

C. James Taylor, Peter C. Young
and Arun Chotai



T RUE
D I G I T A L
C O N T R O L

Statistical Modelling and Non-Minimal State Space Design

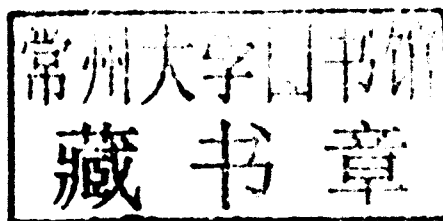
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TRUE DIGITAL CONTROL

STATISTICAL MODELLING AND NON-MINIMAL STATE SPACE DESIGN

C. James Taylor, Peter C. Young and Arun Chotai

Lancaster University, UK



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TRUE DIGITAL CONTROL

To Ting-Li

To Wendy

In memory of Varsha

Preface

This book develops a *True Digital Control* (TDC) design philosophy that encompasses data-based (statistical) model identification, through to control algorithm design, robustness evaluation and implementation. Treatment of both stochastic system identification and control design under one cover highlights the important connections between these disciplines: for example, in quantifying the model uncertainty for use in closed-loop stochastic sensitivity analysis. More generally, the foundations of linear state space control theory that are laid down in early chapters, with *Non-Minimal State Space* (NMSS) design as the central worked example, are utilised subsequently to provide an introduction to other selected topics in modern control theory. MATLAB[®]¹ functions for TDC design and MATLAB[®] scripts for selected examples are being made available online, which is important in making the book accessible to readers from a range of academic backgrounds. Also, the CAPTAIN Toolbox for MATLAB[®], which is used for the analysis of all the modelling examples in this book, is available for free download. Together, these contain computational routines for many aspects of model identification and estimation; for NMSS design based on these estimated models; and for offline signal processing. For more information visit: <http://www.wiley.com/go/taylor>.

The book and associated software are intended for students, researchers and engineers who would like to advance their knowledge of control theory and practice into the state space domain; and control experts who are interested to learn more about the NMSS approach promoted by the authors. Indeed, such non-minimal state feedback is utilised throughout this book as a unifying framework for generalised digital control system design. This includes the *Proportional-Integral-Plus* (PIP) control systems that are the most natural outcome of the NMSS design strategy. As such, the book can also be considered as a primer for potentially difficult topics in control, such as optimal, stochastic and multivariable control.

As indicated by the many articles on TDC that are cited in this book, numerous colleagues and collaborators have contributed to the development of the methods outlined. We would like to pay particular thanks to our good friend Dr Wlodek Tych of the Lancaster Environment Centre, Lancaster University, UK, who has contributed to much of the underlying research and in the development of the associated computer algorithms. The first author would also like to thank Philip Leigh, Matthew Stables, Essam Shaban, Vasileios Exadaktylos, Eleni Sidiropoulou, Kester Gunn, Philip Cross and David Robertson for their work on some of the practical examples highlighted in this book, among other contributions and useful discussions while they studied at Lancaster. Philip Leigh designed and constructed the Lancaster forced

¹ MATLAB[®], The MathWorks Inc., Natick, MA, USA.

ventilation test chamber alluded to in the text. Vasileios Exadaktylos made insightful suggestions and corrections in relation to early draft chapters of the book. The second author is grateful to a number of colleagues over many years including: Charles Yancey and Larry Levens, who worked with him on early research into NMSS control between 1968 and 1970; Jan Willems who helped with initial theoretical studies on NMSS control in the early 1970s; and Tony Jakeman who helped to develop the *Refined Instrumental Variable* (RIV) methods of model identification and estimation in the late 1970s. We are also grateful to the various research students at Lancaster who worked on PIP methods during the 1980s and 1990s, including M.A. Behzadi, Changli Wang, Matthew Lees, Laura Price, Roger Dixon, Paul McKenna and Andrew McCabe; to Zaid Chalabi, Bernard Bailey and Bill Day, who helped to investigate the initial PIP controllers for the control of climate in agricultural glasshouses at the Silsoe Research Institute; and to Daniel Berckmans and his colleagues at the University of Leuven, who collaborated so much in later research on the PIP regulation of fans for the control of temperature and humidity in their large experimental chambers at Leuven.

