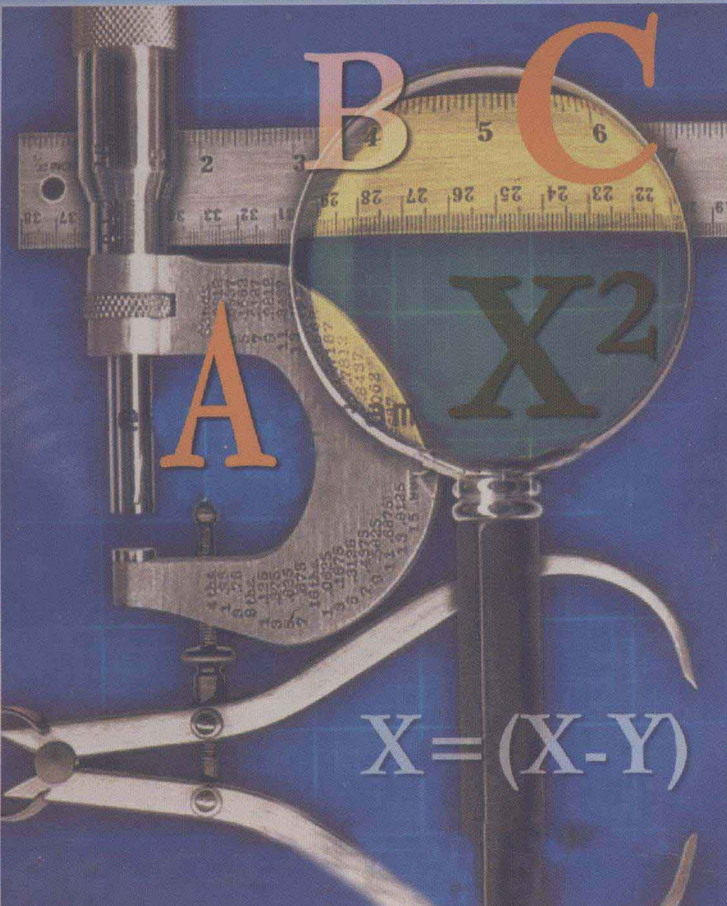

A Textbook of

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Preface

Ability to solve problems is an essential competence for every student. Mathematics courses acknowledge it as a primary aim in recognition of the unique value of the mathematical approach. Algebra provides a new and refined approach to the study of abstract mathematical relationship through the use of a new language and a new symbolism. In this effective means of problem solving, the pupils are prepared to think in terms of symbols of algebra, to understand the relationships among the factors of their environment and thus answers to specific problems are sought through examination and study of functional relationships. Given time and situation, algebra should emerge as a logical postulation system, resting on assumptions and elements and capable of being developed in a structural and logical manner.

Geometry occupies a place alongside arithmetic in school curriculum and aims at inculcating in every pupil a comprehensive knowledge of geometric facts, concepts and processes, an acquaintance with nature of deductive reasoning and application of concepts to the better interpretation and appreciation of environment. Geometry should aim to develop the habits of independent and careful thinking, observing, and comparing in pupils so that they may be able to discover new ideas, statements, truths, concepts and theorems. Teachers should therefore give thoughtful and conscientious

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consideration to the most effective means (techniques) of attaining these goals of instruction and must be skilled in the methods of application and generalisation of the subject matter.

This book brings to the fore each and every aspect, concept, technique, theorem and principle of algebra and geometry to make the subject teachers successful. In this endeavour the book delves deep into basic needs and aims, course content, problem-solving, various types of equations, preparatory course, graphs, linear algebra, analytic geometry, and techniques of induction, deduction, synthesis, analysis, locus and dependence. Written in easy to understand manner, the book encompasses numerous problem-solving illustrations.

R.K. Jacob

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1

The Teaching of Algebra and Geometry

Why teach algebra? Algebra has been most important labour-saving device invented by man. It has also acquired a reputation, among teachers, pupils, and parents alike, as one of the most difficult and troublesome courses in the secondary curriculum.

Perhaps the two facts are not unrelated. Algebra provides a new and refined approach to the study of abstract mathematical relationship through the use of a new language and a new symbolism. Although the answers to specific problems are sought, attention is focused on the examination and study of processes and functional relationships. Indispensable as algebra is for understanding the quantitative aspects of our environment, by its very nature it is disassociated from concrete experiences.

