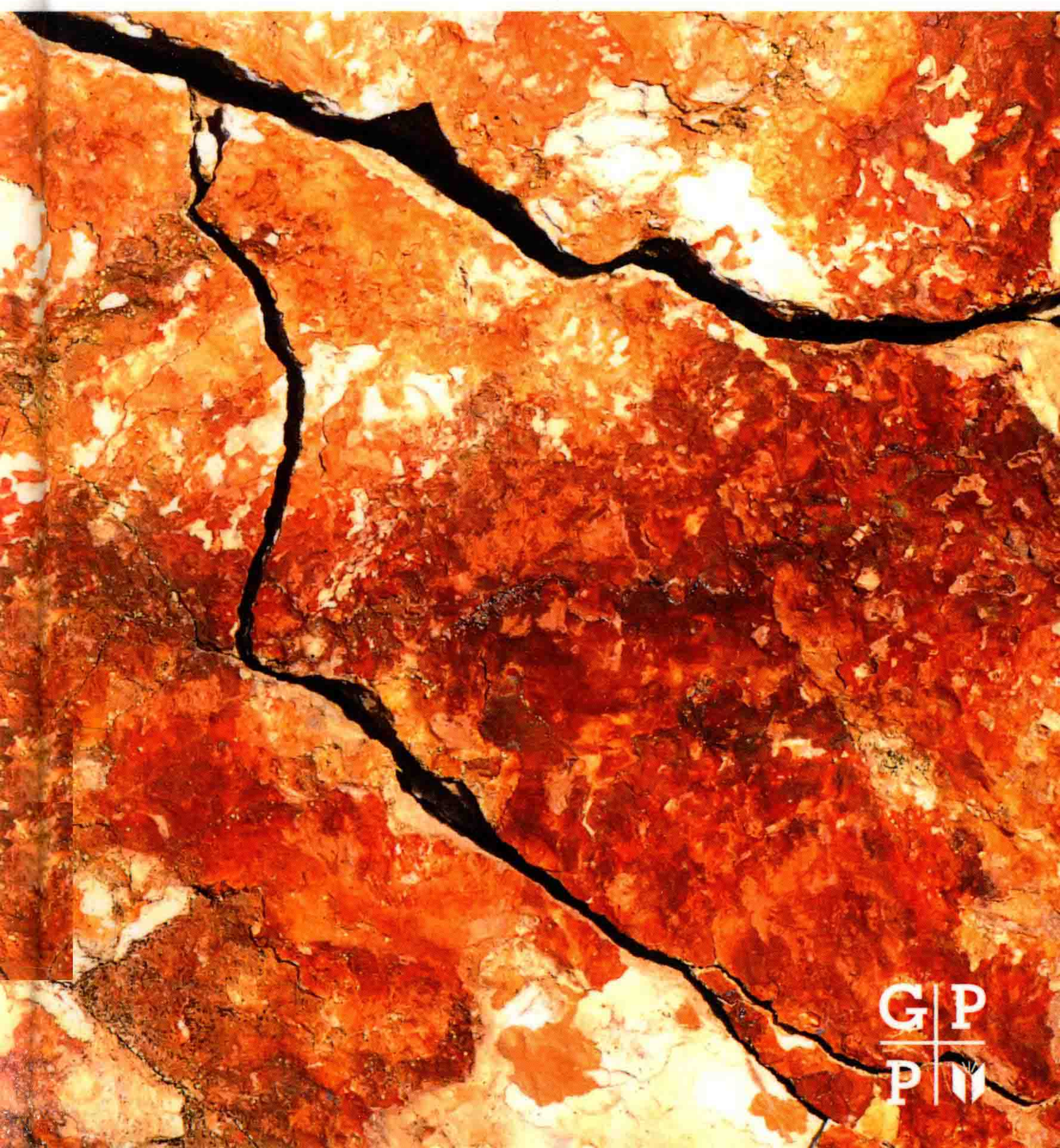
Ching H. Yew and Xiaowei Weng

Mechanics of Hydraulic Fracturing

— Second Edition —



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Second Edition

by

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Mechanics of Hydraulic Fracturing

