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GROWTH THEORY

VOLUME II

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Growth Theory Volume II

Optimal Growth Theories

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Volume II

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Introduction

This three-volume work provides a comprehensive selection of the most important articles on growth theory. The readings in Volume I address theories that attempt to explain the stylized facts of growth. Volume II focuses on normative models of the growth process. Volume III integrates the positive analysis found in the first volume with the welfare approach found in the second volume. Taken together, the volumes depict the development of growth models from the early aggregative theory without explicitly optimizing agents to the current practice of formulating growth models with an explicit microeconomic foundation for consumption and investment decisions. Both the questions and methods of the new equilibrium approach to growth theory are adapted from optimal growth theory. In this sense the descriptive and normative theories are intertwined and elements of both points of view may be found in each of the three volumes.

The problem of optimally allocating resources is a central theme in economic analysis. Optimal growth theory is the analysis of this question when the consumption-investment decision is the focal point of the decision process. The purpose of this volume is to present some selections of the models and problems treated in the theory of optimal economic growth. The majority of the readings focus on neoclassical theories with an explicit utility criterion ranking alternative consumption paths over an infinite horizon. The other selections deal with finite horizon planning models without an explicit consumption criterion. The terminal stocks realized by the prospective plan are the objects used to rank alternative development programmes.

Two seminal articles, one by Ramsey (1928) and the other by von Neumann (1937), were inspired by the problem of optimal growth. Their impact has exceeded, however, by far the narrow confines of optimal growth theory. Ramsey's paper has influenced development, optimal planning theory, financial economics, monetary theory, macrodynamics, public finance, and resource economics. Von Neumann's paper influenced the development of activity analysis models of production, linear programming, game theory, and general competitive analysis. His treatment of durable capital provides the basis for modern developments in multisector capital theory. Consequently, we have included those articles in this volume in order to provide the framework for understanding the other selected papers. Indeed, most of the papers in this volume are really commentaries on these two classic articles.

The readings are divided into four groups. Part I deals with one-sector models, Part II covers multisector models, Part III reviews the sensitivity, capital deepening properties, and comparative dynamics results for optimal paths, and Part IV is devoted to the classic turnpike theorem. Parts I to III fit primarily the neoclassical tradition derived from Ramsey's work whereas Part IV represents work following in the spirit of von Neumann. The multisector work cast in Ramsey's framework (Part II) also shares many characteristics of the von Neumann model. The readings in these sections are subdivided by whether or not the analysis is conducted in a deterministic or stochastic framework. Of course, much of importance could not be included in this volume due to space limitations. For example, the implications for optimal growth with respect to the alternative efficiency and maximin objectives are not covered by the readings. However, the selections should provide a suitable background for understanding the problems and methods associated with optimal growth by those and other criteria.

The deterministic selections in Part I present various explorations of the Ramsey model. Ramsey raised the question of optimal growth as the problem of determining a savings path that would maximize a utility sum over time. He cast the problem as a continuous time variational problem over an infinite horizon. In discrete time, the discounted version of his model takes the form of solving the optimization problem

$$(\mathcal{P}) \qquad V(k) = \sup \{ U(\mathbf{c}) \equiv \textstyle\sum_{t=1}^{\infty} \delta^{t-1} u(c_t) \colon \mathbf{c} \in \mathcal{A}(k) \}$$

