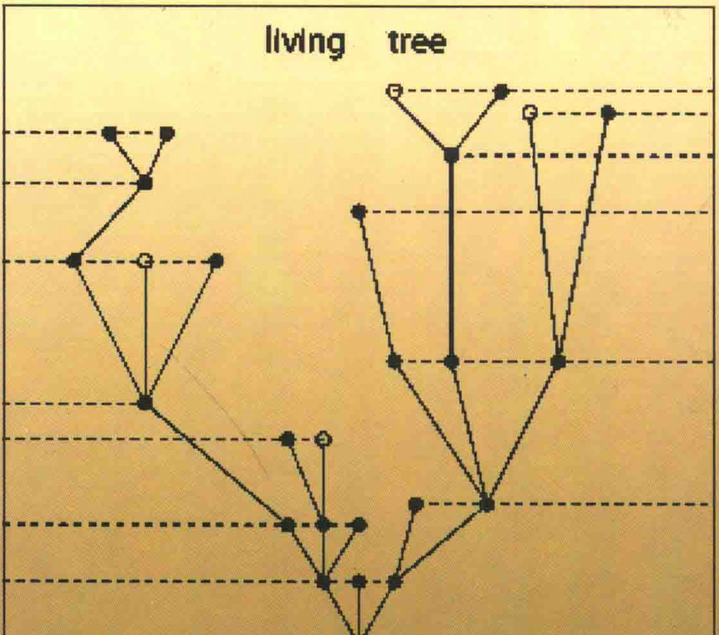
Neutron Fluctuations

A TREATISE ON THE PHYSICS OF BRANCHING PROCESSES



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A TREATISE ON THE PHYSICS OF BRANCHING PROCESSES

Dedicated to
MARIA
and
ANGELA

'Some books are to be tasted, others to be swallowed, and some few to be chewed and digested'
(Francis Bacon, 1564–1626)

PREFACE

Thorough descriptions of branching processes can be found in almost every book and monograph that deals with stochastic processes [1–5]. Moreover, in the monographs by T.E. Harris [6] and B.A. Sevast'yanov [7], nearly every problem of the theory is discussed with mathematical rigour. There are innumerable publications available about the applications of the theory of branching processes in the different fields of natural sciences such as physics [8], nuclear engineering [9–11], and biology [12]. With regard to the fluctuations in branching processes concerning nuclear chain reactions, these are synonymous with zero power neutron noise, or neutron fluctuations in zero power systems. In this respect, already in 1964, earlier than the books by Stacey [9] and Williams [11] appeared in print, a remarkable general work, amounting to a monograph, was published by D.R. Harris [13] on this topic.

However, it is somewhat surprising that no monograph has been published since 1974 on neutron fluctuations. There appears to be a need for a self-contained monograph on the theory and principles of branching processes that are important both for the studies of neutron noise and for the applications, and which at the same time would treat the recent research problems of neutron noise by accounting for new developments. The ambition to fill this gap constitutes the motivation for writing this book.

This book was thus written with two objectives in mind, and it also consists of two parts, although the objectives and parts slightly overlap. The first objective was to present the theory and mathematical tools used in describing branching processes which can be used to derive various distributions of the population with multiplication. The theory is first developed for reproducing and multiplying entities in general, and then is applied to particles and especially neutrons in particular, including the corresponding detector counts. Hence, the text sets out by deriving the basic forward and backward forms of the master equations for the probability distributions and their generating functions induced by a single particle. Various single and joint distributions and their special cases are derived and discussed. Then the case of particle injection by an external source (immigration of entities) is considered. Attention is given to the case when some entities (particles) are born with some time delay after the branching event. Moments, covariances, correlations, extinction probabilities, survival times and other special cases and special probabilities are discussed at depth.

All the above chapters concern an infinite homogeneous material. In Chapter 7 space dependence is introduced. A one-dimensional case is treated as an illustration of a simple space-dependent process, in which a number of concrete solutions can be given in closed compact form.