Preface to the First Edition

This book is intended as a reference book for research engineers and advanced graduate students in petroleum or mechanical engineering. For more than 40 years, hydraulic fracturing has been employed to enhance the production of oil and gas from underground reservoirs. Hydraulic fracturing is a complex operation in which the fluid is pumped at a high pressure into a selected section of the wellbore. The high pressure creates a fracture from the wellbore extending into the rock formation containing oil or gas. One of the important features needed in fracture design is the ability to predict the geometry and the characteristics of the hydraulically induced fracture. Many fracture simulators have been developed for this purpose. This book discusses the underlying mechanics of creating a fracture from the wellbore and the propagation of hydraulic fracture in the reservoir. The propagation of hydraulic fractures in a reservoir at great depth is a complicated phenomenon. Due to limitations in test facilities and lack of a scale law, it is difficult to simulate the propagation of hydraulic fractures in a laboratory specimen. Unfortunately, the characteristics and geometry of a hydraulic fracture at a great depth are verifiable only at great expense. The reliability of a fracture model is therefore dependent on the soundness of its underlying mechanics. It is this author's opinion that, if the underlying mechanics in the simulator are correct, the prediction should not be far from reality.

The book is divided into three parts. The first part, Chapters One, Two and Three, concerns the development of fracture simulators for vertical wellbores. Important contributions from many authors are reviewed in these chapters. The major presentation focuses on the 3D fracture model developed at the University of Texas at Austin. The second part, Chapters Four, Five, and Six, concerns the initiation and propagation of a hydraulic fracture from deviated or horizontal wellbores. Development of directional perforation, link up of mini-fractures from perforated holes and turning of hydraulic fractures are presented here. The third part of the book, Chapter Seven, reviews the result from important experiments conducted in laboratories and in the field. Effort has been made to include a list of comprehensive literature citations in each chapter. However, it is impractical to list all available literature. I apologize sincerely for any omissions.

I am fortunate for having the opportunity to work with a group of talented graduate students, Drs. I. S. Ashour, H. R. Gu, M. G. Hsu, Y. Li, G. F. Liu, S. Ouyang, X. W. Weng, and X. C. Zhang. In fact, most of the materials presented in this book are collections from our joint publications and from their theses and dissertations. I wish to acknowledge the support and guidance of my friends, Drs. G. R. Coulter, W. C. Maurer, D. E. Nierode, C. M. Pearson, T. K. Perkins, R. W. Pittman, A. W. El Rabaa, J. H. Schmidt, and J. Shlyapobersky in the oil and gas industries. It was Dr. Nierode who introduced me to hydraulic fracturing when I was a summer visiting faculty

member at Exxon Production Research Company in 1979. Thanks also to my colleagues, Professors G. F. Carey, A. D. Hill, and R. S. Schechter. It was a pleasure working with these gentlemen in the Stimulation, Logging, and Formation Damage Research Program in the Department of Petroleum Engineering at the University of Texas at Austin.

As mentioned at the beginning, the book is intended as a reference book and not as a text. Thus, the description of phenomena and derivation of equations may not be in depth or in detail as the reader may wish. However, if the reader could obtain a clear picture and understanding of the underlying mechanics of hydraulic fracturing, I would consider the book a success. It is my sincere hope that this book may inspire further research and development into this fascinating subject.

Ching H. Yew

Preface to the Second Edition

The technology of hydraulic fracturing has advanced rapidly during the recent two decades. Although the principle and mechanics of fracturing remain unchanged, the advancement of new knowledge and technology calls for an expansion and revision of the book.

In this new edition, the content and text from the first edition remain mostly unchanged except some editing. In order to incorporate the new development and practice into this revised edition, we added a chapter on Fracture Propagation in Naturally Fractured Formation (Chapter Seven), and a separate chapter on Stress Shadow (Chapter Eight). The section on pseudo 3D simulators in Chapter Three is modified to include new development. And, new sections on the behavior of hydraulic fractures from a horizontal well, and on recent experimental studies are added to Chapters Six and Nine. In these added chapters and sections, the behavior of hydraulic fractures from a horizontal well in unconventional ultra-low permeability shale reservoirs are presented and discussed. The discussions are focused on new development and concept on fracture behavior and on topics related to fracture propagation and well stimulations.

