The background of the cover is a photograph of a person skydiving over a lush green valley. The skydiver is in the center, wearing a red and white jumpsuit, with arms and legs spread out. Below them, a small village with several houses and a church is visible, surrounded by rolling green hills and dense forests. The sky is a clear, bright blue. In the top left corner, there is a faint, stylized graphic of a gear or a mechanical part.

Applied Numerical Methods
with **MATLAB**[®]
for Engineers and Scientists

Steven C. Chapra

Third Edition

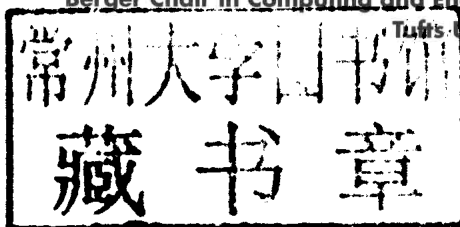
Applied Numerical Methods

with MATLAB[®] for Engineers and Scientists

Third Edition

Steven C. Chapra

Berger Chair in Computing and Engineering
Tufts University





APPLIED NUMERICAL METHODS WITH MATLAB FOR ENGINEERS AND SCIENTISTS, THIRD EDITION

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ABOUT THE AUTHOR

To

My brothers,
John and Bob Chapra

ABOUT THE AUTHOR

Steve Chapra teaches in the Civil and Environmental Engineering Department at Tufts University, where he holds the Louis Berger Chair in Computing and Engineering. His other books include *Numerical Methods for Engineers* and *Surface Water-Quality Modeling*.

Steve received engineering degrees from Manhattan College and the University of Michigan. Before joining the faculty at Tufts, he worked for the Environmental Protection Agency and the National Oceanic and Atmospheric Administration, and taught at Texas A&M University and the University of Colorado. His general research interests focus on surface water-quality modeling and advanced computer applications in environmental engineering.

He has received a number of awards for his scholarly contributions, including the Rudolph Hering Medal, the Meriam/Wiley Distinguished Author Award, and the Chandler-Misener Award. He has also been recognized as the outstanding teacher among the engineering faculties at both Texas A&M University (1986 Tenneco Award) and the University of Colorado (1992 Hutchinson Award).

Steve was originally drawn to environmental engineering and science because of his love of the outdoors. He is an avid fly fisherman and hiker. An unapologetic nerd, his love affair with computing began when he was first introduced to Fortran programming as an undergraduate in 1966. Today, he feels truly blessed to be able to meld his love of mathematics, science, and computing with his passion for the natural environment. In addition, he gets the bonus of sharing it with others through his teaching and writing!

Beyond his professional interests, he enjoys art, music (especially classical music, jazz, and bluegrass), and reading history. Despite unfounded rumors to the contrary, he never has, and never will, voluntarily bungee jump or sky dive.

If you would like to contact Steve, or learn more about him, visit his home page at <http://engineering.tufts.edu/cee/people/chapra/> or e-mail him at steven.chapra@tufts.edu.

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PREFACE

This book is designed to support a one-semester course in numerical methods. It has been written for students who want to learn and apply numerical methods in order to solve problems in engineering and science. As such, the methods are motivated by problems rather than by mathematics. That said, sufficient theory is provided so that students come away with insight into the techniques and their shortcomings.

MATLAB[®] provides a great environment for such a course. Although other environments (e.g., Excel/VBA, Mathcad) or languages (e.g., Fortran 90, C++) could have been chosen, MATLAB presently offers a nice combination of handy programming features with powerful built-in numerical capabilities. On the one hand, its M-file programming environment allows students to implement moderately complicated algorithms in a structured and coherent fashion. On the other hand, its built-in, numerical capabilities empower students to solve more difficult problems without trying to “reinvent the wheel.”

The basic content, organization, and pedagogy of the second edition are essentially preserved in the third edition. In particular, the conversational writing style is intentionally maintained in order to make the book easier to read. This book tries to speak directly to the reader and is designed in part to be a tool for self-teaching.

That said, this edition differs from the past edition in three major ways: (1) two new chapters, (2) several new sections, and (3) revised homework problems.

