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Derek J.S.Robinson

A Course in the Theory of Groups

Second Edition

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With 40 Illustrations

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Preface to the Second Edition

In preparing this new edition I have tried to keep the changes to a minimum, on the principle that one should not meddle with a relatively successful text. Thus the general form of the book remains the same. Naturally I have taken the opportunity to correct the errors of which I was aware. Also the text has been updated at various points, some proofs have been improved, and lastly about thirty additional exercises are included.

There are three main additions to the book. In the chapter on group extensions an exposition of Schreier's concrete approach via factor sets is given *before* the introduction of covering groups. This seemed to be desirable on pedagogical grounds. Then S. Thomas's elegant proof of the automorphism tower theorem is included in the section on complete groups. Finally an elementary counterexample to the Burnside problem due to N.D. Gupta has been added in the chapter on finiteness properties.

I am happy to have this opportunity to thank the many friends and colleagues who wrote to me about the first edition with comments, suggestions and lists of errors. Their efforts have surely led to an improvement in the text. In particular I thank J.C. Beidleman, F.B. Cannonito, H. Heineken, L.C. Kappe, W. Möhres, R. Schmidt, H. Snevily, B.A.F. Wehrfritz, and J. Wiegold. My thanks are due to Yu Fen Wu for assistance with the proofreading. I also thank Tom von Foerster of Springer-Verlag for making this new edition possible, and for his assistance throughout the project.

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Preface to the First Edition

"A group is defined by means of the laws of combinations of its symbols," according to a celebrated dictum of Cayley. And this is probably still as good a one-line explanation as any. The concept of a group is surely one of the central ideas of mathematics. Certainly there are few branches of that science in which groups are not employed implicitly or explicitly. Nor is the use of groups confined to pure mathematics. Quantum theory, molecular and atomic structure, and crystallography are just a few of the areas of science in which the idea of a group as a measure of symmetry has played an important part.

The theory of groups is the oldest branch of modern algebra. Its origins are to be found in the work of Joseph Louis Lagrange (1736–1813), Paulo Ruffini (1765–1822), and Évariste Galois (1811–1832) on the theory of algebraic equations. Their groups consisted of permutations of the variables or of the roots of polynomials, and indeed for much of the nineteenth century all groups were finite permutation groups. Nevertheless many of the fundamental ideas of group theory were introduced by these early workers and their successors, Augustin Louis Cauchy (1789–1857), Ludwig Sylow (1832–1918), Camille Jordan (1838–1922) among others.

The concept of an abstract group is clearly recognizable in the work of Arthur Cayley (1821–1895), but it did not really win widespread acceptance until Walther von Dyck (1856–1934) introduced presentations of groups.

The stimulus to study infinite groups came from geometry and topology, the influence of Felix Klein (1849–1925), Sophus Lie (1842–1899), Henri Poincaré (1854–1912), and Max Dehn (1878–1952) being paramount. Thereafter the standard of infinite group theory was borne almost single-handed by Otto Juljevič Schmidt (1891–1956) until the establishment of the Russian school headed by Alexander Gennadievič Kuroš (1908–1971).

In the meantime the first great age of finite group theory had reached its climax in the period immediately before the First World War with the work of Georg Frobenius (1849–1917), William Burnside (1852–1927), and Issai Schur (1875–1936). After 1928, decisive new contributions were made by Philip Hall (1904–1982), Helmut Wielandt, and, in the field of group representations, Richard Dagobert Brauer (1901–1977). The subsequent intense interest in the classification of finite simple groups is very largely the legacy of their work.

This book is intended as an introduction to the general theory of groups. Its aim is to make the reader aware of some of the main accomplishments of group theory, while at the same time providing a reasonable coverage of basic material. The book is addressed primarily to the student who wishes to learn the subject, but it is hoped that it will also prove useful to specialists in other areas as a work of reference.

An attempt has been made to strike a balance between the different branches of group theory, abelian groups, finite groups, infinite groups, and to stress the unity of the subject. In choice of material I have been guided by its inherent interest, accessibility, and connections with other topics. No book of this type can be comprehensive, but I hope it will serve as an introduction to the several excellent research level texts now in print.

The reader is expected to have at least the knowledge and maturity of a graduate student who has completed the first year of study at a North American university or of a first year research student in the United Kingdom. He or she should be familiar with the more elementary facts about rings, fields, and modules, possess a sound knowledge of linear algebra, and be able to use Zorn's Lemma and transfinite induction. However, no knowledge of homological algebra is assumed: those homological methods required in the study of group extensions are introduced as they become necessary. This said, the theory of groups is developed from scratch. Many readers may therefore wish to skip certain sections of Chapters 1 and 2 or to regard them as a review.

A word about the exercises, of which there are some 650. They are to be found at the end of each section and must be regarded as an integral part of the text. Anyone who aspires to master the material should set out to solve as many exercises as possible. They vary from routine tests of comprehension of definitions and theorems to more challenging problems, some theorems in their own right. Exercises marked with an asterisk are referred to at some subsequent point in the text.

Notation is by-and-large standard, and an attempt has been made to keep it to a minimum. At the risk of some unpopularity, I have chosen to write all functions on the right. A list of commonly used symbols is placed at the beginning of the book.

While engaged on this project I enjoyed the hospitality and benefited from the assistance of several institutions: the University of Illinois at

Urbana-Champaign, the University of Warwick, Notre Dame University, and the University of Freiburg. To all of these and to the National Science Foundation I express my gratitude. I am grateful to my friends John Rose and Ralph Strebel who read several chapters and made valuable comments on them. It has been a pleasure to cooperate with Springer-Verlag in this venture and I thank them for their unfailing courtesy and patience.