where $\mathcal{A}(k) = \{ \mathbf{c}, \mathbf{k}\colon\ c_t \geq 0, k_t \geq 0, k_0 \leq k, \text{ and } c_t + k_t \leq f(k_{t-1}) \text{ for } t= 1,2,\ldots\}$, $\mathbf{c} = \{c_t\}_{t=1}^{\infty}$, $\mathbf{k} = \{k_t\}_{t=1}^{\infty}$, and k is a given positive number. The interpretation of this problem is that of a central planner choosing nonnegative consumption and capital sequences that realize a maximum discounted utility subject to given technological constraints and a capital endowment. The function U gives the planner's utility from the consumption stream c, u is the one period reward or felicity function, and δ is the discount factor ($0 < \delta < 1$). The technology is given by a one-sector model in which there is a single all-purpose capital-consumption good produced from capital and fixed labour according to the production function f (the labour input is normalized at one unit in each period and suppressed in this notation). The function V gives the optimal value for each starting stock k. The set $\mathcal{A}(k)$ constitutes the set of attainable consumption and capital sequences. The maintained assumption on problem ($\mathcal{P}$) is that u is strictly concave, monotone increasing, twice continuously differentiable, and either $u(0) = -\infty$, or the right hand limit of u' at the origin is infinite, i.e. $u'(0+) = +\infty$. Problem ($\mathcal{P}$) is said to be *classical* whenever f is strictly concave, $f(0) = 0$, f is twice continuously differentiable with $f' > 0$ and $f'(0+) = +\infty$, and $f'(\infty) < 1$. Otherwise problem ($\mathcal{P}$) is said to be *nonclassical*. The objective function U is in *time additive* form.

There are several questions that may be asked about problem ($\mathcal{P}$). Consider the classical model. First, the analyst must establish the existence of an optimum. An elementary existence argument is available for the discrete time case. However, in the classical case with $\delta = 1$, Ramsey's modification of the definition of optimality is required to accomplish this task. Uniqueness of the optimum follows from the strict concavity properties of u and f. The dynamics of the optimal solution may be described as follows: there is a unique stationary capital stock , $k(\delta)$, satisfying $\delta f'(k) = 1$, and consumption level $c(\delta) = f(k(\delta)) - k(\delta)$ such that if $0 < k < k(\delta)$, the optimal capital and consumption sequences are monotonically increasing with limit points $k(\delta)$ and $c(\delta)$ respectively. If $k > k(\delta)$, those paths decrease to the same limit point, and if $k = k(\delta)$, the optimal programme is to remain in that initial state with permanent consumption at level $c(\delta)$.

The underlying parameters for the model ($\mathcal{P}$) are the values of k and δ. A comparative dynamics analysis of ($\mathcal{P}$) is concerned with the implications for optimal consumption-investment behaviour when these parameters undergo an alteration. The monotonicity property implies that for a fixed discount factor that all optimal programmes tend to the steady state programme. Put differently, the long-run dynamics of an optimal accumulation programme do not depend on the value of the initial capital stock. A change in δ affects the steady state as well as the entire path of capital and consumption starting from a fixed initial stock. An increase in δ leads to an increase in steady state capital and consumption. Moreover, steady state utility also rises with δ. This response is the prototype for the general *non-paradoxical property* investigated in multisector theories. This model also enjoys a *dynamic capital deepening property*: an increase in δ implies a larger capital stock at each time before the change as well as eventually larger consumption. The initial capital expansion is paid for by a reduction in consumption over an initial segment of the planning horizon.

The deterministic selections in Part I display various generalizations within the one sector format. The paper by Cass, representative of research in the mid-1960s on the classical Ramsey problem, uses the Pontriagin Maximum Principle to derive the shadow prices supporting the optimum as a decentralized competitive equilibrium. He also describes the continuous time dynamics analogous to the discrete time results summarized above. The contribution of Beals and Koopmans generalizes the classical model by allowing for a broader class of objectives, the so-called *recursive* objective functions. The interest in this case arises because the planner's rate of time preference may depend in a stationary state on the particular level of stationary consumption. Their article describes sufficient conditions for the sequence of optimal capital stocks to be monotonic. Dechert and Nishimura show this monotonicity property is valid in a discounted nonclassical model where the production function does not exhibit diminishing marginal returns to capital at the outset. This specification has potential application to understanding optimal growth with non-convex technologies, as well as to optimal harvesting models of fisheries.