Finally, we would like to express our sincere gratitude to the UK Engineering and Physical Sciences, Biotechnology and Biological Sciences, and Natural Environmental Research Councils for their considerable financial support for our research and development studies at Lancaster University.

C. James Taylor, Peter C. Young and Arun Chotai
Lancaster, UK

List of Acronyms

ACF	AutoCorrelation Function
AIC	Akaike Information Criterion
AML	Approximate Maximum Likelihood
AR	Auto-Regressive
ARIMAX	Auto-Regressive Integrated Moving-Average eXogenous variables
ARMA	Auto-Regressive Moving-Average
ARMAX	Auto-Regressive Moving-Average eXogenous variables
ARX	Auto-Regressive eXogenous variables
BIC	Bayesian Information Criterion
BJ	Box–Jenkins
CAPTAIN	Computer-Aided Program for Time series Analysis and Identification of Noisy systems
CLTF	Closed-Loop Transfer Function
CT	Continuous-Time
DARX	Dynamic Auto-Regressive eXogenous variables
DBM	Data-Based Mechanistic
DC	Direct Current
DDC	Direct Digital Control
DF	Directional Forgetting
DT	Discrete-Time
DTF	Dynamic Transfer Function
EKF	Extended or generalised Kalman Filter
EWP	Exponential-Weighting-into-the-Past
FACE	Free-Air Carbon dioxide Enrichment
FIR	Finite Impulse Response
FIS	Fixed Interval Smoothing
FPE	Final Prediction Error
GBJ	Generalised Box–Jenkins
GPC	Generalised Predictive Control
GRIVBJ	Generalised RIVBJ or RIVCBJ
GRW	Generalised Random Walk
GSRIV	Generalised SRIV or SRIVC
IPM	Instrumental Product Matrix
IRW	Integrated Random Walk

IV	Instrumental Variable
IVARMA	Instrumental Variable Auto-Regressive Moving-Average
KF	Kalman Filter
LEQG	Linear Exponential-of-Quadratic Gaussian
LLS	Linear Least Squares
LLT	Local Linear Trend
LPV	Linear Parameter Varying
LQ	Linear Quadratic
LQG	Linear Quadratic Gaussian
LTR	Loop Transfer Recovery
MCS	Monte Carlo Simulation
MFD	Matrix Fraction Description
MIMO	Multi-Input, Multi-Output
MISO	Multi-Input, Single-Output
ML	Maximum Likelihood
MPC	Model Predictive Control
NEVN	Normalised Error Variance Norm
NLPV	Non-Linear Parameter Varying
NMSS	Non-Minimal State Space
NSR	Noise-Signal Ratio
NVR	Noise Variance Ratio
PACF	Partial AutoCorrelation Function
PBH	Popov, Belevitch and Hautus
PEM	Prediction Error Minimisation
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PIP	Proportional-Integral-Plus
PRBS	Pseudo Random Binary Signal
RBF	Radial Basis Function
RIV	Refined Instrumental Variable
RIVAR	Refined Instrumental Variable with Auto-Regressive noise
RIVBJ	Refined Instrumental Variable for Box-Jenkins models
RIVCBJ	Refined Instrumental Variable for hybrid Continuous-time Box-Jenkins models
RLS	Recursive Least Squares
RML	Recursive Maximum Likelihood
RW	Random Walk
RWP	Rectangular-Weighting-into-the-Past
SD	Standard Deviation
SDARX	State-Dependent Auto-Regressive eXogenous variables
SDP	State-Dependent Parameter
SE	Standard Error
SISO	Single-Input, Single-Output
SP	Smith Predictor
SRIV	Simplified Refined Instrumental Variable
SRIVC	Simplified Refined Instrumental Variable for hybrid Continuous-time models
SRW	Smoothed Random Walk

