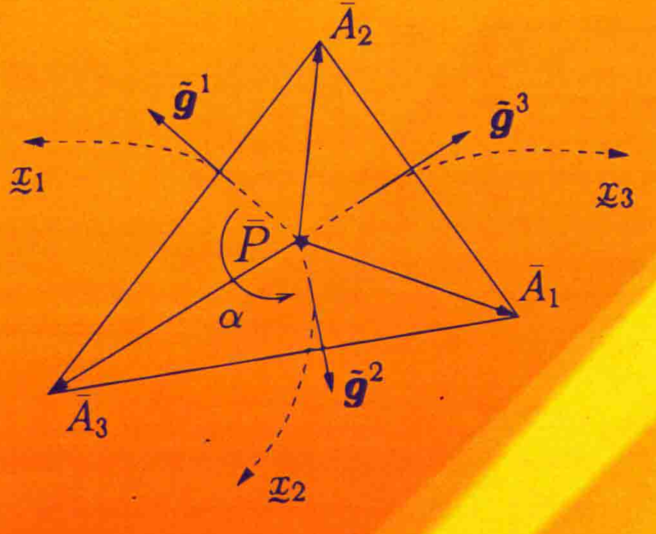
Advanced Mechanics of Continua



Karan S. Surana

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To

My beloved family

Abha, Deepak, Rishi and Yogini

PREFACE

Continuum mechanics is the study of the evolution of deformation of continuous matter, called a *continuum*, in which the behaviors of the matter at molecular, atomic or subatomic scales are neglected. In this study, we only consider phenomena in which the evolution of the continuous matter always remains in thermodynamic equilibrium. This allows us to utilize conservation laws, balance principles, thermodynamic principles and relations in deriving mathematical descriptions of the evolving state of deforming continua. Continuum mechanics provides the mathematical foundation and general mathematical framework for developing mathematical descriptions of the deformation of continuous matter. This mathematical framework then can be specialized to, for example, the study of solids in the theory of elasticity and the study of fluids in fluid mechanics and gas dynamics. Treatment of conservation and balance laws and thermodynamic principles for solids and fluids, and the derivations of constitutive theories for solids and fluids provide essential core knowledge for further studies in these areas in more specialized courses. For students of engineering and mathematical physics, the study of continuum mechanics provides essential core knowledge in developing sound mathematical and theoretical foundations for further study and research in theoretical as well as applied sciences.

There are many books on this subject; some present the subject at the highest level of abstraction and then there are those in which only the most elementary aspects of the subject are considered. In most thorough writings on this subject, complexities of notations and compactness of presentations leave even the most competent graduate student entangled and confused. The purpose of this book is to provide comprehensive and complete treatment of the subject of continuum mechanics using simplicity of notations, detailed and consistent derivations with clarity to the point that even self study of the subject may be possible. The notations used in this book in many instances are different than those used currently but are selected to be natural for the information they represent. For example, we denote Jacobian of deformation by $\boldsymbol{J}$, stretch tensor by $\boldsymbol{S}$, right and left stretch tensors by $\boldsymbol{S}_r$ and $\boldsymbol{S}_l$ and so on.

Chapter 1 provides a brief introduction of the subject. Einstein's notation, index notations, matrix and vector notations, basic definitions and concepts, mathematical preliminaries, tensor calculus and transformations using co- and contra-variant bases and differential calculus are presented in Chapter 2. Since matrix and vector notations are by far most easily understood by the students in engineering and physics, emphasis is placed on their use and deliberate attempts are made whenever possible to present material in subsequent chapters using this notation.

Chapter 3 contains kinematics of motion and measures of deformation in Lagrangian as well as Eulerian descriptions. Strain measures for finite deformation are classified as covariant and contravariant measures, and they are derived using Lagrangian as well as Eulerian descriptions. Covariant measures of strains, namely, Cauchy strain, Finger strain and Green's strain, are derived in the Lagrangian as well as the Eulerian descriptions. Contravariant measures of Cauchy, Finger and Almansi strains are also derived using the Eulerian and Lagrangian descriptions. This is a significant departure from conventional writings in which strain measures are only classified as either Lagrangian or Eulerian. Tensorial nature of strain measures and influence of rotation of frames on various measures are established. Physical meaning of the components of strains is illustrated. Polar decomposition of deformation including theorems and their proofs is also presented. Strain measures are also derived using the left and right stretch tensors. Invariants of strain tensors and relationships between deformed and undeformed areas and volumes are also derived.

