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Jacqueline Stedall

Mathematics Emerging

A Sourcebook 1540–1900

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For Tom, who loves mathematics,

and

for Ellie, who loves old books.

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Apart from the exceptions mentioned below, I have provided fresh translations for all the extracts in this book. Those from Latin or French are my own, those from German were done with invaluable help from Annette Imhausen and further assistance from Peter Neumann. The translation of the cuneiform tablet in 12.1.1 was provided by Eleanor Robson, who also kindly read and checked Chapter 1. For extracts from Bolzano I have used the translations made by Steve Russ in *The mathematical works of Bernard Bolzano*, which were so carefully done that I cannot possibly improve on them. To everyone mentioned here I extend my warmest thanks.

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INTRODUCTION

Mathematics has the longest and richest history of any subject in the academic curriculum. It has been practised in every society and culture, with written records reaching back in some cases as far as four thousand years, so that those who learn mathematics in schools and universities today are studying a subject whose origins lie in civilizations and historical periods often very remote from our own. This book will focus on just a small part of the story, in a sense the most recent chapter of it: the mathematics of western Europe from the sixteenth to the nineteenth centuries. This is because this book evolved from a course designed to provide mathematics undergraduates with some historical background to the material that is now taught universally to students in their final years at school and the first years at college or university: the core subjects of calculus, analysis, and abstract algebra, along with others such as mechanics, probability, and number theory. All of these evolved into their present form in a relatively limited area of western Europe from the mid sixteenth century onwards, and it is there that we find the major writings that relate in a recognizable way to contemporary mathematics. Hence the relatively narrow focus of this book.

There are many different ways of approaching the history of mathematics. The most common are by topic or by period, and in this book you will find both. The first six chapters deal mainly with the seventeenth century, the next six with the eighteenth, and the final six with the nineteenth, but the divisions are very far from rigid, and several chapters deliberately overstep these boundaries. Within that broad framework each chapter focuses on a particular topic and outlines its history, as far as possible through extracts from primary sources. The aim of this book is not to present a conventional account or a predigested version of history, but to encourage you, the reader, to develop critical historical thinking for yourself. It therefore offers mathematics as it was originally written, and invites you to read it for yourselves, to ask your own questions, and make your own judgements.

It can be argued, of course, that the selection of sources and the surrounding commentary already lend support to one interpretation rather than another. That is true but unavoidable, and I very much hope that readers will counter it by feeling free to discuss and argue about anything and everything that is written in these pages. It may also be objected that brief extracts from otherwise lengthy books and articles can never give more than a partial picture. That is also true, but for beginners a complete picture would be overwhelming. Selected themes and examples, on the other hand, can serve as a helpful introduction to what is available, and those who enjoy what is offered in this book will have no difficulty pursuing particular authors or topics much further if they wish to do so. More and more source material is becoming available electronically, but the sheer amount of it can be bewildering and I believe there is still a place for a book like this that can serve as a guide to some of the key mathematical literature of the past.

Sourcebooks in mathematics over the last eighty years have always tended to reflect current trends in the history of mathematics, and this one is no exception. One of the earliest such books was David Eugene Smith's *A source book in mathematics* (1929), a collection of translations grouped under the general headings of number, algebra, geometry, probability, and (lumped together) calculus, functions, and quaternions. Within each section the extracts appear in random order, so that, for instance, Chebyshev on the totality of primes (1851) is immediately followed by Napier on the invention of logarithms (1614). The book therefore presents the reader with a collection of raw material but obscures any sense of historical development. Later popular source books, for example Struik's *A source book in mathematics, 1200–1800* (1969) or Calinger's *Classics of mathematics* (1982) are better arranged, and the latter in particular has useful introductions to each chapter, and extensive suggestions for further reading, but remains primarily an anthology, a collection of diverse and unrelated texts. Such books are based on the premise that the selected extracts will somehow speak for themselves, but unfortunately they do not. The originals were written in particular historical, personal, and mathematical circumstances that influenced both their style and their content, and without some knowledge of that context the reader is likely at best to lack understanding and at worst to be misled. Further, and crucially, mathematics is not a subject carried on by mathematicians working in isolation, in which every few years or centuries some new discovery emerges for us to wonder at; on the contrary, all mathematicians rely on the work of their predecessors and all owe an enormous debt to the past. How mathematics is communicated from one generation or culture to another is not a side issue, but an integral part of mathematics as a human activity.