SVF	State Variable Feedback
TDC	True Digital Control
TF	Transfer Function
TFM	Transfer Function Matrix
TVP	Time Variable Parameter
YIC	Young Information Criterion

List of Examples, Theorems and Estimation Algorithms

Examples

2.1	Transfer Function Representation of a First Order System	19
2.2	Transfer Function Representation of a Third Order System	21
2.3	Poles, Zeros and Stability	25
2.4	Proportional Control of a First Order TF Model	28
2.5	Integral Control of a First Order TF Model	30
2.6	Proportional-Integral Control of a First Order TF Model	31
2.7	Pole Assignment Design Based on PI Control Structure	33
2.8	Limitation of PI Control Structure	35
2.9	Continuous- and Discrete-Time Rainfall-Flow Models	36
3.1	State Space Forms for a Third Order TF Model	42
3.2	State Variable Feedback based on the Controllable Canonical Form	46
3.3	State Variable Feedback Pole Assignment based on the Controllable Canonical Form	48
3.4	State Variable Feedback based on the Observable Canonical Form	51
3.5	Determining the TF from a State Space Model	54
3.6	Eigenvalues and Eigenvectors of a State Space Model	56
3.7	Determining the Diagonal Form of a State Space Model	58
3.8	Rank Tests for a State Space Model	60
4.1	Non-Minimal State Space Representation of a Second Order TF Model	67
4.2	Ranks Test for the NMSS Model	69
4.3	Regulator Control Law for a NMSS Model with Four State Variables	70
4.4	Pole Assignment for the Fourth Order NMSS Regulator	72
4.5	Unity Gain NMSS Regulator for the Wind Turbine Simulation	73
4.6	Mismatch and Disturbances for the Fourth Order NMSS Regulator	75
4.7	Transformations between Minimal and Non-Minimal	80
4.8	The Order of the Closed-loop Characteristic Polynomial	81
4.9	Numerical Comparison between NMSS and Minimal SVF Controllers	83
4.10	Model Mismatch and its effect on Robustness	84
5.1	Proportional-Integral-Plus Control of a First Order TF Model	90
5.2	Implementation Results for Laboratory Excavator Bucket Position	91

5.3	Non-Minimal State Space Servomechanism Representation of a Second Order TF Model	96
5.4	Rank Test for the NMSS Model	97
5.5	Proportional-Integral-Plus Control System Design for NMSS Model with Five State Variables	100
5.6	Pole Assignment Design for the NMSS Model with Five State Variables	106
5.7	Implementation Results for FACE system with Disturbances	108
5.8	PIP-LQ Design for the NMSS Model with Five State Variables	114
5.9	PIP-LQ Control of CO ₂ in Carbon-12 Tracer Experiments	117
6.1	Simulation Response for Feedback and Forward Path PIP Control	128
6.2	Simulation Experiment with Integral 'Wind-Up' Problems	133
6.3	Incremental Form for Carbon-12 Tracer Experiments	134
6.4	SP-PIP Control of Carbon-12 Tracer Experiments	140
6.5	SP-PIP Control of Non-Minimum Phase Oscillator	141
6.6	Kalman Filter Design for Noise Attenuation	147
6.7	Command Input Anticipation Design Example	156
6.8	Generalised Predictive Control and Command Anticipation PIP Control System Design	161
7.1	Multivariable TF Representation of a Two-Input, Two-Output System	168
7.2	Multivariable PIP-LQ control of a Two-Input, Two-Output System	178
7.3	Multivariable PIP-LQ control of an Unstable System	179
7.4	Multivariable PIP-LQ Control of a Coupled Drive System	183
7.5	PIP-LQ control of the Shell Heavy Oil Fractionator Simulation	188
7.6	Pole Assignment Decoupling of a Two-Input, Two-Output System	194
8.1	Estimation of a Simple ARX Model	206
8.2	Estimation of a Simple TF Model	208
8.3	Estimation of a Simple FIR Model	211
8.4	Poles and Zeros of the Estimated ARX [3 3 1] Model	211
8.5	SRIV Estimation of a Simple TF model	226
8.6	A Full RIVBJ Example	228
8.7	A More Difficult Example (Young 2008)	229
8.8	Hair-Dryer Experimental Data	235
8.9	Axial Fan Ventilation Rate	240
8.10	Laboratory Excavator Bucket Position	241
8.11	Multivariable System with a Common Denominator	244
8.12	Multivariable System with Different Denominators	246
8.13	Continuous-Time Estimation of Hair-Dryer Experimental Data	251
8.14	Control of CO ₂ in Carbon-12 Tracer Experiments	257
9.1	Proportional-Integral-Plus Design for a Non-Minimum Phase Double Integrator System	273
9.2	Simulation Experiments for Non-Minimum Phase Double Integrator	275
9.3	Estimation of a Simulated DARX Model	287
9.4	State-Dependent Parameter Representation of the Logistic Growth Equation	291
9.5	SDP-PIP Control of the Logistic Growth Equation	295

