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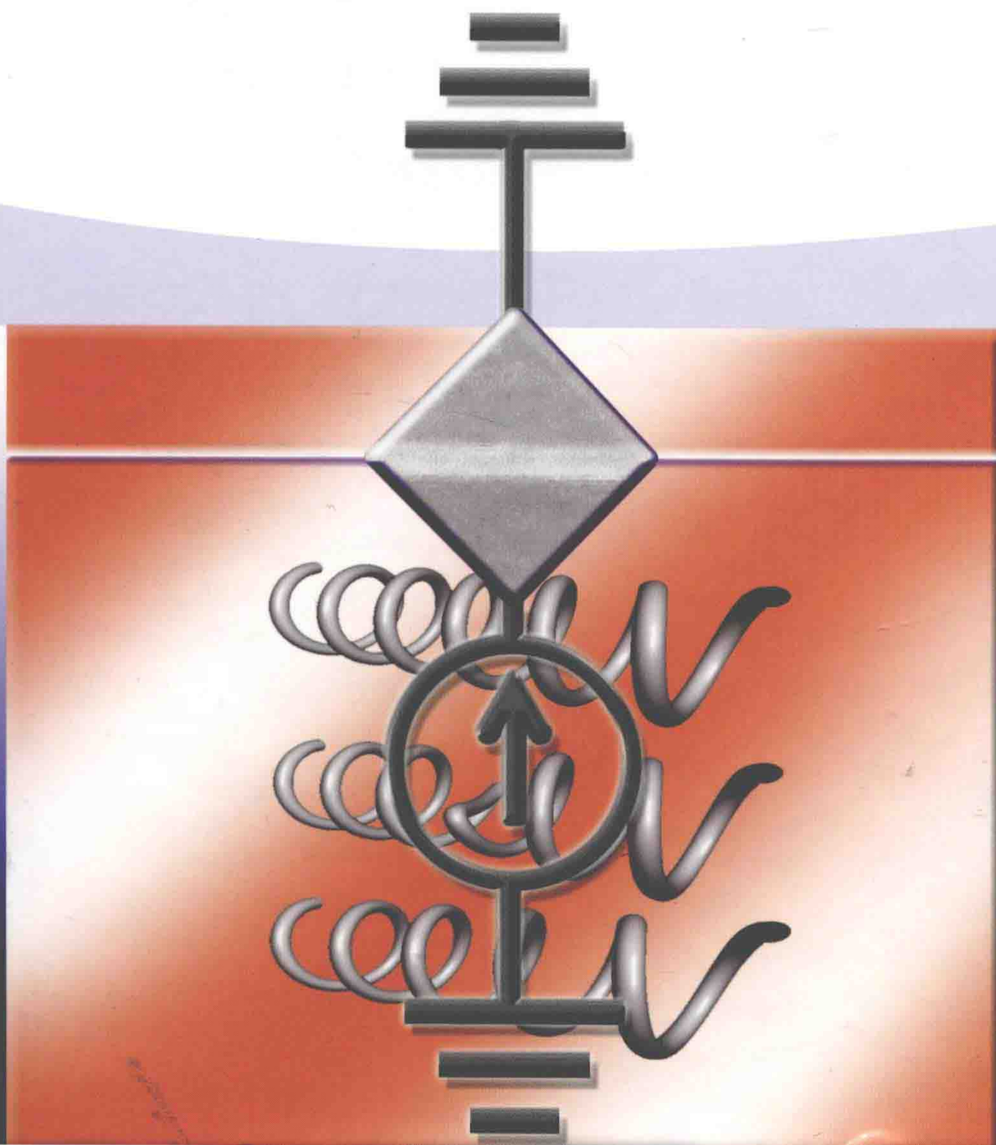

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One-Dimensional Superconductivity in Nanowires

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Cover Figure
Artistic rendering of an 8 nm wide, 20 μm long nanowire fabricated on an InP semiconducting stencil. Electrical contacts to the nanowire (left and right pads) are realized during the nanowire deposition. At the center, a QPS junction is current biased by a current source: The number of windings of the phase of the order parameter decreases because of a phase slippage event. The diagram does not indicate the actual nanowire connection in a circuit. Rendering of the windings courtesy of A. Del Maestro.