Uncertainty about the future is a fundamental feature of dynamic economic problems. Mirman and Epstein's studies explore modifications of the classical model to permit exogenous technological uncertainty. Mirman surveys the basic results obtained in the classical setup. The focus is placed on the convergence of the distribution of capital stocks to a stationary distribution of capital stocks. Epstein introduces a stochastic version of the recursive objective function that is also consistent with the von Neumann and Morgenstern axioms on expected utility. He then works out the stochastic steady state and convergence theory for the technologically-shocked Ramsey model.

The multisector optimal growth contributions selected for Part II illustrate some of the issues which arise when the planner must make static allocation decisions across sectors as well as intertemporal portfolio decisions based on the availability of many assets as a means of holding wealth. All of the studies in this part assume the planner maximizes a time additive utility function with a fixed positive discount rate. The first two papers, by Benhabib and Nishimura, are formulated in continuous time and the remaining are cast in discrete time. The concavity property of the one sector production function is appropriately generalized in the reprinted multisector studies. The first paper gives sufficient conditions on the production technology and magnitude of the planner's discount rate for a unique steady state to be asymptotically stable for all initial vectors of starting stocks. The second paper shows the same class of models may exhibit a Hopf bifurcation as the planner's discount rate increases. This means that an optimum path may be a nontrivial periodic orbit. Convergence to the stationary state from an arbitrary capital endowment is not, in general, possible. In Volume III, perfect foresight competitive equilibria and optimal programmes will be explicitly linked; the Benhabib and Nishimura bifurcation result implies the existence of perfect foresight equilibrium cycles.

McKenzie examines the stability property of a general multisector model where there are independent industries producing goods according to a constant returns to scale technology. He links the technological structure, the magnitude of the planner's discount rate, as well as the dynamics of the undiscounted model in order to prove asymptotic convergence of an optimal path to the optimal stationary state. Boldrin and Montrucchio's essay shows chaotic trajectories may arise even in a two-sector model generating factor intensity reversals along the optimum provided the planner's discount rate is 'large enough'. Consequently, if the one sector technology is abandoned for a model with two or more sectors, optimal growth paths may exhibit complex dynamics. This potential for complex dynamics in two-sector models also has interest in the investigation of the relationship between chaos and real GNP time series data.

Technological and resource uncertainty in multisector models of accumulation are studied in the paper by Radner as well as the one by Majumdar and Radner. Both essays utilize an activity analysis production sector. Radner's contributions are the demonstration of the existence of a stationary stochastic accumulation programme analogous to the steady state of an undiscounted model and the proof that an optimal programme may be supported by a stochastic process of shadow prices. The Majumdar and Radner analysis offers an extension of standard Markovian dynamic programming to the class of stationary stochastic environments. The authors provide a concrete family of models for which various characteristics on the optimal policy are verifiable given the fundamental parameters specifying the planner's preferences and the available technology. This work has importance in applications to the stochastic convergence problem in the multisector uncertainty framework.

Part III is devoted to a variety of sensitivity and comparative dynamics questions. Brock demonstrates a type of turnpike property for the classical Ramsey one-sector model allowing for general exogenous technical change. In essence, he shows that if the finite planning horizon is sufficiently long, the initial segment of an optimal programme will stay 'near' the infinite horizon optimum starting from the same initial stocks. This result is obtained independently of the assignment of the terminal stocks.

The implementation of Samuelson's Correspondence Principle in comparative dynamics is the subject of the next two selections. Burmeister and Long study the steady state variations in consumption due to a change in the planner's discount rate by examining the linear approximation dynamics of the optimum in a neighbourhood of the original stationary optimal programme. They show for some multisector economies that an efficient steady state is 'paradoxical' if and only if more than half of the characteristic roots of the linear approximation system have positive real parts. The Hahn instability problem is also linked to the existence of paradoxical steady states. Becker's paper explores the possibility of dynamic capital deepening in a discrete time aggregate model of capital accumulation. The model includes the familiar Ramsey story as a special case. He points out that the dynamic capital deepening property holds provided the optimal programme has the monotonicity property. Moreover, if an optimal cycle of period two exists, then dynamic capital deepening may not hold in the general aggregate model.