Theorems

4.1	Controllability of the NMSS Representation	69
4.2	Transformation from Non-Minimal to Minimal State Vector	77
5.1	Controllability of the NMSS Servomechanism Model	96
5.2	Pole Assignability of the PIP Controller	105
6.1	Relationship between PIP and SP-PIP Control Gains	139
6.2	Equivalence Between GPC and (Constrained) PIP-LQ	160
7.1	Controllability of the Multivariable NMSS Model	174
9.1	Controllability of the δ -operator NMSS Model	269
	The Theorem of D.A. Pierce (1972)	327

Estimation Algorithms

Ie	<i>en bloc</i> Least Squares	203
I	Recursive Least Squares (RLS)	205
Iie	<i>en bloc</i> Instrumental Variables (IV)	216
II	Recursive IV	217
IIIe	<i>en bloc</i> Refined Instrumental Variables (RIV)	223
III	Recursive RIV	223
IIIs	Symmetric RIV	225

Contents

Preface	xiii
List of Acronyms	xv
List of Examples, Theorems and Estimation Algorithms	xix
1 Introduction	1
1.1 Control Engineering and Control Theory	2
1.2 Classical and Modern Control	5
1.3 The Evolution of the NMSS Model Form	8
1.4 True Digital Control	11
1.5 Book Outline	12
1.6 Concluding Remarks	13
References	14
2 Discrete-Time Transfer Functions	17
2.1 Discrete-Time TF Models	18
2.1.1 <i>The Backward Shift Operator</i>	18
2.1.2 <i>General Discrete-Time TF Model</i>	22
2.1.3 <i>Steady-State Gain</i>	23
2.2 Stability and the Unit Circle	24
2.3 Block Diagram Analysis	26
2.4 Discrete-Time Control	28
2.5 Continuous to Discrete-Time TF Model Conversion	36
2.6 Concluding Remarks	38
References	38
3 Minimal State Variable Feedback	41
3.1 Controllable Canonical Form	44
3.1.1 <i>State Variable Feedback for the General TF Model</i>	49
3.2 Observable Canonical Form	50
3.3 General State Space Form	53
3.3.1 <i>Transfer Function Form of a State Space Model</i>	53
3.3.2 <i>The Characteristic Equation, Eigenvalues and Eigenvectors</i>	55
3.3.3 <i>The Diagonal Form of a State Space Model</i>	57