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To Maria Elena, Mattia, Gabriele, Giulia and to my parents:
Thank you for your love and support.

Fabio

To my family: Ying, Emily and Austin, my sister Margaret, my brother Winston, my father Bertrand Tsu-Shen, and my late mother, Virginia Pang-Ying.

Albert

Preface

This book is devoted to the topic of superconductivity in very narrow metallic wires. Interest in such wires is driven by the continuing drive for miniaturization in the electronics industry, where the reduction of heat dissipation by the use of interconnects which are superconducting may be necessary. This has led to the invention of new methods of producing very narrow wires with good quality and uniformity in dimensions, and opened up the possibility of novel device paradigms.

The superconducting state is a state of coherent pairs of electrons, held together by the mechanism of the Cooper instability. When the lateral dimensions are small, a new pathway opens up for the superconductor to produce dissipation, associated with the enhanced rate at which fluctuations can occur in the complex order parameter – a quantity which describes the magnitude and phase of the Cooper pairs. In many regimes, the fluctuations of the phase of the Cooper pairs is often the dominant process, in analogy with what occurs in dissipative Josephson junctions. This new pathway is related with the motion of vortices in type-II superconductors, which gives rise to dissipation in 2D (two-dimensional) and bulk superconductors, in which a rapid change of phase occurs as a vortex passes by. Here, because of the physical dimension of the system (comparable to 10 nm), the vortex quickly passes across the entire narrow wire, producing dissipation in the process.

Although nanowires are small in their transverse dimensions, they are still much larger than the Fermi wavelength (λ_F) of the electronic system, which is of the order of a few angstrom in all metals. In conventional superconductors such as Al, Pb, Sn, Nb, MoGe, and so on, in thin film or nanowire form, the coherence length is $\xi \approx 5-100$ nm, typically 10–1000 times the Fermi wavelength. The system we will consider is therefore in a sort of mixed-dimensional regime. From the condensate perspective, the system is 1D (one-dimensional), in the sense that the transverse dimensions are smaller than the Cooper pair size: for this reason, the wave function, or alternatively, the order parameter, describing the Cooper pairs, is uniform, and thus position-independent, in the transverse directions. From the fermionic quasiparticle excitation perspective, the system is effectively 3D (three-dimensional; λ_F is much smaller than the lateral dimension) and there is a large number of transverse channels (from $\approx$ 100 in a multiwalled carbon nanotube, to $\approx$ 3000 in an $\approx$ 8 nm diameter aluminum nanowire), analogous to transverse modes in a wave guide.

In this limit, the dominant collective excitations are no longer pair-breaking excitations across the superconducting energy gap, but rather "phase-slips," which are topological defects in the ground state configuration. Phase slips are related to the motion of vortices in type-II superconductors which give rise to dissipation in 2D and bulk superconductors: in a 1D system, they produce a sudden change of phase by 2π across a core region of reduced superconducting correlation which gives rise to a voltage pulse.

Another intriguing aspect arises from the fact that even in wires which are not ballistic along the wire length, the typical level spacing in the transverse direction can significantly exceed the superconducting gap energy scale. Thus, the possibility of singularity in the density of the electronic state in the normal system associated with each transverse channel can cause oscillatory behaviors in the superconducting properties.

From the above discussion, it is clear that the regime of interest is delineated by the condition that the size of the Cooper pairs, or the superconducting coherence length, be larger than the transverse directions perpendicular to the length of the wire. In this limit the order behavior of the Cooper pair is largely uniform across the wire length, and only variations along the wire length need to be considered. At temperatures well below the superconducting transition temperature, the scale for the observation of dimensionality effects is in the 10–100 nm range (5–50 nm in radius): in this regime, new physics have been predicted, including universal scaling laws in the conductance of the wire.

To access the regime where quantum processes become dominant, however, a more stringent requirement is necessary, that of a sufficiently large probability of fluctuations to occur: this more stringent criterion places the scale requirement in the 10 nm (5 nm in radius) range. Nanowires in this regime may either be a single monolithic wire, such as nanowires made of MoGe, Al, Nb, In, PbIn, Sn, and so on, or coupled wires such as in carbon nanotube bundles.

