D. Sornette

Critical Phenomena in Natural Sciences

Chaos, Fractals, Selforganization and Disorder: Concepts and Tools

Springer

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With 89 Figures

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An ever increasing number of scientific disciplines deal with complex systems. These are systems that are composed of many parts which interact with one another in a more or less complicated manner. One of the most striking features of many such systems is their ability to spontaneously form spatial or temporal structures. A great variety of these structures are found, in both the inanimate and the living world. In the inanimate world of physics and chemistry, examples include the growth of crystals, coherent oscillations of laser light, and the spiral structures formed in fluids and chemical reactions. In biology we encounter the growth of plants and animals (morphogenesis) and the evolution of species. In medicine we observe, for instance, the electromagnetic activity of the brain with its pronounced spatio-temporal structures. Psychology deals with characteristic features of human behavior ranging from simple pattern recognition tasks to complex patterns of social behavior. Examples from sociology include the formation of public opinion and cooperation or competition between social groups.

In recent decades, it has become increasingly evident that all these seemingly quite different kinds of structure formation have a number of important features in common. The task of studying analogies as well as differences between structure formation in these different fields has proved to be an ambitious but highly rewarding endeavor. The Springer Series in Synergetics provides a forum for interdisciplinary research and discussions on this fascinating new scientific challenge. It deals with both experimental and theoretical aspects. The scientific community and the interested layman are becoming ever more conscious of concepts such as self-organization, instabilities, deterministic chaos, nonlinearity, dynamical systems, stochastic processes, and complexity. All of these concepts are facets of a field that tackles complex systems, namely synergetics. Students, research workers, university teachers, and interested laymen can find the details and latest developments in the Springer Series in Synergetics, which publishes textbooks, monographs and, occasionally, proceedings. As witnessed by the previously published volumes, this series has always been at the forefront of modern research in the above mentioned fields. It includes textbooks on all aspects of this rapidly growing field, books which provide a sound basis for the study of complex systems.

To Anne, Jaufray and Paul

Preface: Variability and Fluctuations

Life is fundamentally risky, reflecting the pervasive out-of-equilibrium nature of the surrounding world. Risk is synonymous with uncertainty about the future, leading not only to potential losses and perils, but also to gains. This uncertainty results from the numerous dynamical factors entering our life, giving it spice and color as well as its dangerous flavor. Life consists of a succession of choices that have to be made with often limited knowledge and in a complex and changing environment. These choices result in a sequence of often unpredictable outcomes, whose accumulation defines the specific trajectory characterizing each individual, somewhat similar to the apparent random trajectory of a leaf carried by a turbulent wind. The notion of risk is probably one of the most general concepts pervading all the facets of our life [617, 230].

Risk is a companion to most of our daily activities, professional or private. Crossing a street or driving a car involves risk that is quantified by the statistics of traffic accidents and police reports and which impacts on our insurance premium. Staying at home is also risky: falling, burning, electrocution, plane crash, earthquakes, hurricanes, etc. Risk is present in the choice of a career, in the selection of a college and university program as well as in the effect of social interactions on the development of children. Any choice is intrinsically risky, since the existence of a choice implies several alternatives that are all thought to be possible outcomes, albeit with possibly different likelihood. In industry, companies have to face a multitude of risks: R&D, choice of a niche, capital, production, sales, competition, etc., encompassing all types of risks that, ideally, have to be optimized at each instant. The apparent random nature of price variations in both organized and emerging stock markets leads to risky investment choices, with impact on the global economy and our welfare (retirement funds).

The Earth provides its share of risks, partly overcome with the development of technology, but hurricanes, earthquakes, tsunamis, volcanic eruptions and meteorites bring episodic destruction each year, constituting as many Damocles' swords over our heads. Neither is biological risk negligible, with endemic epidemics and the emergence of novel diseases. Human society, with its technical development and population growth, introduces new risks: unemployment, strike, dysfunction of cities, rupture of sensitive technological structures (hydroelectric dams, chemical plants, oil tankers, nuclear plants, etc.). Scientific and technical development and the growing interaction between the different organizational levels of human society introduce an increasing complexity, leading often to an enhanced vulnerability. The weight of human activity has developed to a point where there are growing concerns about new planetary risks such as global warming, ozone-layer depletion, global pollution, demographic overcrowding, and the long-term agricultural and economic sustainability of our finite planet. Paling's little book [550] provides an interesting and stimulating synopsis in which a logarithmic scale is used to quantify all the risks that we have to face, from the largest, which are not always those we think about, to the smallest. This logarithmic scale (similar to the earthquake magnitude scale) reflects the extremely large variability of risk sizes. The concept of risk thus covers the notion of variability and uncertainty.

Our main goal in this book is to present some of the most useful modern theoretical concepts and techniques to understand and model the large variability found in the world. We present the main concepts and tools and illustrate them using examples borrowed from the geosciences. In today's rapidly evolving world, it is important that the student be armed with concepts and methods that can be used outside his/her initial specialization for a better adaptation to the changing professional world. It is probably in the everyday practice of a profession (for instance as an engineer or a risk-controler in a bank) that the appreciation of variabilities and of the existence of methods to address it will be the most useful.