By far the most widely used source book in recent years has been Fauvel and Gray's *The history of mathematics: a reader*, which has done much to overcome the shortcomings of the earlier books. The sources have been imaginatively and astutely chosen to illustrate periods or themes, and there are helpful introductory notes to each chapter and section. The *Reader* was not designed to stand alone, however, but to be read alongside comprehensive study material produced by the Open University, so that to use it in an effective way the reader must refer to that or other supporting material. Jeremy Gray, one of the original editors, is currently compiling the material from the Open University course units into a companion volume to the *Reader*, so that the two books together will provide an excellent introduction to the general history of mathematics.

Nevertheless, I believe there is also room for this present book, not as an alternative to the *Reader* but to complement it. The *Reader* reflects (as all source books are bound to do) the predilections of its editors, but also it was designed for students who could not be assumed to have any mathematical background. It addresses that audience very successfully, but necessarily its emphasis is either on earlier periods of history (up to the seventeenth century), or on topics where technical detail can be presented in a way that is not too daunting. The expected readership of the present book is quite different: students, teachers, or others, who do have some mathematical training, and would like to learn more about what lies behind the mathematics they know. This book therefore offers extracts in which the mathematics stands exactly as it was originally written, in the hope that readers will engage with it for themselves. Just as new mathematics can hardly ever be learned without taking pencil and paper in hand, so there is no better way of entering into the mind of a mathematician than by trying to think as he did (I am afraid there is no 'she' in this book). Some of the mathematics is easy, some is more difficult, but that is the nature of the subject, and it would present an unrealistically anodyne picture if the hard bits were all edited out. It is not necessary to follow every single step of every argument, but it is already a useful historical exercise to observe just where the difficulties arise, and whether or how they were overcome.

The feature of this book that otherwise distinguishes it most sharply from its predecessors is that almost every source is given in its original form, not just in the language in which it was first written, but as far as practicable in the layout and typeface in which it was read by contemporaries. Every researcher knows the thrill of handling old books and journals, and while it is impossible in any modern book to convey a sense of dustiness, or crinkled pages, or battered bindings, or the unmistakable smell of an old library, I hope the extracts will offer some sense of what it is like to see and handle the originals. Modern typeface is clean, clear, and regular, but this was not so in the seventeenth or eighteenth centuries when everything had to be painstakingly set by hand by printers who probably understood little or nothing of the mathematics they were dealing with. Paper was of variable quality, ink was spread unevenly, so that sometimes it bled through the page but at other times failed to make a mark at all, and page edges were often rough and irregular; beyond that, time and age have added further blemishes, some of which are visible in these extracts. The printed page of two or three hundred years ago had a much more homespun look than a modern page, sometimes rough and awkward, but at other times quite beautiful, and difficult to copy or surpass even with all the possibilities of modern technology. The reasons for reproducing the pages in their original form, however, are more than aesthetic. It is only by studying the originals that one sees exactly how notation was invented and used, how equations were laid out, where and how the diagrams were set, and so on. Later editors and translators take liberties with all these things, perhaps producing clearer text, but at the same time distancing us from the original. In this book, sources originally written in English have not been transcribed except where the original presents problems of legibility, and diagrams throughout have been left in their original form and context.

Unfortunately for many of the readers to whom this book is addressed, most of the sources are not in English, which became an international language of intellectual discourse only in the twentieth century. In the earlier chapters, the predominant language is Latin, later giving way to French and then to German. Because it is too much to expect modern readers to know all or even one of these, translations are provided for all the extracts, but it is my hope that anyone with even a rudimentary knowledge of the other languages will try their hand at the originals, with or without the offered translation alongside. Translation is at the best of times a subtle and difficult task for which there is no single correct outcome, and just as in the choice and arrangement of sources, new ways of thinking about the history of mathematics are reflected in changing attitudes to translation. Many older translations reveal the content of a text and can be a useful starting point, but the translators were sometimes very free in their interpretations of the original, and unfortunately have all too often been copied without question in later volumes: Calinger, for instance, and even Fauvel and Gray, draw quite heavily on translations to be found in Smith or Struik. The translations in the present book, apart from two exceptions noted in the Acknowledgements below, have been made entirely afresh, with the following precepts in mind: (i) that vocabulary and sentence structure should remain as true as possible to the original; (ii) that mathematical notation should not be changed unless it cannot be reproduced, or is actually misleading; in this last case any changes have been clearly noted. These rules may sound straightforward but in practice they are not.