We wish to thank Ms. Katie Hammon of Elsevier for contacting us on taking up the task of revising, and for her assistance during the course of preparing the manuscript. We wish also to express our sincere thanks to our friends and colleagues in industry and in academy for their support and encouragement. Special thanks go to Hongren Gu, Olga Kresse, Charles Cohen and Ruiting Wu whose work had contributed much of the materials in the two new chapters, and to Schlumberger for permission to publish these materials.

Ching H. Yew
Xiaowei Weng

Contents

Fracturing of a wellbore and 2D fracture models

1

Introduction

The hydraulic fracturing process has been employed to enhance the production of oil and gas from underground reservoirs for many decades. In the process, the frac-fluid is pumped at a high pressure into a selected section of wellbore. This fluid pressure creates one or more fractures extending into the rock medium that contains oil or gas. Since the fracturing operation is conducted at a great depth, the minimum compressive in situ stress is typically in horizontal direction, the hydraulically induced fracture is a vertical fracture.

The dimension and propagation characteristics of a hydraulic fracture are important information in design of fracturing operations. Knowing the properties of reservoir rock, frac-fluid, and the magnitude and direction of in situ stresses, one seeks an accurate prediction of the dimension (opening width, length, and height) of the hydraulically induced fracture for a given pumping rate and time. Many fracture models have been developed for this purpose. The initiation of a hydraulic fracture from a vertical wellbore and two-dimensional fracture models are discussed in the following sections.

Fracturing of a wellbore

Consider an uncased vertical wellbore (or an open hole) under the action of horizontal in situ stresses σ_{min} and σ_{max} as shown in Fig. 1-1.

Assume that the rock is an elastic medium and has a tensile failure stress σ_{T}. The breakdown pressure p_{b} for introducing a fracture at the surface of borehole can be calculated by applying elasticity theory [1] to give

$$p_{\mathrm{b}} = 3\sigma_{\mathrm{min}} - \sigma_{\mathrm{max}} + \sigma_{\mathrm{T}} \tag{1-1}$$

where σ_{min} is the minimum in situ stress, σ_{max} the maximum in situ stress, and σ_{T} the tensile failure stress of the rock.

The hydraulically induced fracture is a vertical fracture and the fracture plane is perpendicular to the minimum horizontal in situ stress σ_{min} as shown. Note that the above equation is independent of hole size and the elastic moduli of rock medium. For a wellbore section at a depth of 10,000 ft, the typical values for the horizontal minimum and maximum in situ stresses are in the order of 5000-7000 psi, respectively. The rock has a tensile failure stress on the order of 500-1500 psi. Equation (1-1) clearly shows that the rock tensile failure stress σ_{T} has a small effect on the magnitude