Whereas the first part treats concepts generally applicable to a large class of branching processes, Part II of this book is specifically devoted to neutron fluctuations and their application to problems of reactor physics and nuclear material management. The emphasis is on the elaboration of neutron fluctuation based methods for the determination of the reactivity of subcritical systems with an external source. First, in Chapter 8, a detailed derivation of the Pál–Bell equation, together with its diffusion theory approximation, is given. The original publication of the Pál–Bell equation constituted the first theoretical foundation of the zero power noise methods which had been suggested earlier by empirical considerations. Thereafter, Chapters 9 and 10 deal with the applications of the general theory to the derivation of the Feynman and Rossi-alpha methods. Chapter 9 concerns the derivation of the classical formulae for traditional systems, whereas Chapter 10 reflects the recent developments of these methods in connection with the so-called accelerator-driven systems, i.e. subcritical cores driven with a spallation source, and/or with pulsed sources. Finally, Chapter 11 touches upon the basic problems and methods of identifying and quantifying small samples of fissile material from the statistics of spontaneous and induced neutrons and photons. This area of nuclear safeguards, i.e. nuclear material accounting and control, is under a rapidly increasing attention due to the general increase of safety and safeguards needs worldwide.

A special new contribution of this book to the field of neutron noise is constituted by Chapter 6, in which the so-called zero power neutron noise, i.e. branching noise, is treated in systems with time-varying properties. Neutron noise in systems with temporally varying properties is called 'power reactor noise'. Neutron fluctuations in low power steady systems and high power systems with fluctuating parameters have constituted two disjoint areas so far which were treated with different types of mathematical tools and were assumed to be valid in non-overlapping operational areas. The results in Chapter 6 are hence the first to establish a bridge between zero power noise and power reactor noise. Due to space limitations, the *Langevin technique* and the theory of the *parametric noise* are not discussed. The interested reader is referred to the excellent monographs by Van Kampen [14] and Williams [11].

Since the *generating functions* play a decisive role in many considerations of this book, the theorems most frequently used in the derivations are summarised in Appendix A.

This book is not primarily meant for mathematicians, rather for physicists and engineers, and notably for those working with branching processes in practice, and in the first place for physicists being concerned with reactor noise investigations and problems of nuclear safeguards. However, it can also be useful for researchers in the field of biological physics and actuarial sciences.

The authors are indebted to many colleagues and friends who contributed to the realisation of this book in one way or another and with whom they collaborated during the years. One of us (I.P.) is particularly indebted to M.M.R. Williams, from whom he learnt immensely on neutron noise theory and with whom his first paper on branching processes was published. He also had, during the years, a very intensive and fruitful collaboration with several Japanese scientists, in particular with Y. Yamane and Y. Kitamura of Nagoya University. Chapters 9 and 10 are largely based on joint publications. Research contacts and discussions on stochastic processes and branching processes with H. Konno of the University of Tsukuba are acknowledged with thanks. Parts of this book were written during an inspiring visit to Nagoya and Tsukuba. The experimental results given in the book come from the Kyoto University Critical Assembly at KURRI, and contributions from the KURRI staff are gratefully acknowledged. The chapter on nuclear safeguards is largely due to a collaboration with Sara A. Pozzi of ORNL, who introduced this author to the field.

Both authors are much indebted to Maria Pázsit, whose contributions by translating early versions of the chapters of Part I from Hungarian can hardly be overestimated. She has also helped with typesetting and editing the LaTeX version of the manuscript, as well as with proofreading. The authors acknowledge with thanks constructive comments on the manuscript from M.M.R. Williams and H. van Dam and thank S. Croft for reading Chapter 11 and giving many valuable comments.

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We had no ambition to cite all published work related to the problems treated in this monograph. The books and papers listed in the 'List of Publications' represent merely some indications to guide the reader. One has to mention the excellent review of source papers in reactor noise by Saito [15]. This review contains practically all of the important publications until 1977 which are in strong relation with the topic of this book.

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- Figures 1 and 3 from the article by A. Enqvist, I. Pázsit and S. Pozzi in *Nucl. Instr. Meth. A*, **566** (2006) 598.

