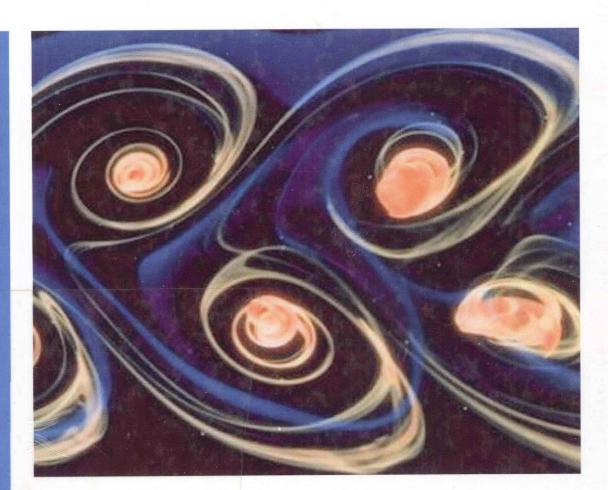
The Dawn of Fluid Dynamics

A Discipline between Science and Technology



Michael Eckert

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and Technology

The author of this book

Dr. Michael Eckert

Deutsches Museum München email: M.Eckert@deutsches-museum.de

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Preface

A leading representative of fluid dynamics defined this discipline as "part of applied mathematics, of physics, of many branches of engineering, certainly civil, mechanical, chemical, and aeronautical engineering, and of naval architecture and geophysics, with astrophysics and biological and physiological fluid dynamics to be added." [1, p. 4]

Fluid mechanics has not always been as versatile as this definition suggests. Fifty years ago, astrophysical, biological, and physiological fluid dynamics was still in the future. A hundred years ago, aeronautical engineering did not yet exist; when the first airplanes appeared in the sky before the First World War, the science that became known as aerodynamics was still in its infancy. By the end of the 19th century, fluid mechanics meant hydrodynamics or hydraulics: the former usually dealt with the aspects of "ideal," i.e., frictionless, fluids, based on Euler's equations of motion; the latter was concerned with the real flow of water in pipes and canals. Hydrodynamics belonged to the domain of mathematics and theoretical physics; hydraulics, by contrast, was a technology based on empirical rules rather than scientific principles. Theoretical hydrodynamics and practical hydraulics pursued their own diverging courses; there was only a minimal overlap, and when applied to specific problems, the results could contradict one another [2].

This book is concerned with the history of fluid dynamics in the twentieth century before the Second World War. This was the era when fluid dynamics evolved into a powerful engineering science. A future study will account for the subsequent period, when this discipline acquired the multifaceted character to which the above quote alluded. The crucial era for bridging the proverbial gap between theory and practice, however, was the earlier period, i.e., the first four decades of the twentieth century. We may call these decades the age of Prandtl, because no other individual contributed more to the formation of modern fluid dynamics. We may even pinpoint the year and the event with which this process began: it started in 1904, when Ludwig Prandtl presented at a conference the boundary layer theory for fluids with little friction. Prandtl's publication was regarded as "one of the most extraordinary papers

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of this century, and probably of many centuries" [1]—it "marked an epoch in the history of fluid mechanics, opening the way for understanding the motion of real fluids" [3].

In order to avoid any misunderstanding: this is not a biography of Prandtl, however desirable an account of Prandtl's life might be. Nor is it a hero story; I do not claim that the emergence of modern fluid dynamics is due solely to Prandtl. If Prandtl and his Göttingen circle's work is pursued here in more detail than that of other key figures of this discipline, it is because the narrative needs a thread to link its parts, and Prandtl's contributions provide enough coherence for this purpose. The history of fluid dynamics in the age of Prandtl, as presented in the following account, is particularly a narrative about how science and technology interacted with another in the twentieth century. How does one account for such a complex process? In contrast to sociological approaches I pursue the history of fluid dynamics not within a theoretical model of science-technology interactions. Nevertheless, the relationship of theory and practice, science and engineering, or whatever rhetoric is used to refer to these antagonistic and yet so similar twins, implicitly runs as a recurrent theme through all chapters of this book. I share with philosophers, sociologists, and other analysts of science studies the concern to better grasp science-technology interactions, but I cannot see how to present the history of fluid dynamics from the perspective of an abstract model. My own approach is descriptive rather than analytical; I approach the history of fluid dynamics from the perspective of a narrator who is more interested in a rich portrayal of historical contexts than in gathering elements for an epistemological analysis. This approach requires deviations here and there from the main alley, so to speak, in order to clarify pertinent contexts, but I am conscious not to lose the narrative thread and regard as pertinent only what contributes to a better understanding of the theory-practice issue. I postpone further reflections to the epilogue, when this issue may be better discussed in view of the empirical material presented throughout the remainder of the book.