It should be noted that, in the strict sense, only systems in 2D or 3D have a true, sharp, thermodynamic phase transition into the superconducting state at a finite temperature T_c. In the 2D case, it is a Berenzinskii–Thouless–Kosterlitz type of second-order phase transition, while in 3D, it is a second-order continuous phase transition in the Ginzburg–Landau sense, at least for type-II superconductors supporting the existence of vortices within the bulk. In contrast, in a 1D nanowire, the fluctuations cause the transition to become smeared, so that the resistance remains finite, albeit small, below the transition. Early theoretical analyses were motivated by experimental observations of such behavior, and attempted to quantify the amount of residual resistance. Thus, naturally, questions arise as to whether the resistance vanishes at zero T, and whether novel excitations are able to limit the supercurrent below the depairing limit.

From a broader point of view, 1D superconducting nanowires are interesting from a variety of perspectives, including many body physics, quantum phase transition (QPT), macroscopic quantum tunneling (MQT) processes, and device applications. The field, by nature, involves rather technically sophisticated methods. This is true from either the experimental or theoretical side.

On the experimental side, there are many technical challenges, and thus it is not for the faint of heart. Such challenges include nanowire fabrication, the delicate nature of nanowires with respect to damage–they act as excellent fuses, and their sensitivity to environmental interference from external noise sources, and so on. At the level of 10 nm in transverse dimensions, corresponding roughly to 40 atoms across, even width fluctuations of a few atoms can have significant influence on the energetics and properties. Thus, to obtain intrinsic behaviors of relevance to a uniform wire, rather than behavior limited to weak-links or a very thin region in a nonuniform wire, fabrication is exceedingly demanding. Arguably, only in recent years has the emergence of fabrication techniques come into existence with sufficient precision for producing unusually uniform nanowires. Thus, substantial progress is occurring on the experimental front.

On the theoretical side, analyses invariably involve sophisticated quantum field theory (QFT), quantum phase transition (QPT), Bogoliubov–de Gennes BdG, Ginzburg–Landau (GL), Gorkov–Eilenberger–Usadel self-consistent (including nonlinearities) techniques, all of which require a rather advanced level of understanding of the theoretical machinery. This is often compounded by the fact that the concept of the quantum-mechanical tunneling in the phase of the superconducting order-parameter is difficult to motivate from a classical perspective, since, unlike coordinates or momenta, the phase is a concept born out of wave mechanics. Instead, the tunneling in the phase degree of freedom is usually introduced via the Feynman path integral type of formulation, as one finds a more natural description, for this phenomenon, in terms of instantons in field theory language.

The goal of this book is to produce a relatively self-contained introduction to the experimental and theoretical aspects of the 1D superconducting nanowire system. The aim is to convey what the important issues are, from experimental, phenomenological, and theoretical aspects. Emphasis is placed on the basic concepts relevant and unique to 1D, on identifying novel behaviors and concepts in this unique system, as well as on the prospects for potentially new device applications based on such new concepts and behaviors. The latest experimental techniques and results in the field are summarized. On the theoretical side, much of the field theoretic methods for analyzing the various quantum phase transitions, such as superconductor–metal transitions, superconductor–insulation transitions, and so on, brought about by disorder, are highly technical and the details are beyond the scope of this book. Nevertheless, an attempt is made to summarize the relevant issues and predictions, to pave the way for understanding the formalisms and issues addressed in available journal literature. It is the hope of the authors that this book will serve as a starting point for those interested in joining this exciting field, as well as serving as a useful reference for active researchers.

To this end, our philosophy is to present the field as an active, exciting, and ongoing discourse, rather than one that is fully established. Thus, many of the concepts and experiments are still fraught with a certain degree of healthy controversy. Thus, an attempt is made to convey a sense of openness to the discourse in the field.

