FROM ACTIONS TO ANSWERS

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PREFACE

The 1989 Theoretical Advanced Study Institute in Elementary Particle Physics (TASI–1989) was held in Boulder, Colorado, June 5 to 30. This year's program, "From Actions to Answers", focussed on computation in theoretical particle physics. Accordingly, the school had large components on collider phenomenology and lattice gauge theory. We also had a number of lectures on current topics in modern mathematical physics (conformal field theory, quantum gravity, and sphalerons). Finally we had an extensive program of seminars on recent experimental results. A grand total of 38 students and 23 lecturers attended the Institute.

The physical high point of the Institute was the summit of Mt. Audobon (13,233 feet above sea level). We can't name a specific intellectual high point. We enjoyed the experience of being able to ask smart people dumb questions during the lectures and the conversations during the coffee breaks while we hid from the thunderstorms.

Organizing a summer school is a nontrivial task and we would like to recognize the people and institutions who helped make it possible. We received funding from the U.S. Department of Energy and the National Science Foundation. The lecturers worked very hard to prepare lectures for the summer school, only to discover that they had to work even harder to write them up for this proceedings. The administrative director of the Institute (and the only one of us who really knew what was going on) was K. T. Mahanthappa. Colorado graduate students Matthew Hecht, Vasilios Koures, and He-Sheng Li were a big help throughout the School. Finally we'd like to thank our tireless secretary Linda Frueh for all her efforts on our behalf.

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I. LATTICE GAUGE THEORY

FIVE WAYS TO BE DISCRETE:

A Nonspecialist's introduction

to Lattice Field Theory

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ABSTRACT

These lectures provide an introduction to lattice field theory. I review the relevant facts about continuum field theory necessary to understand the lattice formalism, then describe gauge theories on the lattice. I next discuss triviality and lattice Higgs models, and conclude with a description of lattice theories in the microcanonical ensemble.

Lecture #1: Introduction and review of continuum field theory

1.1 Prolegomena

A wide range of experimental phenomena can presently be understood on the basis of field theory. Without resort to numerical techniques, however, detailed predictions are often difficult to make. A convenient approach to the problem of nonperturbative analysis of field theory involves its reformulation on a spacetime lattice. A wide variety of numerical and analytical techniques can then be applied to the lattice theory. A prime example of the efficacy of this reformulation is the case of lattice Higgs models, where the possibility of bounding or directly predicting the Higgs mass can be considered[1]. Nonperturbative calculations involving QCD, the generally accepted model of the strong interaction, are also of great importance[2].

These lectures are organized as follows. In the first lecture, a brief review of continuum field theory is given, with particular emphasis on the path integral formalism. The second lecture introduces the concept of a lattice gauge theory. Scalar and fermionic matter fields are discussed in the third and fourth lectures respectively. The fifth lecture covers microcanonical field theory, which leads to techniques of some utility in the numerical simulation of lattice field theories.

1.2 Continuum gauge theories

1.2.1. From a symmetry principle to a Langrangian

Before discussing lattice gauge theories, it is instructive to review briefly the concepts of continuum gauge theories. More detailed reviews can be found in Abers and Lee[3], Taylor[4], Kibble[5], and elsewhere. The following discussion assumes a fair degree of familiarity with classical mechanics; the reader may first wish to refresh his memory on the subject with one of the standard texts such as Goldstein[6].

A fundamental ingredient of any quantum field theory (in the continuum or on a lattice) is the action, which is given by the time integral of the Lagrangian I.

$$S=\int_{-\infty}^{\infty} I\, dt= \int d^4x\, L\{\phi,\partial\phi\} \qquad (1.1)$$

The Lagrangian is in turn given by the space integral of the Lagrange density L, which is a functional of the fields $\{\phi\}$ and their derivatives $\{\partial\phi\}$. The classical equations of motion of this theory are derived by means of Hamilton's principle, which states that the functional variation of the action is a minimum along the classical 'path' or trajectory for a particle or field

$$\delta\int_{t_1}^{t_2} I\, dt = 0 \qquad (1.2)$$

Eq. (2.2) implies that the Lagrange density must obey Euler's equations

$$\frac{\delta L}{\delta\phi} = \partial_\mu \frac{\delta L}{\delta(\partial_\mu\phi)} \qquad (1.3)$$

A simple example is the Lagrange density of a non-interacting scalar field

$$L\,\{\phi,\partial\phi\} = \frac{1}{2}\left[(\partial^\mu \phi^*)(\partial_\mu\phi) - m^2\, \phi^*\phi\right] \qquad (1.4)$$

where m is the mass of the particles of the theory. The field ϕ is complex; as will be seen in a moment, this allows the particles of the theory to carry electric charge. The Euler equation of motion for the field ϕ is

$$(\partial_\mu\partial^\mu + m^2)\phi \equiv (\partial^2 + m^2)\phi = 0 \qquad (1.5)$$

and the corresponding Euler equation for the complex conjugate field ϕ^* is simply the complex conjugate of Eq. (2.5). In this case the two equations are identical.

