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RELIABILITY OF LARGE AND COMPLEX SYSTEMS

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Reliability of Large and Complex Systems

Second Edition

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Reliability of Large and Complex Systems

Notations

E_i	components of series, parallel and '*m* out of *n*' systems
E_{ij}	components of series–parallel and parallel–series systems
T_i	component lifetimes of two-state series, parallel and '*m* out of *n*' systems
T_{ij}	component lifetimes of two-state series–parallel and parallel–series systems
T	a two-state system lifetime
$R(t)$	a component reliability function of a two-state homogeneous system
$F(t)$	a component lifetime distribution function of a two-state homogeneous system
$R^{(i)}(t)$	component reliability functions of two-state non-homogeneous series, parallel and '*m* out of *n*' systems
$F^{(i)}(t)$	component lifetime distribution functions of two-state non-homogeneous series, parallel and '*m* out of *n*' systems
$R^{(i,j)}(t)$	component reliability functions of two-state non-homogeneous series–parallel and parallel–series systems
$F^{(i,j)}(t)$	component lifetime distribution functions of two-state non-homogeneous series–parallel and parallel–series systems
$\overline{\boldsymbol{R}}_n(t)$	a reliability function of a two-state homogeneous series system
$\overline{\boldsymbol{R}}'_n(t)$	a reliability function of a two-state non-homogeneous series system
$\boldsymbol{R}_n(t)$	a reliability function of a two-state homogeneous parallel system

$\boldsymbol{R}'_n(t)$	a reliability function of a two-state non-homogeneous parallel system
$\boldsymbol{R}_n^{(m)}(t)$	a reliability function of a two-state homogeneous '*m* out of *n*' system
$\boldsymbol{R}'_n(m)(t)$	a reliability function of a two-state non-homogeneous '*m* out of *n*' system
$\overline{\boldsymbol{R}}_{k_n l_n}(t)$	a reliability function of a two-state homogeneous parallel–series system
$\overline{\boldsymbol{R}}'_{k_n l_n}(t)$	a reliability function of a two-state non-homogeneous parallel–series system
$\boldsymbol{R}_{k_n l_n}(t)$	a reliability function of a two-state homogeneous series–parallel system
$\boldsymbol{R}'_{k_n l_n}(t)$	a reliability function of a two-state non-homogeneous series–parallel system
$\overline{\mathfrak{R}}(t)$	a limit reliability function of two-state homogeneous series and parallel–series systems
$\overline{\mathfrak{R}}'(t)$	a limit reliability function of two-state non-homogeneous series and parallel–series systems
$\mathfrak{R}(t)$	a limit reliability function of two-state homogeneous parallel and series–parallel systems
$\mathfrak{R}'(t)$	a limit reliability function of two-state non-homogeneous parallel and series–parallel systems
$\mathfrak{R}^{(0)}(t)$	a limit reliability function of a two-state homogeneous '*m* out of *n*' system
$\mathfrak{R}^{(\mu)}(t)$	a limit reliability function of a two-state homogeneous '*m* out of *n*' system
$\overline{\mathfrak{R}}^{(1)}(t,\cdot)$	a limit reliability function of a two-state homogeneous '*m* out of *n*' system
$E(T)$	a mean lifetime of a two-state system
$\sigma(T)$	a lifetime standard deviation of a two-state system
z	a number of reliability states of a multi-state component and a multi-state system
$T_i(u)$	multi-state component lifetimes of series, parallel and '*m* out of *n*' systems in a state subset