The book is organized as follows: Chapter 1 contains a brief history of the field, and a succinct summary of the various theoretical methodologies for understanding conventional superconductivity. These methods are widely used in analyzing 1D superconducting nanowire systems. Chapter 2 is devoted to the basic concepts of 1D superconductivity, including size quantization and its influence on superconducting properties, leading to the phenomenon of shape resonances, the phase-slip phenomenon, which originated from an attempt to explain the broadened temperature transition and the finite voltage along the wire below but near the transition, as well as the conditions and relevant energy scales in molecular systems such as carbon nanotubes. In Chapter 3, the quantum theory based on path integral formulation is summarized. The various types of quantum phase transitions and competing physical scenarios are described. Chapter 4 explores new concepts and potentially new devices based on the idea of a duality between Cooper pairs and the phase slip. Novel QPS junctions are described. These are believed to offer new venues for a current version of the Shapiro steps, as well as a platform for qubits. Nonlinear and nonequilibrium effects based on the Usadel equations are described in Chapter 5.

On the experimental side, the all-important description of the state-of-the-art fabrication methodologies is presented in Chapter 6. Experimental techniques, such as filtering to remove external environmental noise are summarized in Chapter 7. Finally, in Chapter 8, we discuss the current state of experimental progress, and the many open questions, as well as future prospects. To conclude, in Chapter 9, we describe recent experimental results in superconducting nanowire single-photon detector that are now approaching the 1D superconducting limit and devices that are related to 1D superconductivity via the proximity effect: in this class, we find nanotubes and semiconducting nanowires, which have recently indicated of the presence of Majorana fermions.

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Abbreviations and Symbols

Acronyms

APE	Anti-proximity Effect
BdG	Bogoliubov–de–Gennes
BCS	Bardeen–Cooper–Schreiffer
CPR	Current–Phase Relation
cQPS	Coherent Quantum Phase-Slip
DCR	Dark Current Rate
DQM	Dissipative Quantum Mechanics
e-beam	Electron beam
GIO	Giordano expression for the resistance due to quantum phase slips, or to QPS and TAPS
GL	Ginzburg–Landau
HQS	Silsesquioxane, a type of negative electron beam resist
IRFP	Infinite-Randomness Fixed Point
JJ	Josephson Junction
KQPS	Khlebnikov Quantum Phase-Slip
KTB	Kosterlitz–Thouless–Berezinskii
LA	Langer–Ambegaokar
LAMH	Langer–Ambegaokar–McCumber–Halperin
MBE	Molecular-Beam-Epitaxy
MH	McCumber–Halperin
PMMA	Polymethylmethacrylate: probably the most common electron beam resist, mainly used as positive tone resist
PSC	Phase-Slip Center
QPS	Quantum Phase-Slip
RTFIM	Random-Transverse-Field Ising Model
SC	Superconducting
SG	sine-Gordon
SEM	Scanning Electron Microscope
SIT	Superconductor-Insulator Transition

SmNW	Semiconducting Nanowire
SMT	Superconductor-Metal Transition
SNAP	Superlattice Nanowire Pattern Transfer
SNAP	Superconducting Nanowire Avalanche Photo Detector
SNSPD	Superconducting Nanowire Single-Photon Detector
SQUID	Superconducting Interference Device
SSPD	Superconducting Single-Photon Detector
SWNT	Single-Walled Nanotube
SWCNT	Single-Walled Carbon Nanotube
TAP or TAPS	Thermally-Activated Phase Slip

Symbols

w	Width of nanowire or wire
h	Height of nanowire or thin film
d	Diameter of nanowire
L	Length of nanowire
L_x	Length in x-direction of a thin film
L_y	Length in y-direction of a thin film
A	Cross sectional area of a nanowire
V	Volume
T	Temperature
$k_B T$	Thermal energy scale; k_B is the Boltzmann's constant
β	$\frac{1}{k_B T}$
e	fundamental unit of charge: $+1.602 \times 10^{-19}$ C = $+ 4.80 \times 10^{-10}$ esu
μ	(i) Chemical potential (ii) parameter that controls the KTB (Kosterlitz–Thouless–Berezinskii) phase transition
E_F	Fermi energy
v_F	Fermi velocity (speed)
$N(0) \equiv D(E_F)$	Density of states at the Fermi level of both spins, in the normal state
$-e$	Charge of the electron
$(-2e)$	Charge of the Cooper-pair of electrons
m	electron mass
M	Cooper-pair mass $M = 2m$
$D = \frac{1}{3} v_F l_{mfp}$	Diffusion coefficient
l_{mfp}	Mean-free-path
T_c	Critical temperature, or normal to superconducting transition temperature
Δ	Superconducting gap
Δ_0	Zero temperature superconducting gap