3.4	Controllability and Observability	58
3.4.1	<i>Definition of Controllability (or Reachability)</i>	58
3.4.2	<i>Rank Test for Controllability</i>	59
3.4.3	<i>Definition of Observability</i>	59
3.4.4	<i>Rank Test for Observability</i>	59
3.5	Concluding Remarks	61
	References	62
4	Non-Minimal State Variable Feedback	63
4.1	The NMSS Form	64
4.1.1	<i>The NMSS (Regulator) Representation</i>	64
4.1.2	<i>The Characteristic Polynomial of the NMSS Model</i>	67
4.2	Controllability of the NMSS Model	68
4.3	The Unity Gain NMSS Regulator	69
4.3.1	<i>The General Unity Gain NMSS Regulator</i>	74
4.4	Constrained NMSS Control and Transformations	77
4.4.1	<i>Non-Minimal State Space Design Constrained to yield a Minimal SVF Controller</i>	79
4.5	Worked Example with Model Mismatch	81
4.6	Concluding Remarks	85
	References	86
5	True Digital Control for Univariate Systems	89
5.1	The NMSS Servomechanism Representation	93
5.1.1	<i>Characteristic Polynomial of the NMSS Servomechanism Model</i>	95
5.2	Proportional-Integral-Plus Control	98
5.2.1	<i>The Closed-Loop Transfer Function</i>	99
5.3	Pole Assignment for PIP Control	101
5.3.1	<i>State Space Derivation</i>	101
5.4	Optimal Design for PIP Control	110
5.4.1	<i>Linear Quadratic Weighting Matrices</i>	111
5.4.2	<i>The LQ Closed-loop System and Solution of the Riccati Equation</i>	112
5.4.3	<i>Recursive Solution of the Discrete-Time Matrix Riccati Equation</i>	114
5.5	Case Studies	116
5.6	Concluding Remarks	119
	References	120
6	Control Structures and Interpretations	123
6.1	Feedback and Forward Path PIP Control Structures	123
6.1.1	<i>Proportional-Integral-Plus Control in Forward Path Form</i>	125
6.1.2	<i>Closed-loop TF for Forward Path PIP Control</i>	126
6.1.3	<i>Closed-loop Behaviour and Robustness</i>	127
6.2	Incremental Forms for Practical Implementation	131
6.2.1	<i>Incremental Form for Feedback PIP Control</i>	131
6.2.2	<i>Incremental Form for Forward Path PIP Control</i>	134

6.3	The Smith Predictor and its Relationship with PIP Design	137
6.3.1	<i>Relationship between PIP and SP-PIP Control Gains</i>	139
6.3.2	<i>Complete Equivalence of the SP-PIP and Forward Path PIP Controllers</i>	140
6.4	Stochastic Optimal PIP Design	142
6.4.1	<i>Stochastic NMSS Equations and the Kalman Filter</i>	142
6.4.2	<i>Polynomial Implementation of the Kalman Filter</i>	144
6.4.3	<i>Stochastic Closed-loop System</i>	149
6.4.4	<i>Other Stochastic Control Structures</i>	150
6.4.5	<i>Modified Kalman Filter for Non-Stationary Disturbances</i>	151
6.4.6	<i>Stochastic PIP Control using a Risk Sensitive Criterion</i>	152
6.5	Generalised NMSS Design	153
6.5.1	<i>Feed-forward PIP Control based on an Extended Servomechanism NMSS Model</i>	153
6.5.2	<i>Command Anticipation based on an Extended Servomechanism NMSS Model</i>	154
6.6	Model Predictive Control	157
6.6.1	<i>Model Predictive Control based on NMSS Models</i>	158
6.6.2	<i>Generalised Predictive Control</i>	158
6.6.3	<i>Equivalence Between GPC and PIP Control</i>	159
6.6.4	<i>Observer Filters</i>	162
6.7	Concluding Remarks	163
	References	164
7	True Digital Control for Multivariable Systems	167
7.1	The Multivariable NMSS (Servomechanism) Representation	168
7.1.1	<i>The General Multivariable System Description</i>	170
7.1.2	<i>Multivariable NMSS Form</i>	171
7.1.3	<i>The Characteristic Polynomial of the Multivariable NMSS Model</i>	173
7.2	Multivariable PIP Control	175
7.3	Optimal Design for Multivariable PIP Control	177
7.4	Multi-Objective Optimisation for PIP Control	186
7.4.1	<i>Goal Attainment</i>	187
7.5	Proportional-Integral-Plus Decoupling Control by Algebraic Pole Assignment	192
7.5.1	<i>Decoupling Algorithm I</i>	193
7.5.2	<i>Implementation Form</i>	194
7.5.3	<i>Decoupling Algorithm II</i>	195
7.6	Concluding Remarks	195
	References	196
8	Data-Based Identification and Estimation of Transfer Function Models	199
8.1	Linear Least Squares, ARX and Finite Impulse Response Models	200
8.1.1	<i>En bloc LLS Estimation</i>	202
8.1.2	<i>Recursive LLS Estimation</i>	203