Many people and institutions have contributed to this work. Instead of acknowledging their help here individually in the form of a long list of names, I refer readers to the notes in the appendix, where readers may better appreciate how archives and authors of other studies helped to add flesh to the skeleton of my narrative. The only exceptions concern my colleagues from the Deutsches Museum and the Munich Center for the History of Science and Technology, whom I owe thank for years of fruitful collaboration and stimulating discussions, and the Deutsche Forschungsgemeinschaft for funding the Research Group 393, which formed the framework of this study.

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1

Diverging Trends before the Twentieth Century

The flow of water or air around an obstacle is such a familiar phenomenon that we tend to underrate its importance in the history of science and technology. Throughout the centuries, the behavior of a body in a fluid was a fundamental theoretical problem and an obvious practical concern. The motion of celestial bodies, ships, projectiles, and other phenomena involved conceptions of fluid dynamics. Although the development of science from Aristotle to Einstein is usually presented without excursions into the history of fluid dynamics, concepts about motion inevitably involve assumptions about fluid resistance.

1.1 Galileo's Abstraction

In Aristotle's natural philosophy, the medium through which a motion proceeded played a paradoxical role. In order to sustain the motion, a motive agency was required. Aristotle (384–322 BC)imagined that this motive agency resided in the medium: "We must, therefore," Aristotle wrote in Book VIII of his *Physics*, "hold that the original movent gives the power of causing motion to air, or water, or anything else which is naturally adapted for being a movent as well as for being moved" [4, p. 506]. At the start of the motion of a projectile, the medium would be displaced by the projectile, and together with this displacement, a motive force would be passed along the trajectory. Thus, the medium acquired the power to propel the projectile. At the same time the medium would resist the motion: "If air is twice as tenuous as water," Aristotle argued, "the same moving body will spend twice as much time in travelling a certain path in water as in travelling the same path in air" [5, p. 21].

Aristotle dominated pre-modern natural philosophy – but some of his views also served as bones of contention. How could the same medium at the same time propel and resist the motion of a projectile? Most famous among those who criticized this concept was Jean Buridan (1300–1358), who argued that the propulsive property resided in the projectile itself rather than

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in the medium. He called this property impetus: "Whenever some agency sets a body in motion," Buridan wrote, "it imparts to it a certain impetus, a certain power which is able to move the body along in the direction imposed upon it at the outset (...) It is this impetus which moves a stone after it has been thrown until the motion is at an end. But because of the resistance of the air and also because of the heaviness, which inclines the motion of the stone in a direction different from that in which the impetus is effective, this impetus continually decreases" [5, pp. 49–50]. Now the medium through which the motion proceeded was left with just one property: resistance.

The impetus concept marked the emergence of the modern notions of inertia and momentum. But that did not happen at once. Even Galileo Galilei (1564–1642), with whom we associate the revolutionary turn from the medieval philosophy to the "new science" of motion, still mixed Aristotelian concepts with modern concepts of motion. Like his predecessors, Galileo struggled with the role of the medium through which a body moves. His famous *Dialogues Concerning Two New Sciences* reveals what problems were behind the effort to imagine how a body would move without the resistive property of the medium. Galileo lets Salviati ask, for example, "What would happen if bodies of different weight were placed in media with different resistances?" The answer was presented by comparing the motion in air and water: "I found," Salviati continues, "that the differences in speed were greater in those media which were more resistant, that is, less yielding. This difference was such that two bodies which differed scarcely at all in their speed through air would, in water, fall the one with a speed ten times as great as that of the other" [6, p. 68].