$T_{ij}(u)$	multi-state component lifetimes of series–parallel and parallel–series systems in a state subset
$T(u)$	a multi-state system lifetime in a state subset
$R(t,\cdot)$	a multi-state component reliability function of a homogeneous system
$F(t,\cdot)$	a multi-state component lifetime distribution function of a homogeneous system in a state subset
$R^{(i)}(t,\cdot)$	multi-state component reliability functions of homogeneous series, parallel and '*m* out of *n*' systems
$F^{(i)}(t,\cdot)$	multi-state component lifetime distribution functions of homogeneous series, parallel and '*m* out of *n*' systems in a state subset
$R^{(i,j)}(t,\cdot)$	multi-state component reliability functions of homogeneous series–parallel and parallel–series systems
$F^{(i,j)}(t,\cdot)$	multi-state component lifetime distribution functions of homogeneous series–parallel and parallel–series systems in a state subset
$\overline{\boldsymbol{R}}_n(t,\cdot)$	a reliability function of a multi-state homogeneous series system
$\overline{\boldsymbol{R}}'_n(t,\cdot)$	a reliability function of a multi-state non-homogeneous series system
$\boldsymbol{R}_n(t,\cdot)$	a reliability function of a multi-state homogeneous parallel system
$\boldsymbol{R}'_n(t,\cdot)$	a reliability function of a multi-state non-homogeneous parallel system
$\boldsymbol{R}_n^{(m)}(t,\cdot)$	a reliability function of a multi-state homogeneous '*m* out of *n*' system
$\overline{\boldsymbol{R}}_n^{(\overline{m})}(t,\cdot)$	a reliability function of a multi-state homogeneous '*m* out of *n*' system
$\boldsymbol{R}_n'^{(m)}(t,\cdot)$	a reliability function of a multi-state non-homogeneous '*m* out of *n*' system
$\overline{\boldsymbol{R}}_n'^{(\overline{m})}(t,\cdot)$	a reliability function of a multi-state non-homogeneous '*m* out of *n*' system

$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot)$	a reliability function of a multi-state homogeneous parallel−series system
$\overline{\boldsymbol{R}}'_{k_n,l_n}(t,\cdot)$	a reliability function of a multi-state non-homogeneous parallel−series system
$\boldsymbol{R}_{k_n,l_n}(t,\cdot)$	a reliability function of a multi-state homogeneous series−parallel system
$\boldsymbol{R}'_{k_n,l_n}(t,\cdot)$	a reliability function of a multi-state non-homogeneous series−parallel system
$\overline{\Re}(t,\cdot)$	a limit reliability function of multi-state homogeneous series and parallel−series systems
$\overline{\Re}'(t,\cdot)$	a limit reliability function of multi-state non-homogeneous series and parallel−series systems
$\Re(t,\cdot)$	a limit reliability function of multi-state homogeneous parallel and series−parallel systems
$\Re'(t,\cdot)$	a limit reliability function of multi-state non-homogeneous parallel and series−parallel systems
$\Re^{(0)}(t,\cdot)$	a limit reliability function of a multi-state homogeneous '*m* out of *n*' system
$\Re^{(\mu)}(t,\cdot)$	a limit reliability function of a multi-state homogeneous '*m* out of *n*' system
$\overline{\Re}^{(1)}(t,\cdot)$	a limit reliability function of a multi-state homogeneous '*m* out of *n*' system
r	a critical reliability state of a system
$\mathbf{r}(t)$	a risk function of a multi-state system
$M_i(u)$	a multi-state component mean lifetime in a state subset
$\sigma_i(u)$	a multi-state component lifetime standard deviation in a state subset
$\overline{M}_i(u)$	a multi-state component mean lifetime in a state
$M(u)$	a multi-state system mean lifetime in a state subset
$\sigma(u)$	a multi-state system lifetime standard deviation in a state subset
$\overline{M}(u)$	a multi-state system mean lifetime in a state