Yet these inherent difficulties in learning algebra may largely be overcome if we are continually aware of its use and importance in the daily life of man. The need for mathematics by engineers and scientists is widely recognised, but we are likely to forget its importance in the shop, in business and in everyday life. Consider the formula alone, and how it makes possible the expression of symbolic relationships that we need in our thinking. The relation between the age and the value of a house, or between speed and the distance required to stop a car, or, in instalment buying, between the number and amount of payments, the cash price, and the interest rate, is made more understandable and useful by means of formulas. Everyone encounters data presented in tabulate form and is required to interpret, interpolate, and extrapolate in tax schedules, insurance-loan-values tables, Federal Housing Administration (FHA) home-loan schedules, and social or economic data encountered in reading. The average citizen finds it increasingly necessary to interpret graphs, both statistical graphs and graphs of functions, either in his business or in his leisure-time reading. All of these tasks require the understandings that are developed through a study of algebra.

An examination of the vocational fields of human endeavour reveals even greater importance for algebra. Problems arising in the shop provide a wealth of materials for study. The need for statistical procedures by the forester, educator, economist, biologist, and librarian draw on the binomial theorem and many other algebraic concepts and skills. If the distance that a cutting tools advances along the length (*L*) of a piece in a lathe for each revolution of the lathe is called feed (*F*), for given revolutions per minute (rpm), then the time for completion is given by $T = \frac{L}{\text{rpm} \times F}$. Similarly, problems of gear rations, ratios of pulleys, tapers for castings, horsepower, measures of tension, gear boxes, and many other shop situations require algebraic proficiency.

Aims for the Study of Algebra

The contribution of algebra to the general aims of mathematics is therefore unique and fundamental. The aims set up within the province of algebra must be to provide pupils with concepts and methods for using, information about, and appreciation of

1. Symbolism and generalisations
2. More inclusive number systems and their manipulation
3. Functional relations, including formulas and other equations, graphs, tables, variation, proportion, logarithms, laws of relation, and the function concept
4. Use of literal numbers and algebraic equations to formulate and solve problems
5. The place of algebra in the lives of mankind, past and present.

What should a pupil be able to do when he has successfully completed a course in algebra? This question should never be far from our minds, for it clarifies the nature and purposes of the course for the teacher and orients classroom procedures. It reminds us that the pupil must be acquiring the competences, interests, and appreciations needed for citizenship and vocational purposes. Clearly the detailed aims for various classes and even pupils will differ. But insofar as algebra has a unique contribution for the pupil, we can describe what we will look for in such terms as the following:

1. *Symbolism*

The pupil

1. Can explain the meaning of the statement, "Words are actually symbols."
2. Can explain the statement, "Numbers are man-made symbols."
3. Can use, recognise, read, and interpret the symbols +, –, ×, =, exponents, radicals and parentheses.
4. Can use, interpret, explain literal numbers as symbols.
5. Can explain the purposes for using symbols in mathematics.
6. Can translate problem situations into appropriate symbols.

2. *Functions*

a. *Concept*

The pupil

1. Understands the meaning of dependence between quantities.
2. Can recognise and express dependencies.
3. Knows the different ways that dependencies can be represented.

b. *Formulas, equations, variation*

The pupils

1. Knows the purpose for representing relations as formulas.
2. Can formulate a formula from a set of data or observed relations.
3. Can evaluate a formula.
4. Can solve a formula for different variables.
5. Can select and use a formula to solve applied problems.

6. Can use the language of formulas, equations and variations.
7. Can represent variations in formulas and can evaluate the formulas.
8. Can solve linear equations for unknowns.
9. Can interpret solutions of linear equations in terms of their graphs.
10. Can formulate and solve equations for problem situations.

c. *Graphs*

The pupils

1. Can interpret bar, line, and circle charts, and graphs of equations.
2. Can locate points on Cartesian coordinate systems.
3. Understands the connection between number pairs satisfying equations and coordinates of points in planes.
4. Can plot a graph of a linear equation and simple second-degree equations.
5. Can write a linear equation from a straight-line graph.

3. *Familiarity with and ability to use mathematics in literature*

a. *Reference sources*

The pupil

1. Knows where to locate needed facts:
   (a) Tables, (b) formulas, (c) historic facts, (d) explanations, (e) social and economic data.
2. Knows how to use the data he secures.

b. *Current publications*

The pupil

1. Has read current publications involving algebraic data and relations.
2. Can use learnings from this course in reading current literature with understanding.

c. *Work in other courses*

The pupil

1. Can understand the mathematics he encounters in reading for other courses.

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4. *Desirable attitudes toward algebra*

a. *Interest*

The pupil

1. Asks pertinent questions.
2. Pays attention and resists distraction.
3. Volunteers information.
4. Reads other sources outside class.
5. Brings in material from outside.
6. Quotes or records pertinent material from current literature.

b. *Appreciation*

The pupil

1. Works mathematics avocationally—puzzles, oddities, problems.
2. Reads mathematical sources for pleasure.
3. Can defend the importance of mathematics in history and in current life.
4. Understands algebra as an integrated science rather than one with unrelated parts.
5. Can fit topics into the structure of algebra.