List of Most Frequently Used Notations

Symbol	Description
$\mathcal{P}\{\cdots\}$	Symbol of probability
$\mathbf{E}\{\cdots\}$	Symbol of expectation
$\mathbf{D}^2\{\cdots\}$	Symbol of variance
Q	Intensity of a reaction
ν	Number of progeny (neutrons) in one reaction
$\mathcal{P}\{\nu=k\}=f_k$	Probability of $\{\nu=k\}$
$q(z)=\sum_{k=0}^{\infty} f_k z^k$	Basic generating function
$\mathbf{E}\{\nu\}=q'(1)=q_1$	Expectation of the progeny number in one reaction
$\mathbf{E}\{\nu(\nu-1)\}=q''(1)=q_2$	Second factorial moment of the progeny number in one reaction
$Q_a=Qf_0$	Total intensity of absorption
$Q_b=Qf_1$	Intensity of renewal
$Q_m=Q(1-f_0-f_1)$	Intensity of multiplication
$\mathbf{n}(t)$	Number of particles at time t
$\mathcal{P}\{\mathbf{n}(t)=n\|\mathbf{n}(0)=1\}=p_n(t)$	Probability of finding n particles at time t in the case of one starting particle
$g(z,t)=\sum_{n=0}^{\infty} p_n(t)z^n$	Generating function of $p_n(t)$
$\mathbf{q}$	Number of particles produced by one injection (spallation) event
$\mathcal{P}\{\mathbf{q}=j\}=h_j$	Probability of $\{\mathbf{q}=j\}$; probability that there are j emitted neutrons per spallation event
$r(z)=\sum_{j=0}^{\infty} h_j z^j$	Generating function of probability h_j
$\mathbf{E}\{\mathbf{q}\}=r'(1)=r_1$	Expectation of the particle number produced by one injection event; expectation of the number of neutrons emitted per spallation event
$\mathbf{E}\{\mathbf{q}(\mathbf{q}-1)\}=r''(1)=r_2$	Second factorial moment of the particle number produced by one injection event; second factorial moment of the number of neutrons emitted per spallation event
$D_\nu=q_2/q_1^2, \quad D_q=r_2/r_1^2$	Diven factors of ν and $\mathbf{q}$
$s(t)$	Intensity of the injection process at time t
$\mathbf{N}(t)$	Number of particles at time t in the case of particle injection
$\mathcal{P}\{\mathbf{N}(t)=n\|\mathbf{n}(t_0)=0\}=P_n(t\|t_0)$	Probability of finding n particles at time t if the particle injection started at $t_0\leq t$
$G(z,t\|t_0)=\sum_{n=0}^{\infty} P_n(t\|t_0)z^n$	Generating function of $P_n(t\|t_0)$
$\alpha=Q(q_1-1)>0$	Multiplication intensity ($q_1>1$)
$a=-\alpha=Q(1-q_1)>0$	Decay intensity ($q_1<1$)
$m_1(t)$	Expectation of $\mathbf{n}(t)$
$m_2(t)$	Second factorial moment of $\mathbf{n}(t)$
$M_1(t)$	Expectation of $\mathbf{N}(t)$
$M_2(t)$	Second factorial moment of $\mathbf{N}(t)$
$\mathbf{n}_a(t-u,t)$	Number of absorptions in the time interval $[t-u,t]$, $u\geq 0$
$\mathcal{P}\{\mathbf{n}_a(t-u,t)=n\|\mathbf{n}(0)=1\}=p(n,t,u)$	Probability of absorbing n particles in the time interval $[t-u,t]$ in the case of one starting particle