Chapter 4 contains definitions and measures of stress. Using infinitesimal deformation, the stress concept is introduced, the Cauchy principle is derived, and the tensorial nature of the stress matrix is established. The concept of contravariant stress components is introduced for finite deformation from which contravariant Cauchy stress tensor is derived in the Lagrangian and Eulerian descriptions. Covariant Cauchy stress tensors in Lagrangian and Eulerian descriptions are derived in a similar fashion using covariant stress components and contravariant basis. Jaumann stress tensor and its relationship to contravariant and covariant Cauchy stress tensors is presented. First and second Piola-Kirchhoff stress tensors in Lagrangian and Eulerian descriptions are derived using contravariant and covariant Cauchy stress tensors and their importance in deriving convected time derivatives of the covariant and contravariant stress tensors and in balance laws and constitutive theories is discussed. Chapter 5 contains derivations of the rate of deformation, area, volume, strain rate tensors, spin tensor and convected time derivatives of stress tensors (for compressible and incompressible matter) and strain tensors.

Chapter 6 contains derivations of mathematical models in the Eulerian description based on conservation and balance laws. Continuity, momentum and energy equations and entropy inequality are derived using conservation of mass, balance of linear momenta and first and second laws of thermodynamics in the Eulerian description. Such descriptions are useful in fluid mechanics.

Chapter 7 presents derivations of mathematical descriptions for compressible and incompressible matter in the Lagrangian description using conservation and balance laws. Such mathematical models are ideally suited for solid matter in which rate of work results in rate of entropy generation. Separate derivations of the first and second laws of thermodynamics for thermoelastic solids in which rate of work does not result in rate of increase of entropy are also presented. In the derivations of mathematical descriptions presented in this chapter, material point displacements are intrinsic and are monitored. In Chapter 8, axioms and principles of constitutive theory are considered. Various approaches of deriving constitutive theories using entropy inequality and theory of generators and invariants are discussed. Approaches of determining dependent variables in constitutive theories and their arguments are presented for solid matter as well as for fluids. Determination of material coefficients is also considered.

Constitutive theories for stress tensor and heat vector for compressible and incompressible thermoelastic solids using strain energy density function and complementary strain energy density function are presented in Chapter 9. Chapters 10 and 11 consider ordered rate constitutive theories for thermoviscoelastic solids without and with memory. It is shown that all such constitutive theories are ordered rate constitutive theories. Simplifications of these theories for infinitesimal deformation are also considered. Material coefficients are derived for the general case of finite deformation as well as infinitesimal deformation. One-dimensional simple degenerated cases of these theories are considered and compared with those derived based on phenomenological approach.

Chapters 12 and 13 contain derivations of the constitutive theories for homogeneous and isotropic compressible as well as incompressible thermoviscous fluids and thermoviscoelastic fluids (polymers). The constitutive theories are derived using covariant, contravariant and Jaumann measures of stresses, their convected time derivatives and convected time derivatives of Green's strain tensor and Almansi strain tensor. These constitutive theories are also ordered rate theories. Using simplified forms of the general rate theories, rate theories for Newtonian fluids, generalized Newtonian fluids, Maxwell fluids, Oldroyd-B fluids and Giesekus fluids are derived. Derivations of material coefficients are presented for all cases. Chapter 14 contains derivations of the constitutive theories for homogeneous and isotropic compressible as well as incompressible hypo-elastic solids.

Chapter 15 considers various thermodynamic relations and brings together the mathematical models derived in Chapters 6 and 7 and the constitutive theories in Chapters 9–14 to present complete mathematical models that have closure and can be readily used in applications. Principle of virtual work, Hamilton's principle, the derivations of Euler-Lagrange equations, momentum equations and the equations of equilibrium based on the principle of virtual work are considered in Chapter 16. Appendix A provides a list of generators and invariants for various combinations of argument tensors.