Eq. (1.5) is the Klein-Gordon equation, which describes quantum wave mechanics associated with spinless particles of mass m (see, e.g. Bjorken and Drell[7]). For those unfamiliar with the Klein-Gordon equation, it may seem less mysterious when written in component form:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi = 0 \qquad (1.6)$$

which, for m=0, is just the standard wave equation. Then if the usual quantum mechanical identification

$$i\frac{\partial}{\partial t}\rightarrow E$$
$$-i\nabla\rightarrow p \qquad (1.7)$$

is made, the Klein-Gordon equation arises from the relativistic kinematic requirement

$$E^2 - p^2 = m^2 \qquad (1.8)$$

As stated above, the fact that the ϕ is complex allows it to couple to an electromagnetic field. What is remarkable is that the form of the field equations and Lagrangian for scalar electrodynamics can be obtained by 'extending' a symmetry of the Lagrange density L (1.4). In order to understand the mechanism by which the Lagrangian for electromagnetism is generated, it is necessary to appreciate more fully the implications of symmetries of the classical Lagrangian.

To every continuous symmetry of the Lagrangian there corresponds a conservation law. Conversely, for every conserved quantum number there exists a transformation on the fields of the theory which leaves the Lagrangian invariant. These statements are a standard result of classical mechanics known as Noether's theorem. A simple example of this general phenomenon is related to the idea of electric charge. Consider a group of transformations on the fields ϕ,

$$\phi(x)\rightarrow\exp(-iq\theta)\phi(x) \quad \phi^*(x)\rightarrow\exp(+iq\theta)\phi^*(x) \qquad (1.9)$$

where q is to be associated with the real charge of the ϕ field and θ

parameterizes the transformation. The Langrange density equation (1.5) is invariant under this set of transformations, which form the group of unitary transformations in one dimension and are classified in the standard fashion as the group U(1). Note that even though the Lagrange density contains terms involving gradients of the ϕ fields, these terms are invariant under the transformation Eqs. (1.9) since θ is independent of x. Eq. (1.9) describes a global gauge transformation.

For infinitesimal $\theta(=\delta\theta)$, the global transformation Eqs. (1.9) reads

$$\delta\phi=-i(\delta\theta)q\phi, \quad \delta\phi^{*}=+i(\delta\theta)q\phi^{*} \qquad (1.10)$$

The condition that the Lagrange density be invariant under this transformation can be written

$$\delta L=0=\frac{\delta L}{\delta\phi}\,\delta\phi+\frac{\delta L}{\delta(\partial_{\mu}\phi)}\delta(\partial_{\mu}\phi)+\frac{\delta L}{\delta\phi^{*}}\,\delta\phi^{*}+\frac{\delta L}{\delta(\partial_{\mu}\phi^{*})}\,\delta(\partial_{\mu}\phi^{*}) \qquad (1.11)$$

From Eqs. (1.3) and (1.11) it can be seen that the current

$$J^{\mu} \equiv +\frac{i}{2}\left[\frac{\delta L}{\delta(\partial_{\mu}\phi)}\,q\phi-\frac{\delta L}{\delta(\partial_{\mu}\phi)}\,q\phi^{*}\right]$$

$$=(J^{0},\,J^{1},\,J^{2},\,J^{3})\equiv(\rho,\,J) \qquad (1.12a)$$

is conserved, that is:

$$\partial_{\mu}J^{\mu}=0$$

$$=\frac{\partial\rho}{\partial\tau}+\nabla\,.\,J \qquad (1.12b)$$

In the simple example Eq. (1.4) the conserved current is

$$J^{\mu}=\frac{i}{2}\,(\phi\partial_{\mu}\phi^{*}-\phi^{*}\partial_{\mu}\phi)q \qquad (1.13)$$

The connection with electric charge can be made with the second quantized version of the theory (see, e.g., Abers and Lee[3]) where the operator Q

$$Q=\int d^{3}x\;J^{0}=\int d^{3}x\;\rho(x) \qquad (1.14)$$

gives the total charge of the current.

A few simple algebraic tricks have led to a remarkable phenomenon! Simply from the observation that the Lagrange density possesses a certain continuous symmetry [in this case the U(1) symmetry of a redefinition of the phase convention the ϕ field, Eq. (1.9)] it is possible to deduce a conservation law for the theory. Note especially that no property of the Lagrangian other than this symmetry is needed to discover the conserved current of Eq. (1.12a).