Symbol	Description
$\mathbf{N}_a(t-u,t)$	Number of absorptions in the time interval $[t-u,t]$, $u \geq 0$ in the case of particle injection
$\mathcal{P}\{\mathbf{N}_a(t-u,t)=n\|\mathbf{n}(0)=0\}=P(n,t,u)$	Probability of absorbing n particles in the time interval $[t-u,t]$ in the case of particle injection
$m_1^{(a)}(t,u)$	Expectation of the number of absorbed particles in the time interval $[t-u,t]$ in the case of one starting particle
$m_2^{(a)}(t,u)$	Second factorial moment of the number of absorbed particles in the time interval $[t-u,\ t]$ in the case of one starting particle
$M_1^{(a)}(t,u)$	Expectation of the number of absorbed particles in the time interval $[t-u,\ t]$ in the case of particle injection
$M_2^{(a)}(t,u)$	Second factorial moment of the number of absorbed particles in the time interval $[t-u,\ t]$ in the case of particle injection
$\mathbf{D}^2\{\mathbf{N}_a(t-u,t)\}$	Variance of $\mathbf{N}_a(t-u,\ t)$
$\{\mathbf{S}(t)=S_\ell\}, \ell \in \mathcal{Z}^{(+)}$	Medium is in the state S_ℓ at time t
$\mathcal{U}$	Subset of the coordinate-velocity space
$\mathbf{u}=\{\vec{r},\vec{v}\}$	Phase point in the coordinate-velocity space
$\mathbf{n}(t,\mathcal{U})$	Number of neutrons in the subset $\mathcal{U}$ at time t
$p[t_0,\mathbf{u}_0;t,n(\mathcal{U})]$	Probability of finding n neutrons in the subset $\mathcal{U}$ at time t, when one neutron started from the phase point $\mathbf{u}_0$ at time $t_0 \leq t$
$m_1(t_0,\mathbf{u}_0;t,\mathcal{U})$	Expectation of the number of neutrons in the subset $\mathcal{U}$ at time t, when one neutron started from the phase point $\mathbf{u}_0$ at time $t_0 \leq t$
$m_2(t_0,\mathbf{u}_0;\ t,\mathcal{U})$	Second factorial moment of the number of neutrons in the subset $\mathcal{U}$ at time t, when one neutron started from the phase point $\mathbf{u}_0$ at time $t_0 \leq t$
$\mathbf{C}(t)$	Number of the delayed neutron precursors at time t
$\mathbf{Z}(t,t_d)$	Number of the detected neutrons in the time interval $[t_d,\ t]$
λ_c	Intensity of capture
λ_f	Intensity of fission
λ_d	Intensity of detection
λ	Decay constant
S	Source intensity
$p_f(n,m)$	Probability of emitting n neutrons and m precursors in one fission
$g_f(x,y)$	Generating function of $p_f(n,m)$
$\partial g_f(x,y)/\partial x\|_{x=y=1}=\langle\nu_p\rangle$	Average number of prompt neutrons per fission
$\partial g_f(x,y)/\partial y\|_{x=y=1}=\langle\nu_d\rangle$	Average number of delayed neutrons per fission
$\langle\nu\rangle=\langle\nu_p\rangle+\langle\nu_d\rangle$	Average number of neutrons per fission
$\beta=\langle\nu_d\rangle/\langle\nu\rangle$	Effective delayed neutron fraction
ρ	Reactivity
$\Lambda=1/\langle\nu\rangle\lambda_f$	Prompt neutron generation time
$\alpha=(\beta-\rho)/\Lambda$	Prompt neutron decay constant used in Chapters 9 and 10
$\epsilon=\lambda_d/\lambda_f$	Detector efficiency
$P(N,C,Z,t\|t_0)$	Probability of finding N neutrons and C precursors at time t in the system driven by a source, and of counting Z neutrons in the time interval $[0,\ t]$
$G(x,y,v,t\|t_0)$	Generating function of $P(N,C,Z,t\|t_0)$
$Z(t)$	Asymptotic expectation of the number of detected neutrons in the time interval $[0,t]$
$\mu_{ZZ}(t,0\|t_0)$	Modified second factorial moment

Symbol	**Description**
$\lim_{t_0\to-\infty}\mu_{ZZ}(t,0\|t_0)=\mu_{ZZ}(t)$	Asymptotic modified second factorial moment
$Y(t)=\mu_{ZZ}(t)/Z(t)$	$Y(t)$ in the Feynman-alpha formula
$p(n,c,z,T,t)$	Probability that there are n neutrons and c precursors at time t in the system, induced by one initial neutron at $t=0$, and there have been z detector counts in the time interval $[t-T,t]$
$P(N,C,Z,T,t)$	Probability that there are N neutrons and C precursors at time t in the system, induced by a source of intensity S, and that there have been Z detector counts in the time interval $[t-T,\ t]$, provided that there were no neutrons and precursors in the system at time $t=0$ and no neutron counts have been registered up to time $t=0$
$g(x,y,v,T,t)$	Generating function of $p(n,c,z,T,t)$
$G(x,y,v,T,t)$	Generating function of $P(N,C,Z,T,t)$
$\boldsymbol{\nu}$	Total number of neutrons produced in a cascade
$\boldsymbol{\mu}$	Total number of gamma photons produced in a cascade
$\widetilde{\nu}_1$	Neutron singles
$\widetilde{\nu}_2$	Neutron doubles
$\widetilde{\nu}_3$	Neutron triples
$\mathbf{M}$	Leakage multiplication
φ_r	Average number of neutrons generated in a sample
$\mathbf{M}_\gamma$	Gamma multiplication per one initial neutron
$\widetilde{\mu}_1$	Gamma singles
$\widetilde{\mu}_2$	Gamma doubles
$\widetilde{\mu}_3$	Gamma triples
$P(n)$	Number distribution of neutrons generated in a sample
$F(n)$	Number distribution of gamma photons generated in a sample

Contents

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