- 1. New Chapters.** As shown in Fig. P.1, I have developed two new chapters for this edition. Their inclusion was primarily motivated by my classroom experience. That is, they are included because they work well in the undergraduate numerical methods course I teach at Tufts. The students in that class typically represent all areas of engineering and range from sophomores to seniors with the majority at the junior level. In addition, we typically draw a few math and science majors. The two new chapters are:
 - **Eigenvalues.** When I first developed this book, I considered that eigenvalues might be deemed an “advanced” topic. I therefore presented the material on this topic at the end of the semester and covered it in the book as an appendix. This sequencing had the ancillary advantage that the subject could be partly motivated by the role of eigenvalues in the solution of linear systems of ODEs. In recent years, I have begun

PART ONE Modeling, Computers, and Error Analysis	PART TWO Roots and Optimization	PART THREE Linear Systems	PART FOUR Curve Fitting	PART FIVE Integration and Differentiation	PART SIX Ordinary Differential Equations
CHAPTER 1 Mathematical Modeling, Numerical Methods, and Problem Solving	CHAPTER 5 Roots: Bracketing Methods	CHAPTER 8 Linear Algebraic Equations and Matrices	CHAPTER 14 Linear Regression	CHAPTER 19 Numerical Integration Formulas	CHAPTER 22 Initial-Value Problems
CHAPTER 2 MATLAB Fundamentals	CHAPTER 6 Roots: Open Methods	CHAPTER 9 Gauss Elimination	CHAPTER 15 General Linear Least-Squares and Nonlinear Regression	CHAPTER 20 Numerical Integration of Functions	CHAPTER 23 Adaptive Methods and Stiff Systems
CHAPTER 3 Programming with MATLAB	CHAPTER 7 Optimization	CHAPTER 10 LU Factorization	CHAPTER 16 Fourier Analysis	CHAPTER 21 Numerical Differentiation	CHAPTER 24 Boundary-Value Problems
CHAPTER 4 Roundoff and Truncation Errors	CHAPTER 11 Matrix Inverse and Condition	CHAPTER 17 Polynomial Interpolation	CHAPTER 18 Splines and Piecewise Interpolation		
	CHAPTER 12 Iterative Methods	CHAPTER 13 Eigenvalues			

FIGURE P.1

An outline of this edition. The shaded areas represent new material. In addition, several of the original chapters have been supplemented with new topics.

to move this material up to what I consider to be its more natural mathematical position at the end of the section on linear algebraic equations. By stressing applications (in particular, the use of eigenvalues to study vibrations), I have found that students respond very positively to the subject in this position. In addition, it allows me to return to the topic in subsequent chapters which serves to enhance the students' appreciation of the topic.

- **Fourier Analysis.** In past years, if time permitted, I also usually presented a lecture at the end of the semester on Fourier analysis. Over the past two years, I have begun presenting this material at its more natural position just after the topic of linear least squares. I motivate the subject matter by using the linear least-squares approach to fit sinusoids to data. Then, by stressing applications (again vibrations), I have found that the students readily absorb the topic and appreciate its value in engineering and science.

It should be noted that both chapters are written in a modular fashion and could be skipped without detriment to the course's pedagogical arc. Therefore, if you choose, you can either omit them from your course or perhaps move them to the end of the semester. In any event, I would not have included them in the current edition if they did not represent an enhancement within my current experience in the classroom. In particular, based on my teaching evaluations, I find that the stronger, more motivated students actually see these topics as highlights. This is particularly true because MATLAB greatly facilitates their application and interpretation.

2. **New Content.** Beyond the new chapters, I have included new and enhanced sections on a number of topics. The primary additions include sections on animation (Chap. 3), Brent's method for root location (Chap. 6), *LU* factorization with pivoting (Chap. 8), *random numbers* and *Monte Carlo simulation* (Chap. 14), *adaptive quadrature* (Chap. 20), and *event* termination of ODEs (Chap. 23).
3. **New Homework Problems.** Most of the end-of-chapter problems have been modified, and a variety of new problems have been added. In particular, an effort has been made to include several new problems for each chapter that are more challenging and difficult than the problems in the previous edition.

Aside from the new material and problems, the third edition is very similar to the second. In particular, I have endeavored to maintain most of the features contributing to its pedagogical effectiveness including extensive use of worked examples and engineering and scientific applications. As with the previous edition, I have made a concerted effort to make this book as "student-friendly" as possible. Thus, I've tried to keep my explanations straightforward and practical.

Although my primary intent is to empower students by providing them with a sound introduction to numerical problem solving, I have the ancillary objective of making this introduction exciting and pleasurable. I believe that motivated students who enjoy engineering and science, problem solving, mathematics—and yes—programming, will ultimately make better professionals. If my book fosters enthusiasm and appreciation for these subjects, I will consider the effort a success.