Retaining the thought forms of another language is never easy, especially (in this book) translating from Latin or German where word order is markedly different from English. Even from French, which is much closer to English in its modes of expression, the problems are subtle. Lacroix, for instance, in his account of the development of the calculus in the introduction to his *Traité du calcul* wrote (of Newton): 'il appela fluxions les vîtesses qui régulaient ces mouvemens'.[1] Translating literally we have: 'he called fluxions the speeds which regulate these movements', but since the sentence is offering a definition of fluxions it is much more natural to say in English: 'he called the speeds which regulate these movements fluxions'. In other words, English witholds the emphasis on the new concept to the end of the sentence, where French has it up front. Do such fine distinctions matter? They do, because by changing word order it is all too easy to alter the balance of a sentence, and therefore of a thought, in ways that the author did not intend. Strictly literal translations, on the other hand, can be awkward to the point of being unreadable and, as in the example above from Lacroix, can even get the meaning wrong. In the end, every translator has to take a little licence with the original for the sake of fluency. Seventeenth- and eighteenth-century Latin, for example, is often riddled with 'moreovers' and 'therefores', and I confess to having silently eliminated quite a few of them. Nevertheless I have kept these and other stylistic

1. Lacroix 1810, xv.

changes to a minimum and have tried to avoid the temptation to modernize or 'improve' other people's writing. Not the least of my reasons for doing so is to enable the reader to compare originals and translations as directly as possible.

The problems outlined above are common to all translation, but the translation of historical mathematical texts poses yet further levels of difficulty, in the handling of technical words and mathematical notation. Most readers of this book will almost certainly want to re-write some of its mathematics in modern symbolism to understand it more easily; this is a natural and reassuring thing to do, and indeed we have all learned mathematics by precisely this technique of writing it 'in our own words'. A modern translation should not be confused, however, with what the author himself had in mind. When, for instance, we change Cardano's equation

$$1 \textit{ cubum } p : 8 \textit{ rebus, aequalem } 64$$

into

$$x^3 + 8x = 64$$

we are translating an obscure form of words into something we can immediately recognize. But it also takes us a long way from the original, turning Cardano's 'rebus', or 'things', which had its own history and meaning, into our 'x', which has its own quite different history and meaning. To understand Cardano's thought we need to return to his text armed with modern formulas only as a guide. The symbolic version can certainly give us some hints and clues as to the richness of the original, but should never be mistaken for it.

Similar problems arise with words and phrases that were once in common currency but which have now lost their meaning. Sixteenth- and seventeenth-century mathematicians, for instance, frequently used the Latin word '*in*' in phrases like '*A in B*'. Literally, this refers to the construction of a line segment A on ('*in*'), and perpendicular to, a line segment B; together the lines define or 'produce' a rectangular space. The phrase is usually most easily translated as 'A multiplied by B' or 'A times B', but the words 'multiplied by' or 'times' strictly apply to numbers, not lengths, and in themselves carry no geometric overtones. Similarly, the geometric '*applicare ad*' (to lay against) is generally translated as 'to divide by', but again the geometric connotation is lost. Does it matter? Yes, because we cannot really understand meaning if we ignore historical roots, and yet it is almost impossible to preserve or convey those roots in the very different language of today.

In other cases there are words that have remained outwardly the same while their meaning has changed many times over: 'algebraic' and 'analytic', for instance, which can only ever be understood in relation to a particular time and context. There are also words or phrases that simply have no exact counterparts in English. Euler, for example, used the expression *functio integra* (literally a whole or complete function) for a function that can be expressed as a finite sum of positive powers. The same phrase appears in French as *fonction entière* or in German as *ganze Function*, but there is no English equivalent. Cayley tried 'integral function' but that simply confuses matters with the calculus. A technically correct translation is 'polynomial function', but that is unsatisfactory because 'polynomial' means literally 'having many terms', something almost the opposite of the sense Euler was trying to convey of being complete, finished, rounded off.