Obviously, motion in air would be closer to a motion without any influence of the medium. But there were problems in quantitatively measuring differences for bodies with different weights in air. "It occurred to me therefore," Galileo argues with the voice of Salviati, "to repeat many times the fall through a small height in such a way that I might accumulate all those small intervals of time that elapse between the arrival of the heavy and light bodies respectively at their common terminus." With the repetition of the free fall, he meant the repeated swings of a pendulum:

"Accordingly I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine threads, each four or five cubits long. Pulling each ball aside from the perpendicular, I let them go at the same instant, and they, falling along the circumferences of circles having these equal strings for semi-diameters, passed beyond the perpendicular and returned along the same path. This free vibration repeated a hundred times showed clearly that the heavy body maintains so nearly the period of the light body that neither in a hundred swings nor even in a thousand will

the former anticipate the latter by as much as a single moment, so perfectly do they keep step. We can also observe the effect of the medium which, by the resistance which it offers to motion, diminishes the vibration of the cork more than that of the lead, but without altering the frequency of either; even when the arc traversed by the cork did not exceed five or six degrees while that of the lead was fifty or sixty, the swings were performed in equal times" [6, pp. 84–85].

In order to find out how the resistance of air depends on the velocity, Galileo compared the swings of pendulums with equal weights but different amplitudes. He found that the air resistance is proportional to the velocity of the moving body [6, p. 254].

Already Galileo's contemporaries noticed that these conclusions could not have resulted from actual experiments. Marin Mersenne (1588-1648) compared the swings of equal pendulums with different amplitudes: he found that one which started swinging with an amplitude of two feet differed from one with an amplitude of one foot already after thirty periods of oscillation by as much as one full period. In 1639, a year after the publication of the Dialogues Concerning Two New Sciences, he remarked that if Galileo had performed real pendulum experiments and only waited for thirty or forty swings, he would have noticed the difference [7]. Recent pendulum experiments confirmed Mersenne's critique [8,11].

This and other observations of Galileo stirred considerable debate among historians of science - to what extent did Galileo actually perform experiments? Only his pendulum experiments with small amplitude are presumed "real"; those with larger amplitudes are regarded as "imaginary" or "hypothetical," i.e., they were not performed in reality, but (contrary to mere thought experiments) are based on extrapolation from empirical observations [7]. Earlier interpretations tended to categorize Galileo's style of research into one of two extremes: either as deductive in the tradition of Platonic and idealistic natural philosophy, in which the experiment only plays a role as a confirmation of insights gained by mere thinking; or as inductive, with the experiment as the origin of new knowledge. According to more recent historical studies, however, Galileo's science was more complex and does not fit neatly into one category or the other alone [9].

The question whether Galileo actually performed free fall experiments from the leaning tower of Pisa attracted particular scrutiny [10]. As with the pendulum experiments, the problem of resistance plays an important role here, too. "Aristotle says that 'an iron ball of one hundred pounds falling from a height of one hundred cubits reaches the ground before a one-pound ball has fallen a single cubit.' I say," Salviati responds to such an obvious discrepancy with reality, "that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two finger-breadths" [6, p. 64]. However, a modern calculation, which takes into account the air resistance, yields a difference of 1.05 m for the free fall of a 100-lb. iron sphere (with a radius of 11.13 cm) and a 1-lb. iron sphere (with a radius of 2.4 cm) over a distance of 100 cubits (58.4 m). The lighter sphere would be more than one meter behind the heavier one – certainly much more than the "two finger-breadths" in Galileo's argument [11]. If Galileo really performed the tower experiment, why didn't he notice this discrepancy? The puzzle can be resolved by a psycho-physical argument: when an experimenter intends to release simultaneously two different weights from his outstretched hands, the palm with the lighter weight tends to open a bit earlier than the palm with the heavier weight; this difference could have compensated for the difference due to the air resistance [12], [13, Supplement 3].

But Galileo, presumably, was rather motivated by a theoretical argument. The medium had to be "thrust aside by the falling body," Salviati argued. "This quiet, yielding, fluid medium opposes motion through it with a resistance which is proportional to the rapidity with which the medium must give way to the passage of the body." By such reasoning, Galileo related the displaced mass of the medium to the resistance: "And since it is known that the effect of the medium is to diminish the weight of the body by the weight of the medium displaced, we may accomplish our purpose by diminishing in just this proportion the speeds of the falling bodies, which in a non-resisting medium we have assumed to be equal" [6, pp. 74–75].