δ	a permitted level of a multi-state system risk function
τ	a moment of exceeding a permitted multi-state system risk level
$D_{\overline{\mathfrak{R}}_l}$	domains of attraction of limit reliability functions $\overline{\mathfrak{R}}_l(t)$ of two-state homogeneous series system
$\boldsymbol{R}^{(m)}_{k_n,l_1,l_2,\ldots,l_{k_n}}(t)$	a reliability function of a homogeneous two-state series−'m out of k_n' system
$\overline{\boldsymbol{R}}^{(\overline{m})}_{k_n,l_1,l_2,\ldots,l_{k_n}}(t)$	a reliability function of a homogeneous two-state series−'m out of k_n' system
$\boldsymbol{R}^{(m)}_{k_n,l_n}(t)$	a reliability function of a homogeneous and regular two-state series−'m out of k_n' system
$\overline{\boldsymbol{R}}^{(\overline{m})}_{k_n,l_n}(t)$	a reliability function of a homogeneous and regular two-state series−'m out of k_n' system
$\overline{\boldsymbol{R}^{(m_1,m_2,\ldots,m_{k_n})}_{k_n,l_1,l_2,\ldots,l_{k_n}}}(t)$	a reliability function of a two-state 'm_i out of l_i'−series system
$\overline{\overline{\boldsymbol{R}}^{(\overline{m}_1,\overline{m}_2,\ldots,\overline{m}_{k_n})}_{k_n,l_1,l_2,\ldots,l_{k_n}}}(t)$	a reliability function of a two-state 'm_i out of l_i'−series system
$\overline{\boldsymbol{R}^{(m)}_{k_n,l_n}}(t)$	a reliability function of a homogeneous and regular two-state 'm out of k_n'−series system
$\overline{\overline{\boldsymbol{R}}^{(\overline{m})}_{k_n,l_n}}(t)$	a reliability function of a homogeneous and regular two-state 'm out of k_n'−series system
$\mathfrak{R}^{(m)}_i(t)$	a limit reliability function of a homogeneous and regular two-state series−'m out of k_n' system
$\mathfrak{R}^{(\overline{m})}_i(t)$	a limit reliability function of a homogeneous and regular two-state series−'m out of k_n' system
$\overline{\mathfrak{R}^{(m)}_i}(t)$	a limit reliability function of a homogeneous and regular two-state 'm out of k_n'−series system
$\overline{\overline{\mathfrak{R}}^{(\overline{m})}_i}(t)$	a limit reliability function of a homogeneous and regular two-state 'm out of k_n'−series system
$\boldsymbol{R}_{r,k_n,l_n}(t)$	a reliability function of a two-state series−parallel system of order r

$\mathfrak{R}_i(t)$	a limit reliability function of a two-state series–parallel system of order r
$\overline{\boldsymbol{R}}_{r,k_n,l_n}(t)$	a reliability function of a two-state parallel–series system of order r
$\overline{\mathfrak{R}}_l(t)$	a limit reliability function of a two-state parallel–series system of order r
ρ	a factor reducing a component failure rate
$\boldsymbol{R}_n^{(1)}(t)$	a reliability function of a two-state series system with components improved by reducing their failure rates by a factor ρ
$\boldsymbol{R}_n^{(2)}(t)$	a reliability function of a two-state series system with a single hot reservation of its components
$\boldsymbol{R}_n^{(3)}(t)$	a reliability function of a two-state series system with a single cold reservation of its components
$\boldsymbol{R}_n^{(4)}(t)$	a reliability function of a two-state series system with a single mixed reservation of its components
$\boldsymbol{R}_n^{(5)}(t)$	a reliability function of a two-state series system with its single hot reservation
$\boldsymbol{R}_n^{(6)}(t)$	a reliability function of a two-state series system with its single cold reservation
$\mathfrak{R}^{(1)}(t)$	a limit reliability function of a two-state series system with components improved by reducing their failure rates by a factor ρ
$\mathfrak{R}^{(2)}(t)$	a limit reliability function of a two-state series system with a single hot reservation of its components
$\mathfrak{R}^{(3)}(t)$	a limit reliability function of a two-state series system with a single cold reservation of its components
$\mathfrak{R}^{(4)}(t)$	a limit reliability function of a two-state series system with a single mixed reservation of its components
$\mathfrak{R}^{(5)}(t)$	a limit reliability function of a two-state series system with its single hot reservation

$\Re^{(6)}(t)$	a limit reliability function of a two-state series system with its single cold reservation
$T^{(1)}$	a lifetime mean value of a two-state series system with components improved by reducing their failure rates by a factor ρ
$T^{(2)}$	a lifetime mean value of a two-state series system with a single hot reservation of its components
$T^{(3)}$	a lifetime mean value of a two-state series system with a single cold reservation of its components
$T^{(4)}$	a lifetime mean value of a two-state series system with a single mixed reservation of its components
$T^{(5)}$	a lifetime mean value of a two-state series system with its single hot reservation
$T^{(6)}$	a lifetime mean value of a two-state series system with its single cold reservation
$Z(t)$	a complex system operation process
z_b	a complex system operational state
$[p_b(0)]_{1\times\nu}$	a vector of probabilities of an operation process $Z(t)$ at initial states z_b
$[p_{bl}]_{\nu\times\nu}$	a matrix of probabilities of transitions of an operation process $Z(t)$ between operation states z_b and z_l
$p_b(t)$	a transient probability of process $Z(t)$ at an operation state z_b at a moment t
p_b	a limit value of a transient probability $p_b(t)$
θ_{bl}	a conditional sojourn time of a process $Z(t)$ at operational states
$[H_{bl}(t)]_{\nu\times\nu}$	a matrix of conditional distribution functions of sojourn times θ_{bl}
m_{bl}	a mean values of a sojourn time θ_{bl}
θ_b	an unconditional sojourn time of process $Z(t)$ at an operation state z_b
$H_b(t)$	an unconditional distribution function of a sojourn time θ_b at an operation state z_b