In the technologically-shocked classical Ramsey model, Danthine and Donaldson indicate how summary statistics such as the mean may be used to report how the model economy evolves for any realization of the underlying stochastic process in response to a parameter shift. For example, if the discount parameter rises, they show a stochastic analogue of the non-paradoxical property holds for mean consumption and capital. They also prove that the initial impact of this parameter change is to provide more capital on average. In the same vein, Majumdar and Zilcha present stochastic generalizations of Brock's sensitivity results. Their version of the stochastic turnpike theorem states that for any two initial stocks, almost all sample paths of capital accumulation do not deviate from each other by an arbitrarily given amount for more than a fixed number of periods which is independent of the particular realization of the production shocks. Dutta demonstrates a form of dynamic capital deepening for a general class of technologically-shocked classical Ramsey models. He also argues that there is a precise sense in which lengthening the plan horizon for a fixed discount rate or decreasing the discount rate for a fixed (infinite) horizon are equivalent notions of decreasing impatience on the part of the planner.

In his 1937 study von Neumann introduced a disaggregated model of capital accumulation with choice of technique. Alternative processes or activities could be used and each activity could utilize many inputs as well as produce many outputs. For example, joint production could arise due to capital depreciation: an activity might use a particular machine to produce some output commodity as well as a depreciated version of the machine. Labour inputs were treated as intermediate processes tied to the consumption activities and consumption was modelled as a necessary input to produce labour within the system. Von Neumann assumed that primary factors were not constraints on production. He showed the existence of a type of balanced growth equilibrium where all stocks proportionately expanded at a maximal rate. He also demonstrated that this equilibrium could be supported by prices in the sense that activities actually used earned zero profits and no activity earned a positive profit. Moreover, the support prices had an implicit discount rate given by the maximum rate of expansion. The von Neumann equilibrium programme is now called the 'turnpike' since it represents the configuration of stocks with the largest rate of steady growth.

In 1958, Dorfman, Samuelson, and Solow (DOSSO) published the first turnpike theorem. This result described the optimal accumulation programme for a finite horizon planning model with arbitrary starting stocks. Given a prespecified mix of final stocks, the planner's objective was to maximize the size of the terminal stocks at the end of the period of accumulation in the desired proportions. They showed that for a sufficiently long planning horizon, an optimal programme of accumulation would spend most of the time in an arbitrary neighbourhood of the von Neumann equilibrium. An optimal accumulation programme in the DOSSO sense is approximately a balanced growth equilibrium for most of the accumulation period. Hence, the von Neumann equilibrium has an important role in the normative analysis of accumulation plans.

McKenzie's study of maximal programmes in the von Neumann model includes general versions of the turnpike theorem. He uses the 'value loss' technique first introduced by Radner to demonstrate the turnpike theorem. Subsequent refinements of the value loss method have proven useful in studying the turnpike property in models with explicit consumption oriented utility maximization (for example, see McKenzie's paper in Part II). The Tsukui contribution is to show that the von Neumann equilibrium and turnpike property may be explored in an empirical setting and thereby be used in development planning models. He employs a computable dynamic Leontief version of the von Neumann model. Using data for the Japanese economy from 1955–1960, he argues that the computed optimal path moves towards the turnpike even with a short planning horizon. He also indicates that the theoretical optimal programme and the actual path followed during the period in question are 'close' (given the limitations of the data). In this sense, Tsukui's work shows that the turnpike may be relevant to actual planning.

The papers reprinted in Volume II provide the foundation for studying optimal growth theory. Our selections reflect our judgements as to what constitutes a sound foundation. We believe that the subjects treated here are essential for a proper understanding of the equilibrium models dealt with in Volume III as well as the 'new growth theory' associated with Lucas and Romer illustrated in Volume I. The optimal growth models also provide a normative view of Solow's fundamental growth theory.

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