In other words, despite a flawed concept of fluid resistance in terms of buoyancy, Galileo arrived at his goal: the abstraction of a motion in a non-resisting medium. With a vanishing buoyancy, the resistance would vanish too. In this case, with no mass to be displaced, all bodies would fall in the same manner. Galileo's law of free fall certainly has to be rated among the most important accomplishments in the history of science, but it is erroneous to infer from Galileo's abstraction that he "had a correct notion of air resistance," as a widely read book on the history of aerodynamics has claimed [14, p. 8]. Galileo did not aim at a theory of aerodynamics; his predominant concern was Aristotle's natural philosophy. The abstraction of a motion in a non-resisting medium, perceived as a motion in which no medium had to be displaced, touched upon another ancient philosophical belief: Aristotle believed in the impossibility of a vacuum; for Galileo, it was the domain in which the laws of free fall hold. Maybe it is not an exaggeration to state that Galileo's elaborations on the medium through which a body moves only served to justify his abstraction of a motion in empty space.

Against this background it does not come as a surprise that it was a pupil of Galileo, Evangelista Torricelli (1608–1647), who is credited with presenting the first experimental evidence of a vacuum. Torricelli emptied glass tubes

filled with mercury into a container, such that the openings of the tubes were not exposed to the air. Inside the inverted tube, above a remaining column of mercury, there was left an empty space, a "Torricellian vacuum." The height of the mercury column in the tube was found to depend on the ambient air pressure. Torricelli undertook these experiments with another pupil of Galileo (Vincenzio Viviani). Like Galileo himself, his pupils also were primarily interested in refuting Aristotelian dogmas. "Many have said [that vacuum] cannot happen," Torricelli wrote to another follower of Galileo after his experiment; yet, it "may occur with no difficulty, and with no resistance from nature." Thus, he refuted the dogma of a "horror vacui." He concluded, with a now famous quote: "We live submerged at the bottom of an ocean of elementary air which is known by incontestable experiments to have weight." [15, p. 84].

After Torricelli's experiment the old debate among "vacuists" and "plenists" seemed to be decided in favor of the "vacuists," but René Descartes (1586-1650) renewed the belief of a universal filling of space. He denied that Galileo's extrapolation of free fall in empty space was based on sound arguments. According to Descartes' doctrine, all natural phenomena resulted from the motions of infinitely fine weightless particles of an ether that pervaded the entire universe. The particles of ordinary matter, such as air or water, were supposed to have weight, so that their displacement by a moving body would retard its motion. In order to prevent a temporary depletion behind a moving object, the displacement of matter involved a flow around the object, which Descartes imagined as vortical. He extended his doctrine to the entire universe. The solar system was supposed to be an enormous vortex of matter, in which the planets orbited as smaller vortices around the center [16].

Descartes did not produce quantitative results – neither for his cosmogony nor for the domain of earthly physics. Once, he communicated in a letter a formula about the retardation of a free falling body in a medium, whereby the speed approached a limit in the form of an infinite geometrical series, but he did not provide a physical argument for this result [23, p. 110]. Nevertheless, he exerted a remarkable influence on seventeenth century natural philosophy. Christiaan Huygens (1629–1695) pursued several of Descartes' ideas, such as the concept of an attracting force due to vortical motion around a center. Such a force would keep a planet embedded in a vortex in his orbit around the sun. In order to illustrate this force, Huygens arranged a little sphere in a cylindrical vessel filled with water such that it was free to move in a radial direction only. When the vessel was rotated around its axis, the sphere moved inwards against the centrifugal force [16, pp. 76–77].

1.2 Hogs' Bladders in St. Paul's Cathedral

Descartes' concepts of motion also influenced Isaac Newton (1643-1727), but as an opponent rather than as a follower. Alluding to Descartes' Principia Philosophiae, Newton titled his own three-volume treatise on mechanics Philosophiae Naturalis Principia Mathematica. The first volume with "Newton's laws of motion" for a body in a vacuum is celebrated as the foundation of classical mechanics. It is less known that Newton also spent a lot of time developing the laws of motion for a body in a fluid. The entire second volume is dedicated to this problem. It was regarded as "the most original part of the whole work, though also largely incorrect" [17, p. 167]. As for Galileo and Descartes, the debate among "vacuists" and "plenists" was also a major issue for Newton. One of his pupils, Henry Pemberton, wrote in 1728 a book titled A View of Sir Isaac Newton's Philosophy about the second volume of Newton's Principia: "By this theory of the resistance of fluids, and these experiments our author decides the question so long agitated among natural philosophers whether the space is absolutely full of matter. The Aristotelians and Cartesians both assert this plenitude; the Atomists have maintained the contrary. Our author has chosen to determine this question by his theory of resistance" [18, p. 314].