Acknowledgments. Several members of the McGraw-Hill team have contributed to this project. Special thanks are due to Lorraine Buczek, and Bill Stenquist, and Melissa Leick for their encouragement, support, and direction. Ruma Khurana of MPS Limited, a Macmillan Company also did an outstanding job in the book's final production phase. Last, but not least, Beatrice Sussman once again demonstrated why she is the best copyeditor in the business.

During the course of this project, the folks at The MathWorks, Inc., have truly demonstrated their overall excellence as well as their strong commitment to engineering and science education. In particular, Courtney Esposito and Naomi Fernandes of The MathWorks, Inc., Book Program have been especially helpful.

The generosity of the Berger family, and in particular Fred Berger, has provided me with the opportunity to work on creative projects such as this book dealing with computing and engineering. In addition, my colleagues in the School of Engineering at Tufts, notably Masoud Sanayei, Lew Edgers, Vince Manno, Luis Dorfmann, Rob White, Linda Abriola, and Laurie Baise, have been very supportive and helpful.

Significant suggestions were also given by a number of colleagues. In particular, Dave Clough (University of Colorado–Boulder), and Mike Gustafson (Duke University) provided valuable ideas and suggestions. In addition, a number of reviewers provided useful feedback and advice including Karen Dow Ambtman (University of Alberta), Jalal Behzadi (Shahid Chamran University), Eric Cochran (Iowa State University), Frederic Gibou (University of California at Santa Barbara), Jane Grande-Allen (Rice University), Raphael Haftka (University of Florida), Scott Hendricks (Virginia Tech University), Ming Huang (University of San Diego), Oleg Igoshin (Rice University), David Jack (Baylor University), Clare McCabe (Vanderbilt University), Eckart Meiburg (University of California at Santa Barbara), Luis Ricardez (University of Waterloo), James Rottman (University of California, San Diego), Bingjing Su (University of Cincinnati), Chin-An Tan (Wayne State University), Joseph Tipton (The University of Evansville), Marion W. Vance (Arizona State University), Jonathan Vande Geest (University of Arizona), and Leah J. Walker (Arkansas State University).

It should be stressed that although I received useful advice from the aforementioned individuals, I am responsible for any inaccuracies or mistakes you may find in this book. Please contact me via e-mail if you should detect any errors.

Finally, I want to thank my family, and in particular my wife, Cynthia, for the love, patience, and support they have provided through the time I've spent on this project.

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PEDAGOGICAL TOOLS

Theory Presented as It Informs Key Concepts. The text is intended for Numerical Methods users, not developers. Therefore, theory is not included for “theory’s sake,” for example no proofs. Theory is included as it informs key concepts such as the Taylor series, convergence, condition, etc. Hence, the student is shown how the theory connects with practical issues in problem solving.

Introductory MATLAB Material. The text includes two introductory chapters on how to use MATLAB. Chapter 2 shows students how to perform computations and create graphs in MATLAB’s standard command mode. Chapter 3 provides a primer on developing numerical programs via MATLAB M-file functions. Thus, the text provides students with the means to develop their own numerical algorithms as well as to tap into MATLAB’s powerful built-in routines.

Algorithms Presented Using MATLAB M-files. Instead of using pseudocode, this book presents algorithms as well-structured MATLAB M-files. Aside from being useful computer programs, these provide students with models for their own M-files that they will develop as homework exercises.

Worked Examples and Case Studies. Extensive worked examples are laid out in detail so that students can clearly follow the steps in each numerical computation. The case studies consist of engineering and science applications which are more complex and richer than the worked examples. They are placed at the ends of selected chapters with the intention of (1) illustrating the nuances of the methods, and (2) showing more realistically how the methods along with MATLAB are applied for problem solving.

Problem Sets. The text includes a wide variety of problems. Many are drawn from engineering and scientific disciplines. Others are used to illustrate numerical techniques and theoretical concepts. Problems include those that can be solved with a pocket calculator as well as others that require computer solution with MATLAB.

Useful Appendices and Indexes. Appendix A contains MATLAB commands, and Appendix B contains M-file functions.

Textbook Website. A text-specific website is available at www.mhhe.com/chapra. Resources include the text images in PowerPoint, M-files, and additional MATLAB resources.

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