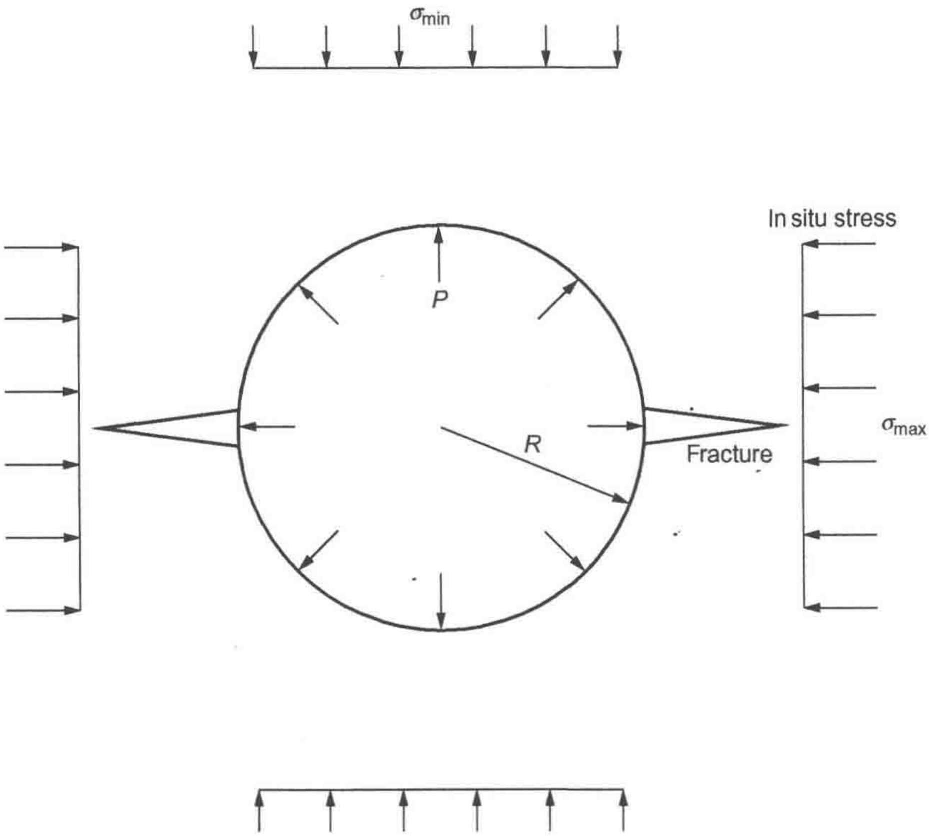

Figure 1-1 Horizontal section of a vertical wellbore under the action of in situ stresses and borehole pressure.

of breakdown pressure, and the hole breakdown pressure is mainly to overcome the compressive circumferential hoop stress produced by in situ stresses.

It is clear that the applied wellbore pressure first balances the reservoir pressure (or pore pressure), then overcomes the compressive circumferential hoop stress, causing a tensile stress on the hole surface. A fracture is initiated when this surface stress reaches the tensile failure stress of the rock medium.

The hydraulically induced fracture propagates from the wellbore into reservoir as pumping continues. A typical downhole pressure record (i.e., the pressure measured inside the hole near the opening of hydraulic fracture) is sketched in Fig. 1-2.

The hydraulically induced fracture propagates into the reservoir as pumping continues, and at the same time the frac-fluid leaks off from the fracture surface into the surrounding rock medium. It is important to observe that the opening of the fracture is maintained by the net pressure (fluid pressure minus the minimum in situ stress), while the fluid leak-off rate from the fracture surface is caused by the differential between fluid pressure and reservoir pressure.

Referring to Fig. 1-2 again, the maximum pressure is the initial breakdown pressure p_b. The pressure drops, but not always in the field, when a fracture is initiated at the borehole surface. The near constant portion of the pressure curve is the propagation

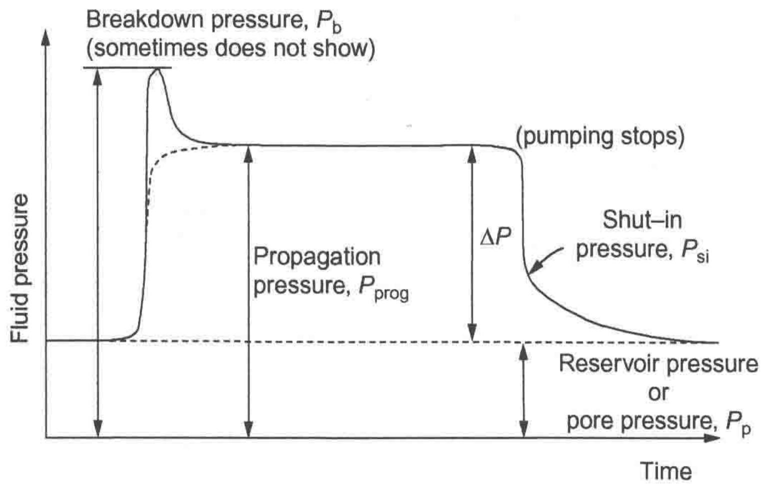

Figure 1-2 A down-hole pressure record.

pressure p_{prog}. This is the pressure that causes the propagation of hydraulic fracture into the reservoir. When pumping stops, the pressure drops instantly to a lower value, due to the vanishing frictional pressure loss in the pipe, perforation entrance and near-borehole area, and then continues to decrease slowly to the reservoir pressure due to fluid leaking off from the fracture and borehole as shown in the figure. The transition point is called the shut-in pressure p_{si} (or the instantaneous shut-in pressure, ISIP). However, fluid continues to leak off from fracture surface and the fracture opening width continues to decrease. The fluid pressure inside the fracture eventually reaches to an equilibrium with the minimum in situ stress and at this point the hydraulic fracture closes. The fracture closure pressure, which can be determined from the pressure decline analysis, is taken as a measure of the minimum in situ stress. Although the ISIP is somewhat higher than the fracture closure pressure, the ISIP can be easily identified from the measured pressure-time curve. Field engineers often use ISIP to estimate the magnitude of the minimum horizontal in situ stress. Unfortunately, the situation is somewhat more complicated in field conditions. The underlying control factors for this pressure drop are discussed by McLennan and Roegiers [2].