If the universe were filled with a material substance, as taught by Descartes and his school, then the planets would encounter a resistance along their orbits around the sun. Descartes' vortex conception could not escape that fundamental problem and therefore would have given rise to contradictions if it had been formulated in a quantitative manner. Newton presented an alternative concept with his theory of universal gravitation, which assumed an empty space between the celestial bodies—or a "bodiless" medium that would not exert a noticeable resistance: "And therefore the celestial spaces, thro' which the globes of the Planets and Comets are perpetually passing towards all parts, with the utmost freedom, and without the least sensible diminution of their motion, must be utterly void of any corporeal fluid, excepting perhaps some extremely rare vapours, and the rays of light." This was Newton's conclusion at the end of the section "Of the motion of fluids and the resistance made to projected bodies" [19, vol. 2, proposition 40, pp. 161–162].

From the outset, Newton assumed: "In mediums void of all tenacity, the resistances made to bodies are in the duplicate ratio of the velocities." Galileo's relation "that the resistance is in the ratio of the velocity," according to Newton, was "more a mathematical hypothesis than a physical one" [19, vol. 2, proposition 4, p. 11]. Others had already made the same assumption of a quadratic velocity dependence, which seemed to be more in agreement with empirical observations (see below); but Newton was the first natural philosopher who attempted to justify this relation on the basis of a physical model.

His concept is too complex for a short summary. It may suffice to hint at Newton's argument for an "elastic fluid" like air, which he conceived as a gas of particles. Based on certain assumptions about the mutual collisions of these particles, Newton obtained quantitative results about the resistance of such a fluid. "But whether elastic fluids do really consist of particles so repelling each other," he concluded, "is a physical question. We have here demonstrated mathematically the property of fluids consisting of particles of this kind, that hence philosophers may take occasion to discuss that question" [19, vol. 2, proposition 23, theorem 18, p. 79]. Newton explicitly envisioned different sources of resistance, "as from the expansion of the particles after the manner of wool, or the boughs of trees, or any other cause, by which the particles are hindered from moving freely among themselves; the resistance, by reason of the lesser fluidity of the medium, will be greater than in the corollaries above" [19, vol. 2, proposition 34, theorem 17, p. 117]. He also developed a notion of viscosity: "The resistance, arising from the want of lubricity in the parts of fluid, is, ceteris paribus, proportional to the velocity with which the parts of the fluid are separated from each other" [19, vol. 2, proposition 51, p. 184].¹

Newton did not content himself with establishing theorems. "In order to investigate the resistances of fluids from experiments, I procured a square wooden vessel (...) this I filled with rain-water: and having provided globes made up of wax, and lead included therein, I noted the times of descents." Thus, Newton described the beginning of a series of experiments on fluid resistance. He used a pendulum with an oscillation period of a half-second for the measurement of time, and meticulously compared the various outcomes with his theoretical formulae: "Three equal globes, weighing 141 grains in air and 4 3/8 in water, being let fall several times, fell in the times of 61, 62, 63, 64 and 65 oscillations, describing a space of 182 inches," he described one of these experiments. "And by the theory they ought to have fallen in 64 1/2 oscillations, nearly." He noticed that sometimes "the globes in falling oscillate a little" and believed that for this reason the resistance was "somewhat greater than in the duplicate ratio of the velocity." But in general he regarded the outcome as an experimental verification of his square law formula for the resistance of a "globe moving though a perfectly fluid compressed medium." After a series of 12 experiments he concluded "that the theory agrees with the phaenomena of bodies falling in water; it remains that we examine the phaenomena of bodies falling in air" [19, vol. 2, proposition 40, pp. 145–155].

¹⁾ In modern terms, this is equivalent to a linear relation between shear stress and strain rate: we call fluids with such viscous behavior "Newtonian." However, Newton did not investigate the relation between stress and strain. See [20, pp. 258-259].