m_b	a mean value of an unconditional sojourn time θ_b at an operation state z_b
$\hat{\theta}_b$	an operation process total sojourn time $\hat{\theta}_b$ at the particular operation state z_b during the fixed system opetation time
$\widehat{m}_b$	an expected value of an operation process total sojourn time $\hat{\theta}_b$ at the particular operation state z_b during the fixed system opetation time
$T_{ij}^{(b)}(u)$	a conditional lifetime in a subset of reliability state of a component E_{ij} of a multi-state complex system at an operation states z_b
$[R_{ij}(t,\cdot)]^{(b)}$	a conditional reliability function of a component E_{ij} of a multi-state complex system at an operation states z_b
$[R_{ij}(t,u)]^{(b)}$	a coordinate of a conditional reliability function of a component E_{ij} of a multi-state complex system at an operation states z_b
$T^{(b)}(u)$	a conditional lifetime in a subset of reliability states of a multi-state complex system at operational states z_b
$[\boldsymbol{R}(t,\cdot)]^{(b)}$	a conditional reliability function of a multi-state complex system at operational states z_b
$[\boldsymbol{R}(t,u)]^{(b)}$	a coordinate of a conditional reliability function of a multi-state complex system at operational states z_b
$M_b(u)$	a multi-state complex system conditional mean lifetime in a subset of reliability states
$T(u)$	an unconditional lifetime in a subset of reliability states of a non-homogeneous multi-state complex system
$\boldsymbol{R}(t,\cdot)$	an unconditional reliability function of a multi-state complex system
$\boldsymbol{R}(t,u)$	a coordinate of an unconditional reliability function of a multi-state complex system
$\boldsymbol{r}(t)$	a risk function of a multi-state complex system
$M(u)$	a multi-state complex system mean lifetime in a subset of reliability states

$\sigma(u)$	a multi-state complex system lifetime standard deviation in a subset of reliability states
$\overline{M}(u)$	a multi-state complex system mean lifetime in a particular reliability state
δ	a permitted level of a multi-state complex system risk function
τ	a moment of exceeding a permitted multi-state complex system risk level δ
$\dot{p}_b$	an optimal limit value of a transient probability $p_b(t)$
$\dot{m}_{bl}$	an optimal mean values of a sojourn time θ_{bl}
$\dot{m}_b$	an optimal mean value of an unconditional sojourn time θ_b at an operation state z_b
$\dot{\hat{m}}_b$	an optimal expected value of an operation process total sojourn time $\hat{\theta}_b$ at the particular operation state z_b during the fixed system opetation time
$\dot{\boldsymbol{R}}(t,\cdot)$	an optimal and unconditional reliability function of a multi-state complex system
$\dot{\boldsymbol{r}}(t)$	an optimal risk function of a multi-state complex system
$\dot{M}(u)$	an optimal multi-state complex system mean lifetime in a subset of reliability states
$\dot{\sigma}(u)$	an optimal multi-state complex system lifetime standard deviation in a subset of reliability states
$\dot{\overline{M}}(u)$	an optimal multi-state complex system mean lifetime in a particular reliability state
$\dot{\tau}$	an optimal moment of exceeding a permitted multi-state complex system risk level δ
$[\Re(t,\cdot)]^{(b)}$	a conditional limit reliability function of a multi-state complex system at operational states z_b
$[\Re(t,u)]^{(b)}$	a coordinate of a conditional limit reliability function of multi-state complex system at operational states z_b
$\Re(t,\cdot)$	an unconditional limit reliability function of a multi-state complex system
$\Re(t,u)$	a coordinate of an unconditional limit reliability function of multi-state complex system

Preface

The book is concerned with the application of limit reliability functions to the reliability evaluation of large systems. Two-state and multi-state large systems composed of independent components are considered. The main emphasis is on multi-state systems with degrading (ageing) components because of the importance of such an approach in safety analysis, assessment and prediction, and analysing the effectiveness of operation processes of real technical systems.