8.1.3	<i>Statistical Properties of the RLS Algorithm</i>	205
8.1.4	<i>The FIR Model</i>	210
8.2	General TF Models	211
8.2.1	<i>The Box–Jenkins and ARMAX Models</i>	212
8.2.2	<i>A Brief Review of TF Estimation Algorithms</i>	213
8.2.3	<i>Standard IV Estimation</i>	215
8.3	Optimal RIV Estimation	218
8.3.1	<i>Initial Motivation for RIV Estimation</i>	218
8.3.2	<i>The RIV Algorithm in the Context of ML</i>	220
8.3.3	<i>Simple AR Noise Model Estimation</i>	222
8.3.4	<i>RIVAR Estimation: RIV with Simple AR Noise Model Estimation</i>	223
8.3.5	<i>Additional RIV Algorithms</i>	226
8.3.6	<i>RIVAR and IV4 Estimation Algorithms</i>	227
8.4	Model Structure Identification and Statistical Diagnosis	231
8.4.1	<i>Identification Criteria</i>	232
8.4.2	<i>Model Structure Identification Procedure</i>	234
8.5	Multivariable Models	243
8.5.1	<i>The Common Denominator Polynomial MISO Model</i>	243
8.5.2	<i>The MISO Model with Different Denominator Polynomials</i>	246
8.6	Continuous-Time Models	248
8.6.1	<i>The SRIV and RIVBJ Algorithms for Continuous-Time Models</i>	249
8.6.2	<i>Estimation of δ-Operator Models</i>	253
8.7	Identification and Estimation in the Closed-Loop	253
8.7.1	<i>The Generalised Box–Jenkins Model in a Closed-Loop Context</i>	254
8.7.2	<i>Two-Stage Closed-Loop Estimation</i>	255
8.7.3	<i>Three-Stage Closed-Loop Estimation</i>	256
8.7.4	<i>Unstable Systems</i>	260
8.8	Concluding Remarks	260
	References	261
9	Additional Topics	265
9.1	The δ -Operator Model and PIP Control	266
9.1.1	<i>The δ-operator NMSS Representation</i>	267
9.1.2	<i>Characteristic Polynomial and Controllability</i>	268
9.1.3	<i>The δ-Operator PIP Control Law</i>	269
9.1.4	<i>Implementation Structures for δ-Operator PIP Control</i>	270
9.1.5	<i>Pole Assignment δ-Operator PIP Design</i>	271
9.1.6	<i>Linear Quadratic Optimal δ-Operator PIP Design</i>	272
9.2	Time Variable Parameter Estimation	279
9.2.1	<i>Simple Limited Memory Algorithms</i>	281
9.2.2	<i>Modelling the Parameter Variations</i>	282
9.2.3	<i>State Space Model for DTF Estimation</i>	284
9.2.4	<i>Optimisation of the Hyper-parameters</i>	287