The existence of a conserved current is not the only novelty, however. By a systematic extension of this symmetry of the Lagrangian, it is possible to create (almost from nothing!) a set of fundamental laws which appear to describe our universe. Of course such an approach has great aesthetic appeal. The method by which this extension is made is now reviewed.

Consider the physical meaning of the symmetry described by Eq. (1.9). This symmetry arises because the phase of the field ϕ is not an observable quantity, and therefore an overall constant (in this case θ) can be added to its intrinsic phase without affecting the predictions made by the theory. In other words, the overall phase of the wave function is simply a matter of convention. It seems peculiar however that the phase convention chosen at one point should constrain the choice of convention at all the points of spacetime. Such a concept does not appear to be consistent with the localized field description that underlies the usual physical theories. Instead, it might be expected that the choice of phase convention could be made independently at all points of spacetime. (This view is usually attributed to Yang and Mills[8].)

The Lagrangian Eq. (1.4) is not invariant under this more general transformation however. The property is easy to demonstrate. If a transformation of the form of Eq. (1.9) but with $\theta(x)$ taken to be a function of x applied to the field $\phi(x)$ it transforms as

$$\phi(x)\rightarrow\phi(x)\exp[-iq\theta(x)] \qquad (1.15)$$

Terms in the Lagrange density which do not depend on the derivatives of the fields ϕ are invariant under the more general (local) gauge transformations, e.g.,

$$m^2\ \phi^*(x)\phi(x)\ \rightarrow\ m^2\ \phi^*(x)\phi(x) \qquad (1.16)$$

However, terms involving the gradients $\partial_\mu\phi$ of the ϕ fields are not invariant since under the transformation given by Eq. (1.15):

$$\partial_\mu\phi\rightarrow[\partial_\mu\phi(x)-iq\partial_\mu\theta(x).\phi(x)]\exp[-iq\theta(x)] \qquad (1.17)$$

The Lagrange density can however be made invariant under a local gauge transformation by the introduction of a new field (which can be identified with the electromagnetic field) in a fashion which is usually referred to as minimal coupling. This procedure is equivalent to the replacement of the gradient operator in the Lagrange density the the so-called 'covariant derivative' operator

$$D_\mu \equiv \partial_\mu-ieqA_\mu(x) \qquad (1.18)$$

If, under the local gauge transformation of Eq. (1.15), the field $A_\mu(x)$ transforms in the fashion

$$A_\mu(x)\rightarrow-e^{-1}\partial_\mu\theta(x)+A_\mu(x) \qquad (1.19a)$$

then the covariant derivative operating on the field ϕ transforms as follows:

$$D_\mu\phi(x)\rightarrow\exp[-iq\theta(x)]D_\mu\phi(x) \qquad (1.19b)$$

Therefore the modified kinetic term

$$L_{kin}=\tfrac{1}{2}(D_\mu\phi)^*(D^\mu\phi)-\tfrac{1}{2}m^2\phi^*\phi \qquad (1.20)$$

is invariant under the local gauge transformation given by Eq. (1.19) and is an acceptable candidate for part of a locally gauge-invariant Lagrange density.

There should, of course, by kinetic terms in the Lagrange density which couple A_μ only to itself. These terms constitute the part of the action involving only the pure gauge field. One quantity involving only the A_μ is the second-rank tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad (1.21)$$

which is invariant under the gauge transformation in Eq. (1.19). Thus the expression for the Lagrange density of scalar electrodynamics, L_{se}, is

$$L_{se} = L_{kin} + L_{EM} \qquad (1.22a)$$

$$L_{EM} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= \tfrac{1}{2} (E^2 - B^2) \qquad (1.22b)$$

is also gauge invariant and can be proposed as a Lagrange density for a field theory. (The factor $-\tfrac{1}{4}$ is included by convention). The Lagrange density of Eq. (1.22b) is recognizable as that of pure electromagnetism, while that of Eq. (1.22b) is the standard form of the Lagrange density of scalar electrodynamics (see, e.g., Bjorken and Drell[7]. In addition, the transformation law given by Eqs. (1.19) is just the canonical 'gauge transformation' of electrodynamics. The results of the above analysis are thus not new. What is new is the derivation of the Lagrange density for scalar electrodynamics from a simple principle - the extension of a global symmetry of a Lagrange density to a local one.

This concept can be applied to many types of symmetry. For example, if the familiar Lorentz symmetry of special relativity is extended to a local symmetry, the result is essentially the Einstein theory of general relativity (see Kibble[5]). In particle physics, the relevant symmetries to be considered involve Lie groups. As shown above, the theory of electromagnetism is found by extending a U(1) symmetry. The current theories of what are called weak and strong interactions are in turn partially based upon the Lie groups SU(2) and SU(3). These theories are discussed in more detail later on; first it is necessary to take a mathematical detour to see how to extend the above analysis to groups other than U(1).

The gauge transformation for a general internal symmetry Lie group can be written as