ω_D	Debye frequency
n_s	Superconducting carrier density
g	Gorkov coupling constant, take to be positive $g = VV$
u	Amplitude of electron-like component of a Bogoliubov quasiparticle
v	Amplitude of hole-like component of a Bogoliubov quasiparticle
ξ	Superconducting coherence length, usually in the dirty limit of $\xi_{cln} \sim \xi_{bulk} \sim \xi_{BCS} \gg l_{mfp}$
ξ_0	Zero temperature superconducting coherence length
ξ_{bulk}	Superconducting coherence length is bulk 3D material
ξ_{cln}	Superconducting coherence length in the clean limit
ξ_{BCS}	The BCS superconducting coherence length
λ_L	London penetration depth
λ_n or λ_i	Eigenvalues of index n or index i
E_C	Coulomb charging energy $E_C = (2e)^2/C$, where C is the capacitance
E_J	Josephson energy
C	(i) Capacitance per unit length of a nanowire (ii) capacitance of a Josephson junction
L_{kin}	Kinetic inductance per unit length of a nanowire
c_{pl}	Mooij–Schön plasmon mode propagation speed
ψ or $\psi^\dagger$	Electron annihilation or creation operator
ψ	(i) Ginzburg–Landau superconducting order parameter (dimensionless) (ii) Gross–Pitaevskii superconducting order parameter
Ψ	Ginzburg–Landau superconducting order parameter
τ_{GL}	Ginzburg–Landau relaxation time: $\tau_{GL} = \frac{\pi}{8}\frac{\hbar}{T_c - T}$
ϕ_0	Superconducting flux quantum: $\phi_0 = hc/(2e)$ (cgs); $\phi_0 = h/(2e)$ (SI)
R_Q	Superconducting quantum resistance $R_Q \equiv h/(2e)^2 \approx 6.453\,k\Omega$
A	Vector potential
V	(i) electrostatic potential (ii) strength of electron-phonon coupling in the Gorkov coupling constant $g = VV$
$\mathbf{E}$	Electric field
$\mathbf{B}$	Magnetic field
J	Electrical current density
J_s	Electrical current density of the superfluid
J_c	Critical electrical current density
j	Reduced electrical current density
j_c	Reduced critical electrical current density
I	Electrical current
I_c	Critical electrical current

I_s	Switching current, at which a nanowire switches from a superconducting state to a normal state, typically somewhat smaller than the I_c in the depairing limit
I_{bias}	Externally applied bias current
a	(i) Lattice constant (ii) Proportionality constant in the Giordano expression for the quantum phase-slip rate or quantum phase-slip resistance
φ	Phase of the complex superconducting order-parameter
H_c	Thermodynamic critical (magnetic) field
R_{QPS}	The quantum phase-slip contribution to the resistance
R_{TAP}	The thermally-activated phase-slip contribution to the resistance
Γ_{QPS}	The quantum phase-slip (tunneling) rate
Γ_{TAP}	The phase-slip rate due to thermal-activation
$\Gamma^{\pm}$	The phase-slip rate: + corresponds to increasing the phase-winding by one unit, − to decreasing by one unit
Γ_{inst}	Quantum tunneling rate of an instanton
Ω	Attempt frequency for the phase-slip process
$\Omega^{\pm}$	Attempt frequency for the phase-slip process: + corresponds to increasing the phase-winding by one unit, − to decreasing by one unit
Z	(i) Partition function (ii) Mooij–Schön plasmon propagation impedance
S	Action, in almost all cases, the Euclidean action in the imaginary-time formulation
S_D	Drude contribution to the action
S_J	Josephson contribution to the action
S_L	London contribution to the action
S_{em}	Electromagnetic field contribution to the action
S_{diss}	Dissipation contribution to the action
S_{bias}	Contribution to the action from a biasing current
S_{bdry}	Boundary contribution to the action
S_{1D}	Action for a 1D superconducting nanowire
S_{QPS}	Action due to a single or multiple quantum phase-slip
F	Free energy
ΔF	Free energy barrier for the creation of a phase-slip
$\Delta F^{\pm}$	Free energy barrier for the creation of a phase-slip: + corresponds to increasing the phase-winding by one unit, − to decreasing by one unit
$\hat{G}$	The Gorkov Green's functions in matrix form in Nambu space
$\check{G}$	The Gorkov Green's functions in the Keldysh formulation, with both the advanced and retarded, as well as the Keldysh component. The matrix is in the direct product space of forward and backward branches of the Keldysh contour, with the Nambu space.