Finally, in translating mathematics it is often all too tempting to introduce words that now have a precise technical meaning, as though that was what an author in the past would have used if he could. Thus, for example, a recent translation of Cantor's proof of the countability of algebraic numbers, uses the terms 'set' and 'one-to-one', both of them thoroughly familiar to modern mathematicians, but nowhere to be seen in Cantor in 1874; on the contrary, Cantor had to explain his new meaning as best he could *without* the help of such words and in everyday vocabulary.[2] To read the present into the language of the past is to imagine that modern mathematics was somehow always in existence, just waiting for someone to notice it. This attitude is not only ahistorical but it belittles the struggles and insights of those who had to grapple for the first time with strange and difficult ideas, and express them in whatever words they could muster.

The process of change in style and language continues even now, and with increasing rapidity. Those who were educated in the 1960s or 1970s will still be intimately familiar with some of the terms and concepts to be found in the extracts in this book, in a way that those educated even twenty or thirty years later will not. As a reader of this book you need to be constantly aware of the problems of language and translation, and if you have any knowledge at all of the source languages, look at the originals and decide for yourself how to interpret them. Discuss the problems with others, and make whatever changes or improvements you see fit to the translations offered.

The same direction to discuss and improve applies to every other feature of this book. Everyone who reads it with any seriousness is likely sooner or later to complain that this or that source should have been included but is not: in that case, follow it up and make the missing source the starting point of an alternative or more complete story. Around every extract in the book there is some commentary to explain how it fits into a broader picture, or to elucidate those parts where the mathematics may seem to modern eyes (and perhaps to contemporary eyes also) particularly difficult or obscure, but these relatively brief remarks should by no means be taken as all there is to say. Compare them with interpretations made by other historians; weigh up the points of agreement or disagreement; make your own judgements based on the source material available to you here and elsewhere; become not just a reader but a historian.

Part of this process will be to understand that mathematics does not separate itself into sections as neatly as might appear from the eighteen chapter divisions of this

2. Ewald 1996, II, 840.

book. There are already many cross-references from one chapter to another, as there are bound to be in a subject where apparently unrelated topics have a way of turning out to belong to the same larger picture. I hope you will find other connections, both within this book and to material outside it. I also hope that you will look for relationships and influences not just between one piece of mathematics and another, but between mathematicians themselves. Mathematics, perhaps more than any other academic subject, develops out of insights and understanding accumulated over time and passed from person to person, sometimes through books and printed papers, but also through letters, lectures, and conversations. The significance of personal meetings and friendships between mathematicians is often overlooked, but they are very much part of the intricate social history of mathematics.[3] The brief biographical notes at the end of the book are intended to help illustrate this.

Mathematics is a thoroughly human endeavour in yet another sense, and this time with the emphasis on endeavour. Mathematics, for obvious reasons, is generally taught as a set of tried and tested theorems, carefully built up in a sensible and convincing way from accepted starting points, but it will not take you long as you peruse the extracts in this book to see that mathematics was not discovered or invented that way at all. Here you will find mathematicians groping in the dark, experimenting with new ideas, making hypotheses and guesses, proving correct theorems wrongly, and even on occasion proving incorrect theorems wrongly too. Mathematics is for everyone, beginner or expert, a process of discovery that is prone to error, false starts, and dead ends. Some past mistakes have been included in this book quite deliberately to demonstrate that not even for an Euler, a Lagrange, or a Cauchy was it all plain sailing.

My hope is that through the pages of this book you will see the emergence of mathematics that is universally taught today, as it took shape in the minds of those who created it. Often, especially in the earlier years of the seventeenth century, it was developed in contexts that now appear strange and perhaps unwelcoming, and expressed in language that can seem difficult and obscure. Yet gradually it becomes more recognizable and familiar, until by the early nineteenth century we are in touch with those who directly formed much of our present day curriculum. If this book helps you to a better understanding of some of the mathematics you know, it will have served one good purpose; if it leads to you to read historical texts with perception and judgement it will have served another, no less valuable.

3. For a remarkable example of a mathematician working alone see Simon Singh's account of Andrew Wiles in *Fermat's last theorem* (2002); for an equally remarkable example of mathematicians working in collaboration see Mark Ronan's *Symmetry and the Monster* (2006).

CONTENTS