Notation

$G, H, \ldots$	Sets, groups, rings, etc.
$\mathfrak{X}, \mathfrak{Y}, \ldots$	Classes of groups
$\alpha, \beta, \gamma, \ldots$	Functions
$x, y, z, \ldots$	Elements of a set
$x\alpha$ or x^{α}	Image of x under α
x^y	$y^{-1}xy$
$[x, y]$	$x^{-1}y^{-1}xy$
$H \simeq G$	H is isomorphic with G
$H \leq G, H < G$	H is a subgroup, a proper subgroup of the group G.
$H \lhd G$	H is a normal subgroup of G
H sn G	H is a subnormal subgroup of G
$H_1 H_2 \cdots H_n$	Product of subsets of a group
$\langle X_\lambda \mid \lambda \in \Lambda \rangle$	Subgroup generated by subsets X_λ of a group
$\langle X \mid R \rangle$	Group presented by generators X and relators R
$d(G)$	Minimum number of generators of G
$r_p(G), r_0(G), r(G)$	p-rank, torsion-free rank, (Prüfer) rank of G

G^n, nG	Subgroup generated by all g^n or ng where $g \in G$
$G[n]$	Subgroup generated by all $g \in G$ such that $g^n = 1$ or $ng = 0$.
$\|S\|$	Cardinality of the set S
$\|G : H\|$	Index of the subgroup H in the group G
$\|x\|$	Order of the group element x
$C_G(H)$, $N_G(H)$	Centralizer, normalizer of H in G
H^G, H_G	Normal closure, core of H in G
Aut G, Inn G	Automorphism group, inner automorphism group of G
Out G	Aut G/Inn G, outer automorphism group of G
Hol G	Holomorph of G
$\mathrm{Hom}_\Omega(G, H)$	Set of Ω-homomorphisms from G to H
$\mathrm{End}_\Omega\, G$	Set of Ω-endomorphisms of G
$H_1 \times \cdots \times H_n$, $H_1 \oplus \cdots \oplus H_n$, $\underset{\lambda \in \Lambda}{\mathrm{Dr}}\, H_\lambda$	Set product, direct products, direct sums
$H \ltimes N$, $N \rtimes H$	Semidirect products
$\underset{\lambda \in \Lambda}{\mathrm{Cr}}\, H_\lambda$	Cartesian product, Cartesian sum
$H \wr K$	Wreath product
$H_1 * \cdots * H_n$, $\underset{\lambda \in \Lambda}{\mathrm{Fr}}\, H_\lambda$	Free products
$H \otimes K$	Tensor product
$G' = [G, G]$	Derived subgroup of a group G
G_{ab}	G/G'
$G^{(\alpha)}$	Term of the derived series of G
$\gamma_\alpha G$, $\zeta_\alpha G$	Terms of the lower central series, the upper central series of G
ζG	Center of G
Fit G	Fitting subgroup of G
Frat G	Frattini subgroup of G

$M(G)$	Schur multiplicator of G
$O_\pi(G)$	Maximal normal π-subgroup of G
$l_\pi(G)$	π-length of G
$\mathrm{St}_G(X)$, X_G	Stabilizer of X in G
Sym X	Symmetric group on X
S_n, A_n	Symmetric, alternating groups of degree n
D_n	Dihedral group of order n
Q_{2^n}	Generalized quaternion group of order 2^n
$\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$	Sets of integers, rational numbers, real numbers, complex numbers
$\mathbb{Z}_n$	$\mathbb{Z}/n\mathbb{Z}$
R^*	Group of units of a ring R with identity
RG	Group ring of a group G over a ring R with identity element
I_G, $\bar{I}_G$	Augmentation ideals
$\mathrm{GL}(V)$	Group of nonsingular linear transformations of a vector space V
$\mathrm{GL}(n, R)$, $\mathrm{SL}(n, R)$	General linear and special linear groups
$\mathrm{PGL}(n, R)$, $\mathrm{PSL}(n, R)$	Projective general linear and projective special linear groups
$T(n, R)$, $U(n, R)$	Groups of triangular, unitriangular matrices
$B(n, e)$	Free Burnside group with n generators and exponent e
M^G, χ^G	Induced module, induced character
max, min	Maximal, minimal conditions
E_{ij}	Matrix with (i, j) entry 1 and other entries 0.

Contents

CHAPTER 1

Fundamental Concepts of Group Theory

In this first chapter we introduce the basic concepts of group theory, developing fairly rapidly the elementary properties that will be familiar to most readers.

1.1. Binary Operations, Semigroups, and Groups

A binary operation on a set is a rule for combining two elements of the set. More precisely, if S is a nonempty set, a *binary operation* on S is a function $\alpha: S \times S \to S$. Thus α associates with each ordered pair (x, y) of elements of S an element $(x, y)\alpha$ of S. It is better notation to write $x \circ y$ for $(x, y)\alpha$, referring to "$\circ$" as the binary operation.

If $\circ$ is *associative*, that is, if:

(i) $(x \circ y) \circ z = x \circ (y \circ z)$ is valid for all x, y, z in S,

then the pair $(S, \circ)$ is called a *semigroup*.

Here we are concerned with a very special type of semigroup. A semigroup $(G, \circ)$ is called a *group* if it has the following properties:

(ii) There exists in G an element e, called a *right identity*, such that $x \circ e = x$ for all x in G.
(iii) To each element x of G there corresponds an element y of G, called a *right inverse* of x, such that $x \circ y = e$.

While it is clear how to define left identity and left inverse, the existence of such elements is not presupposed; indeed this is a consequence of the group axioms—see 1.1.2.