Many technical systems belong to the class of complex systems as a result of the large number of components they are built of and their complicated operating processes. This complexity very often causes evaluation of system reliability and safety to become difficult. As a rule these are series systems composed of large numbers of components. Sometimes the series systems have either components or subsystems reserved and then they become parallel–series or series–parallel reliability structures. We meet large series systems, for instance, in piping transportation of water, gas, oil and various chemical substances. Large systems of these kinds are also used in electrical energy distribution. A city bus transportation system composed of a number of communication lines, each serviced by one bus, may be a model series system, if we treat it as not failed, when all its lines are able to transport passengers. If the communication lines have at their disposal several buses we may consider it as either a parallel–series system or an '*m* out of *n*' system. The simplest example of a parallel system or an '*m* out of *n*' system may be an electrical cable composed of a number of wires, which are its basic components, whereas the transmitting electrical network may be either a parallel–series system or an '*m* out of *n*'–series system. Large systems of these types are also used in telecommunication, in rope transportation and in transport using belt conveyers and elevators. Rope transportation systems like port elevators and ship-rope elevators used in shipyards during ship docking and undocking are model examples of series–parallel and parallel–series systems.

Taking into account the importance of the safety and operating process effectiveness of such systems, it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time without repair gives the possibility for more precise analysis of their reliability, safety and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system.

In the case of large systems, the determination of the exact reliability functions of the systems and the system risk functions leads us to very complicated formulae that are often useless for reliability practitioners. One of the important techniques in this situation is the asymptotic approach to system reliability evaluation. In this approach, instead of the preliminary complex formula for the system reliability function, after assuming that the number of system components tends to infinity and finding the limit reliability of the system, we obtain its simplified form.

The mathematical methods used in the asymptotic approach to the system reliability analysis of large systems are based on limit theorems on order statistics distributions considered in very wide literature [3,9–11,15,22,23,28–31,37–41,44,45, 125,143,149,151,169,172]. These theorems have generated the investigation concerned with limit reliability functions of the systems composed of two-state components [5,7,25–29,48–50,58–74,93,94,119–125,135,136,148,152,167,175]. The main and fundamental results on this subject that determine the three-element classes of limit reliability functions for homogeneous series systems and for homogeneous parallel systems have been established by Gniedenko [41]. These results are also presented, sometimes with different proofs, for instance in subsequent works [7,15,23,30,65,93]. The generalizations of these results for homogeneous '*m* out of *n*' systems have been formulated and proved by Smirnow [151], where the seven-element class of possible limit reliability functions for these systems has been fixed. Some partial results obtained by Smirnow may be found in Ref. [93] and additionally with the solution of the speed of convergence problem in Ref. [31]. As has been done for homogeneous series and parallel systems, classes of limit reliability functions have been fixed by Chernoff and Teicher [23] for homogeneous series–parallel and parallel–series systems. Their results were concerned with the so-called 'quadratic' systems only. They have fixed limit reliability functions for the homogeneous series–parallel systems with the number of series subsystems equal to the number of components in these subsystems, and for the homogeneous parallel–series systems with the number of parallel subsystems equal to the number of components in these subsystems. These results may also be found for instance in later works [7,65].

All the results so far described have been obtained under the linear normalization of the system lifetimes. Of course, there is the possibility of looking for limit reliability functions of large systems under other than linear standardization of their lifetimes. In this context, the results obtained by Pantcheva [143] and Cichocki [27] are exemplary. Pantcheva [143] has fixed the seven-element classes of limit reliability functions of homogeneous series and parallel systems under power standardization for their lifetimes. Cichocki [27] has generalized Pantcheva's results to hierarchical series–parallel and parallel–series systems of any order.

The book contains the results described above and their newest generalizations for large two-state systems and their developments for multi-state systems'