

# Fundamental Math and Physics for Scientists and Engineers

David Yevick • Hannah Yevick

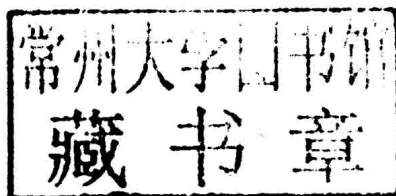
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# FUNDAMENTAL MATH AND PHYSICS FOR SCIENTISTS AND ENGINEERS

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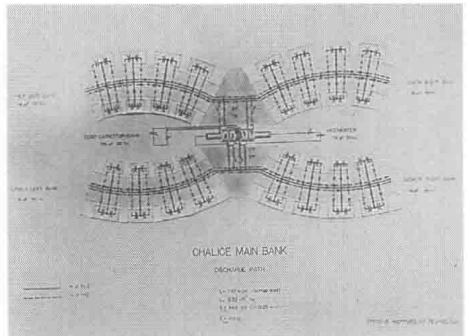
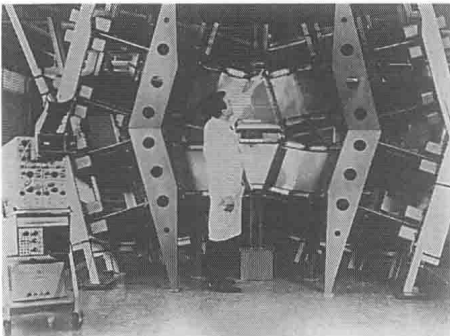
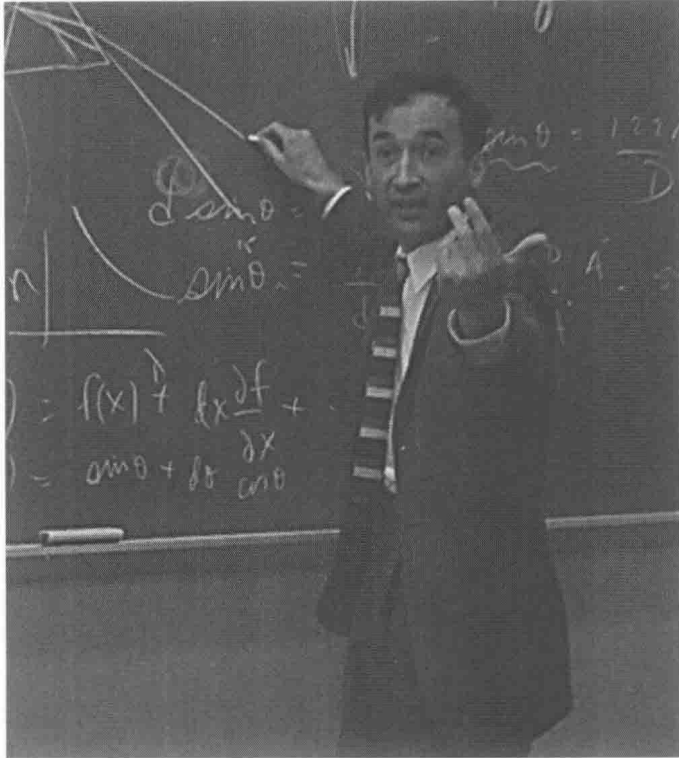
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**FUNDAMENTAL MATH AND PHYSICS  
FOR SCIENTISTS AND ENGINEERS**



*To George*









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# 1

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## INTRODUCTION

Unique among disciplines, physics condenses the limitlessly complex behavior of nature into a small set of underlying principles. Once these are clearly understood and supplemented with often superficial domain knowledge, any scientific or engineering problem can be succinctly analyzed and solved. Accordingly, the study of physics leads to unsurpassed satisfaction and fulfillment.

This book summarizes intermediate-, college-, and university-level physics and its associated mathematics, identifying basic formulas and concepts that should be understood and memorized. It can be employed to supplement courses, as a reference text or as review material for the GRE and graduate comprehensive exams.

Since physics incorporates broad areas of science and engineering, many treatments overemphasize technical details and problems that require time-consuming mathematical manipulations. The reader then often loses sight of fundamental issues, leading to gaps in comprehension that widen as more advanced material is introduced. This book accordingly focuses exclusively on core material relevant to practical problem solving. Fine details of the subject can later be assimilated rapidly, effectively placing leaves on the branches formed by the underlying concepts.

Mathematics and physics constitute the language of science. Hence, as with any spoken language, they must be learned through repetition and memorization. The central results and equations indicated in this book are therefore indicated by shaded text. These should be rederived, transcribed into a notebook or review cards with a summary of their derivation and memorized. Problems from any source should be solved in conjunction with this book; however, undertaking time-consuming problems



without recourse to worked solutions that indicate optimal calculational procedures is not recommended.

Finally, we wish to thank our many inspiring teachers, whose numerous insights guided our approach, in particular Paul Bamberg, Alan Blair, and Sam Treiman, and, above all, our father and grandfather, George Yevick, whose boundless love of physics inspired generations of students.