Equation (1-1) is derived from the assumption that the rock is an elastic medium. However, most reservoir rocks are porous medium through which fluid can flow. The pressure difference between fracture and reservoir causes the fluid to flow from the fracture into reservoir, that is, fluid leak off. The experimental study carried out by Haimson and Fairhurst [3,4] and Medlin and Masse [5] have demonstrated that the porosity and pore fluid have an influence on the hole breakdown pressure. By applying the poroelasticity theory, Schmitt and Zoback [6] have modified Eq. (1-1) to the form as follows:

For a formation impermeable to frac-fluid,

$$p_b = 3\sigma_{min} - \sigma_{max} + \sigma_T - \beta p_b \tag{1-2}$$

For a formation permeable to frac-fluid,

$$p_b = \frac{3\sigma_{min} - \sigma_{max} + \sigma_T - \alpha p_p\left(\frac{1-2v}{1-v}\right)}{1 + \beta - \alpha\left(\frac{1-2v}{1-v}\right)} \tag{1-3}$$

where p_p is the pore pressure; β the pore pressure factor in tensile failure criterion, $1 \geq \beta \geq 0$; v the Poisson's ratio of dry rock; and $\alpha = 1 - \frac{\text{bulk modulus of dry rock}}{\text{bulk modulus of skeleton material}}$, $1 \geq \alpha > 0$. Parameter α is known as the Biot's poroelastic parameter which approaches the upper limit of 1.0 for a compliant rock and less for a stiff low-porosity rock. Schmitt and Zoback [6] have demonstrated that Eqs. (1-2) and (1-3) give a better agreement with experimental data.

The above equations clearly show that the effect of rock porosity and pore pressure is to lower the hole breakdown pressure. They also suggest that the breakdown pressure of the hole is dependent on the filtercake-forming capability of the fluid.

Most wellbores that need fracturing are cased wellbores. To fracture a cased wellbore, the wellbore is first perforated with shaped charges to form a series of perforated holes spiraling along the wellbore surface as shown in Fig. 1-3.

The perforations are typically made at spacings of 4-6 shots per foot and at a phase angle of 60° or 120° as shown in the figure. When the wellbore is pressurized, the perforated holes in (or near) the direction of maximum horizontal in situ stress (σ_{max}) will be fractured first.

The breakdown pressure can be calculated from Eq. (1-1) by replacing the maximum horizontal in situ stress σ_{max} with the vertical stress σ_{Vert}. The mini-fractures