With such a list as a guide, the teacher may select the goals to be achieved in some measure in a given topic, and expand the list to include the special aims of the topic. The content, activities and evaluation procedures may then be selected with a view to their achievement.

Algebra in the Secondary Schools

The increasing need for certain phases of algebra in many important vocations, as well as its historical importance, established the subject as a requirement in most high school curricula in the nineteenth century. The course content was dictated by college-entrance requirements, conceived as essential for the future development of mathematicians and scientists. Increasing non-college high school enrolments led to a decline in the per cent of students in the high school taking algebra prior to World War II, and in some modification of the content of algebra courses. In 1905, 57.51 per cent of all students in the high school were enrolled in algebra courses, while in

1934 about 30 per cent were so enrolled. The shift was partially accounted for by the introduction of two-track plans, with general mathematics offered to meet the general education mathematics requirements, as well as by competition of new courses more attractive than mathematics to many students.

World War II changed the trend—whether permanently it is difficult to say. A limited survey in the spring of 1947 showed that 48 per cent of all ninth-grade pupils in the schools responding to the questionnaire were enrolled in beginning algebra. The same data indicated that about 59 per cent of all tenth-grade pupils enrolled in some mathematics course, with about 27 per cent of all tenth-graders in these schools enrolled in a first course in algebra. Considered with other available evidence this study appears to confirm the belief that the decline in mathematical interest in high school was terminated as the widespread need for mathematical proficiency was impressed on parents, teachers and pupils alike.

The typical algebra class today includes pupils with strikingly diverse objective; some are preparing for scientific training, and others have general education or non-science college training as their goals. In small schools where only one track can be offered, this grouping of pupils with a combination of purposes will probably continue. Many larger schools have reduced this diversity by offering as an alternative a general mathematics course, while the algebra course is recommended for pupils known to require the subject for scientific pursuits.

The survey referred to above indicates to some degree the extent to which the study of algebra is postponed until the tenth grade, as proposed in the alternative curriculum pattern of the Joint Commission. In this plan a general mathematics course is offered in grade nine, which prepares for a rather complete algebra programme in grade ten designed for prospective technical and scientific pursuits.

Course Content and Trends

The content of most algebra courses follows rather closely the recommendations of the National Committee on Mathematical Requirements (1923), and outlined in detail in the *Fifteenth Yearbook*. The typical current secondary algebra courses centre about units such as the following:

1. Use of literal numbers
2. The solution of simple equations and formulas
3. Graphs and the concept of dependence
4. Meaning and use of directed numbers
5. Fundamental operations with algebraic quantities
6. Linear equation in one unknown
7. Linear equation in two unknown
8. Special products and factors
9. Fundamental operations with fractions
10. Fractional equations
11. Ratio, proportion, variation
12. Numerical trigonometry, indirect measurement
13. Powers, roots, radicals
14. Quadratic equations
15. Logarithms.

Although the content of the algebra course is rather well standardised, certain trends are discernible over the past few years, largely as changes in emphasis rather than in major topics. Recent textbooks and curriculums tend to reveal the following shifts in content:

1. A tendency to leave more complicated material until later parts of books or problem lists, or to star it as optional. This material includes most linear equations with fractional and decimal coefficients, the more involved literal equations, quadratic equations with irrational roots, the more complicated equations involving radicals, the construction of formulas from sets of data, the more involved factoring, simplification of more involved radicals, graphing of quadratic functions, and trigonometric ratios. These topics are primarily those that were recommended by the Joint Commission for exclusion from the general education algebra course.
2. Inclusion of problems drawn from more diversified fields.
3. Inclusion of sections on bar, line and circle graphs as well as graphs of equations.
4. Inclusion of more experiences for developing concepts and meanings.
5. Greater attention to methods of solving problems.

Trouble Spots in Teaching

Even with these readjustments in content, several trouble spots remain in the learning of elementary algebra. They are, for the most part, located in these areas: beginning the study of algebra; problem-solving; graphing; solution of equations; signed numbers; the processes of algebra. Fortunately, experience and experimentation have provided effective procedures for the teachers who would avoid these difficulties.