G	The normal components of the Gorkov Green's functions. They represent the diagonal components of the $\hat{G}$ matrix
F	The superconducting components of the Gorkov Green's functions, representing the off-diagonal components of the $\hat{G}$ matrix (sometimes with an extra sign change, and or hermitian conjugation)
$G^{\mathrm{R,A,K}}$	In the Keldysh formulation: Retarded (R) G, Advanced (A) G, and Keldysh (K) G
$F^{\mathrm{R,A,K}}$	In the Keldysh formulation: Retarded (R) F, Advanced (A) F, and Keldysh (K) F
$\hat{g}$	The Eilenberger–Larkin–Ovchinnikov Green's functions, which are the Gorkov ones averaged over an energy variable
$\check{g}$	The Eilenberger–Larkin–Ovchinnikov Green's functions, which are the Gorkov ones averaged over an energy variable, in the Keldysh formulation, with both the advanced and retarded, as well as the Keldysh component. The matrix is in the direct product space of forward and backward branches of the Keldysh contour, with the Nambu space.
g	The normal components of the Eilenberger–Larkin–Ovchinnikov Green's functions, which are the Gorkov ones averaged over an energy variable. They represent the diagonal components of the $\hat{g}$ matrix
f	The superconducting components of the Eilenberger–Larkin–Ovchinnikov Green's functions, which are the Gorkov ones averaged over an energy variable, representing the off-diagonal components of the $\hat{g}$ matrix
σ	Electron spin index
$\sigma_{x,y,z}$	x: x-component of Pauli matrix; y: y-component, z: z-component
$g^{\mathrm{R,A,K}}$	In the Keldysh formulation: Retarded (R) g, Advanced (A) g, and Keldysh (K) g
$f^{\mathrm{R,A,K}}$	In the Keldysh formulation: Retarded (R) f, Advanced (A) f, and Keldysh (K) f
f_{QPS}	QPS fugacity
x_0	Core size of a quantum phase-slip, typically taken to be of order ξ_0
τ_0	Time-scale of a quantum phase-slip, typically $\gtrsim \hbar/\Delta$
D_{N}	Inverse of the compressibility of normal electron fluid
D_{S}	Inverse of the compressibility of superconducting electron fluid
V_{N}	Voltage of the normal fluid
V_{S}	Voltage of the super fluid
r	Normal electron to Cooper-pair conversion resistance
γ	Parameter proportional to the rate of normal electron to Cooper-pair conversion
λ_{Q}	Length scale for the normal electron to Cooper–pair conversion process

$f_{\mathrm{dpl,p}}$	Fugacity of QPS–anti-QPS dipoles, separated by p lattice sites
K or K_{s}	The superconducting stiffness
$K_{\mathrm{s,w}}$ or K_{w}	The superconducting stiffness for a 2D strip of width w
α_{dis}	Dimensionless disorder parameter in the Khlebnikov–Pryadko theory of quantum phase-slips
A_{dis}	A parameter, which characterizes the correlation integral of the disorder potential
C_{w}	Capacitance per unit area of a narrow 2D superconducting strip
$L_{\mathrm{kin,w}}$	Kinetic inductance for a unit area of a narrow 2D superconducting strip
q	Vortex charge-vector
$q^{0;1,1}$	The 0-th, 1st and 2nd components of the (2+1)D q-vector
J_{vor}	Vortex current density-vector
$f_{\mu\nu}$	The field tensor components of an effective photon field
n_{cp}	Cooper-pair number
n_{flux}	The flux number penetration a superconducting loop
ω_{pl}	Plasma frequency, e.g. in a Josephson junction, or that characterizing the vibrations within the local minimum of the free-energy
V_{c}	The critical voltage in a QPS-junction. It is the dual to the critical current I_{c} in a conventional Josephson Junction