Appendix B is helpful in transforming information and mathematical models from Cartesian to cylindrical or spherical coordinate systems.

The material presented in this book is intended for two three-credit hour courses in continuum mechanics. The material in Chapters 1 to 7 is recommended for the first course and the remaining chapters for the follow-up course. The author has successfully used this material in this format for the last six years in the Mechanical Engineering Department at the University of Kansas.

The author's long friendship and collaboration with Professor J. N. Reddy (Texas A&M University) has been extremely enjoyable and fruitful in bringing focus, depth and clarity to the material presented in this book. Many long discussions on the phone and in person on various topics with Professor Reddy, especially in the areas of constitutive theories and the joint research grants from U.S. Army (ARO) resulted in a significant number of joint fundamental publications, which have helped the author in developing and presenting this material in the book with more depth and clarity and yet maintaining simplicity. The author is truly grateful to many of his graduate students: Daniel Nunez, Tristan Moody, Yusshy Mendoza, Aaron Joy and Michael Powell, whose Ph.D. and M.S. theses in various areas of continuum mechanics have helped me immensely in bringing the subject matter in this book to the present level of maturity. My very special thanks to Dr. Daniel Nunez, who was the author's first Ph.D. student to engage in theoretical and continuum mechanics research. His interest in the subject, hard work, many discussions, suggestions, and above all, typing and retyping many times of the entire manuscript single handedly, has helped me immensely in bringing this book to completion. This book would not have been possible without the research grants: W911NF-09-1-0548 (FED0065623), W-911NF-11-1-0471 (FED0061541) and W911NF1210463 from the U.S. Army Research Office (ARO) to the author at the University of Kansas and to Professor J. N. Reddy at Texas A&M University that lead to research in various areas of continuum mechanics, especially constitutive theories. My sincere thanks to Dr. Joseph Myers, Division Chief, Mathematical Sciences Division, Information Science Directorate, ARO, for his interest and support of the research results contained in this book.

This book contains many involved equations, derivations and mathematical details and it is hardly possible to avoid some typographical and other errors. The Author would be grateful to those readers who are willing to draw attention to the errors using the email: kssurana@ku.edu

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ABOUT THE AUTHOR

Karan S. Surana, born in India, went to undergraduate school at Birla Institute of Technology and Science (BITS), Pilani, India and received a B.E. degree in Mechanical Engineering in 1965. He then attended the University of Wisconsin, Madison where he obtained M.S. and Ph.D. in Mechanical Engineering in 1967 and 1970. He worked in industry in research and development in various areas of computational mechanics and software development for fifteen years: SDRC, Cincinnati (1970-1973), EMRC, Detroit (1973-1978) and McDonnell Douglas, St. Louis (1978-1984). In 1984, he joined The University of Kansas, Department of Mechanical Engineering faculty where he is currently serving as Deane E. Ackers University Distinguished professor of Mechanical Engineering.

His areas of interest and expertise are computational mathematics, computational mechanics and continuum mechanics. He is author of over 350 research reports, conference papers and journal papers. He has served as advisor and chairman of 50 M.S. students and 22 Ph.D. students at the University of Kansas in the department of Mechanical Engineering in various areas of Computational Mathematics and Continuum Mechanics. He has delivered many plenary and keynote lectures in various national and international conferences and congresses on computational mathematics, computational mechanics and continuum mechanics. He has served on international advisory committees of many conferences and has co-organized mini-symposia on k-version of finite element method, computational methods and constitutive theories in UNCCM. He is a member of IACM, USACM and fellow and life member of ASME.

His most notable contributions include: large deformation finite element formulations of shells, k-version of finite element method, operator classification and variationally consistent integral forms in methods of approximations and ordered rate constitutive theories for solid and fluent continua. His most recent and present research work is in Polar Continuum Theories for solid and fluent continua and ordered rate constitutive theories for polar continua.

Contents