The problems of method in each of these crucial areas are simply those of directing pupils through the learning sequence. Learning difficulties indicate neglect or failure at one or more of the steps, and they can be prevented by careful planning. The pupil must be introduced to each process in a realistic experience, and his previous concrete experiences with it must be summarised and organised as a useful background. Succeeding experiences must be provided on an increasingly symbolic level until the pupil understands the process, its symbolism, and its relationships to other mathematical processes. At this stage of learning, and not before, he is ready to define rules, to drill, and to study the broad and varied fields of application.

The preceding is a broad statement of teaching strategy. It becomes interesting and valuable if its steps are followed through in these examinations of problem areas. Equally important with the strategy are the detailed procedures that make each step work in each situation—in other words, the tactics. Teaching expertness requires not only an understanding of the broad pattern of learning but skill in the procedures used to guide pupils through it as well.

Beginning the Study of Algebra

The introductory period in beginning algebra is a crucial period for developing not only basic skill and knowledge but also important interests and attitudes. The methods and processes that the pupil is to acquire in algebra must, in this transitional period, be integrated with and built on his previous experiences in arithmetic. While the idea of general number is being acquired, a beginning must be made in vocabulary building, in learning the symbolism, and in using the processes. At the same time the pupil must develop an appreciation of the importance of algebra to himself and to society.

Various successful approaches, all based on the same general principles of learning, have been devised to achieve these results. Let us follow the procedures of Harry Johnson through the first two weeks of his ninth-grade course. He used the formula as a natural tie-in between algebra and the mathematics of the previous grades.

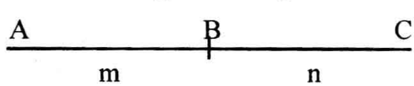
Step 1. *Summary and organisation of previous experiences*

Mr. Johnson brought the formula before the class in a discussion of jet planes and the speed of sound. Most of the pupils were reasonably familiar with the formula d = rt. The discussion was then turned on the question of formulas in general, and what formulas the pupils knew. Mr. Johnson listed them on the board, and had the pupils state in words the rule expressed by each formula. The list was increased throughout the week as the pupils recalled or discovered other formulas. The following ideas were developed in a general way:

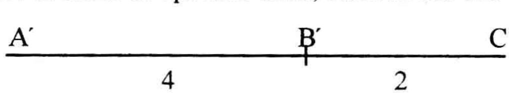
That rules can be turned into formulas, and *vice versa*
That the notation is concise and definite
That the formula is general; the formula for the area of a rectangle, for example, applies to the area of any rectangle, and so on.
That when numbers are substituted for letters, the situation treated changes from general to specific.

Step 2. *Extension of formulas into semisymbolic notation*

So long as the pupils continue to consider the formula as the abbreviation for a rule, their attention is on the rule and its applications, and the idea of general number is not obvious. To move away from concrete settings, and to shift attention to the notation and symbolism, Mr. Johnson applied the use of the formula to working with lines, as semisymbolic devices. Thus: Given the line *AC*, where *AB* in *m* units long and *BC* is *n* units long, how long is the line *AC*?

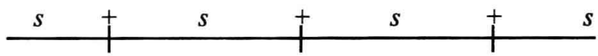


Reference is made to specific lines, such as the one that follows:



where numerical lengths are to be added to obtain the total length. From this concrete example the idea of expressing the length of *AC* as the sum of m and n or (m + n), followed naturally. Similarly, the difference between two lengths was investigated first in numerical examples and then in general.

The idea of the product of an integer and a literal number was illustrated by use of a line composed of four segments, each of length *s*, thus:



The total length is seen to be $s + s + s + s$ or 4s. The fact that the product of two numbers a and b is written as ab is explained at this stage.

Step 3. *Symbolisation and manipulation of relationships*

Next the use of literal numbers was explored and algebraic expressions formulated in many other situations, such as ages two years hence, five years ago, and y years hence and past; also considered were weights, numbers of marbles, distances, angle sizes. Listings of words added to the vocabulary, such as "literal numbers", "square", "cube", "odd and even integers", "exponent", "base", "square", "cube" and "power" were placed on the board. As confidence and facility in use of literal numbers and of symbols of operation developed, new problem situations were introduced and solved.

Step 4. *The equation as related to the formula*

Simple familiar situations were then solved by use of equations. These included problems like finding the third angle of a triangle, given two angles; finding a distance when the total and one part are given; finding the price per gallon when the number of gallons and total price are given; finding the side of a regular polygon when the number of sides and perimeter are given. The equations were solved by intuitive methods, the idea slowly developing that solutions consists of taking the equations apart by a process inverse to the processes that built them.

For example, given the problem:

John's father is 28 yrs. old, which is four years more than twice John's age. How old is John?

We build the equation:

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