These ideas are of utmost importance in the advancement of the traditional scientific disciplines and it is in their context that this book is presented. The notions of variability, fluctuations, disorder, and non-reproducibility, on a deep conceptual level, progressively penetrate the traditional disciplines, which were initially developed using the concepts of averages, or more generally, of representative elements (as in thermodynamics, mechanics, acoustics and optics, etc.). Modern physics deals, for instance, with heterogeneous composite systems and new materials, chaotic and self-organizing behaviors in out-of-equilibrium systems, and complex patterns in the growth and organization of many structures (from that of the universe at the scale of hundreds of megaparsecs to the minute branchings of a snowflake). It is clear that these phenomena are all deeply permeated by the concepts of variability, fluctuations, self-organization and complexity. In the context of natural evolution, let us mention the remarkable illustrations (evolution and baseball) presented by S.J. Gould [276], in which the full distribution (and not only the average) of all possible outcomes/scenarios provides the correct unbiased description of reality. This is in contrast with the usual reductionist approach in terms of a few indicators such as average and variance.

The physical sciences focus their attention on a description and understanding of the surrounding inanimate world at all possible scales. They address the notion of risk as resulting from the intrinsic fluctuations accompanying any possible phenomenon, with chaotic and/or quantum origins. Mathematics has developed a special branch to deal with fluctuations and risk, the theory of probability, which constitutes an essential tool in the book. We begin with a review of the most important notions to quantify fluctuations and variability, namely *probability distribution* and *correlation*. "Innocuous" Gaussian distributions are contrasted with "wild" heavy-tail power law distributions. The importance of characterizing a phenomenon by its full distribution and not only by its mean (which can give a very distorted view of reality) is a recurrent theme. In many different forms throughout the book, the central theme is that of *collective* or *cooperative* effects, i.e. the whole is more than the sum of the parts. This concept will be visited with various models, starting from the sum of random variables, the percolation model, and self-organized criticality, among others.

The first six chapters cover important notions of statistics and probability and show that collective behavior is already apparent in an ensemble of uncorrelated elements. It is necessary to understand those properties that emerge from the law of large numbers to fully appreciate the additional properties stemming from the interplay between the large number of elements and their interactions/correlations. The second part (Chaps. 7–15) discusses the behavior of many correlated elements, including bifurcations, critical transitions and self-organization in out-of-equilibrium systems which constitute the modern concepts developed over the last two decades to deal with complex natural systems, characterized by collective self-organizing behaviors with long-range correlations and sometimes frozen heterogeneous structures. The last two chapters, 16 and 17, provide an introduction to the physics of frozen heterogeneous systems in which remarkable and non-intuitive behaviors can be found.

The concepts and tools presented in this book are relevant to a variety of problems in the natural and social sciences which include the large-scale structure of the universe, the organization of the solar system, turbulence in the atmosphere, the ocean and the mantle, meteorology, plate tectonics, earthquake physics and seismo-tectonics, geomorphology and erosion, population dynamics, epidemics, bio-diversity and evolution, biological systems, economics and so on. Our emphasis is on the concepts and methods that offer a unifying scheme and the exposition is organized accordingly. Concrete examples within these fields are proposed as often as possible. The worked applications are often very simplified models but are meant to emphasize some basic mechanisms on which more elaborate constructions can be developed. They are also useful in illustrating the path taken by progress in scientific endeavors, namely "understanding", as synonymous with "simplifying". We shall thus attempt to present the results and their derivations in the simplest and most intuitive way, rather than emphasize mathematical rigor.

This book derives from a course taught several times at UCLA at the graduate level in the department of Earth and Space Sciences between 1996 and 1999. Essentially aimed at graduate students in geology and geophysics offering them an introduction to the world of self-organizing collective behaviors, the course attracted graduate students and post-doctoral researchers from space physics, meteorology, physics, and mathematics. I am indebted to all of them for their feedback. I also acknowledge the fruitful and inspiring discussions and collaborations with many colleagues over many years, including J.V. Andersen, J.-C. Anifrani, A. Arneodo, W. Benz, M. Blank, J.-P. Bouchaud, D.D. Bowman, F. Carmona, P.A. Cowie, I. Dornic, P. Evesque, S. Feng, U. Frisch, J.R. Grasso, Y. Huang, P. Jögi, Y.Y. Kagan, M. Lagier, J. Laherrère, L. Lamaignère, M.W. Lee, C. Le Floc'h, K.-T. Leung, C. Maveyraud, J.-F. Muzy, W.I. Newman, G. Ouillon, V.F. Pisarenko, G. Saada, C. Sammis, S. Roux, D. Stauffer, C. Vanneste, H.-J. Xu, D. Zajdenweber, Y.-C. Zhang, and especially A. Johansen, L. Knopoff, H. Saleur, and A. Sornette. I am indebted to M.W. Lee for careful reading of the manuscript and to F. Abry and A. Poliakov for constructive comments on the manuscript.

UCLA and Nice, *Didier Sornette*
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