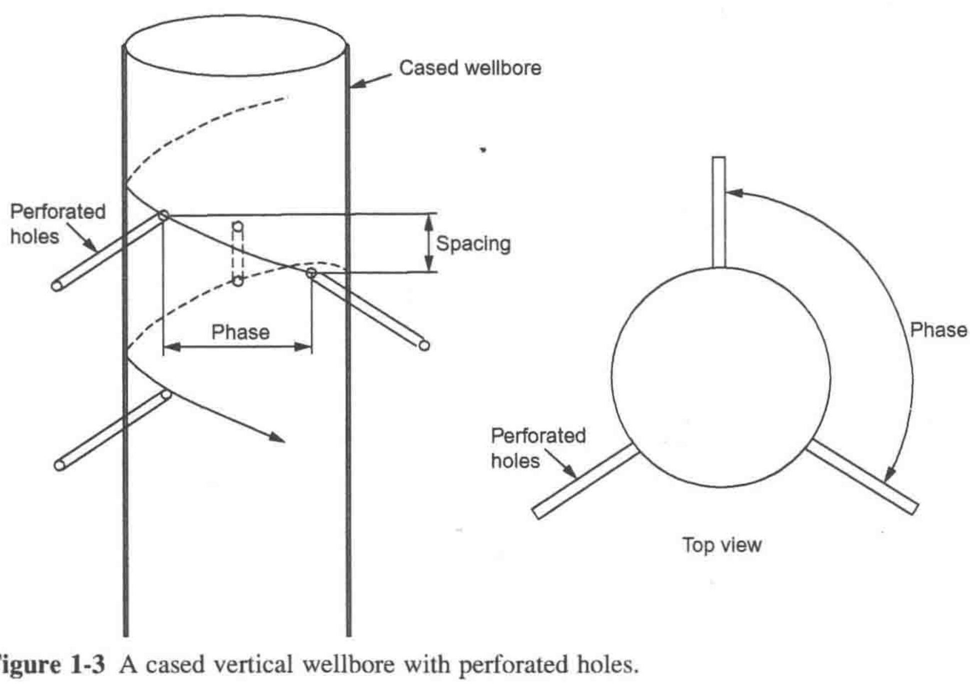

Figure 1-3 A cased vertical wellbore with perforated holes.

initiated from the perforations may or may not link up to form a large hydraulic fracture perpendicular to the minimum in situ stress along the direction of the wellbore axis. In practice, it is desirable for the mini-fractures to link up forming a large fracture along the wellbore. The linking up of mini-fractures will be discussed in Chapter Five.

Constant height fracture models

Since the wellbore is often fractured at a great depth (>5000 ft) where the minimum in situ stress is in the horizontal plane, the fracture is a vertical fracture whose plane is perpendicular to the minimum in situ stress. There are two factors that control the vertical growth of a hydraulic fracture. They are (1) the contrast in material properties, and (2) the contrast in vertical distribution of in situ stress. Warpinski and co-authors [7–9] have made detailed studies of these factors both in the laboratory and in field. They have found that the contrast in in situ stress is the predominant factor that influences the height growth of hydraulic fractures and that the contrast in material properties, unless very large (five times or larger), is not a dominant factor in fracture containment. Laboratory experiments have demonstrated that an in situ stress contrast of 400 psi is sufficient to contain the vertical growth of a hydraulic fracture.

Since the plane of hydraulic fracture is perpendicular to the minimum horizontal in situ stress, the growth of fracture height is controlled by the vertical distribution of the horizontal minimum in situ stress. When the contrast of stresses between adjacent stress zones is large, the growth of fracture height is contained as shown in Fig. 1-4.

There are two basic constant height models: the Khristianovic-Geertsma-de Klerk (KGD) model [10], and the Perkins-Kern-Nordgren (PKN) model [11]. Most of the

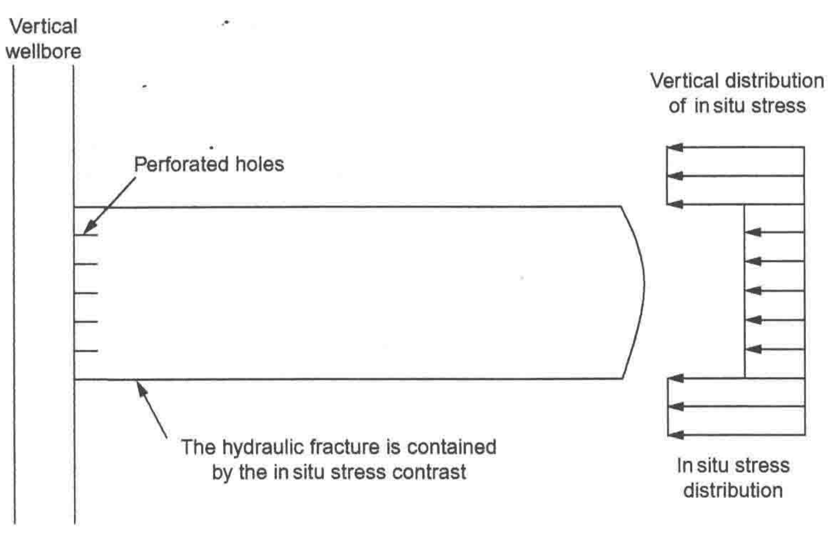

Figure 1-4 The vertical growth of hydraulic fracture is contained by the contrast in in situ stresses.

early hydraulic fractures were designed by applying one of these models. The underlying mechanics in these two models, however, differs significantly.

The KGD model

One wing of the KGD fracture is shown in Fig. 1-5. In addition to the constant height assumption, two other assumptions are (1) the fracture is at a plane strain condition in the horizontal plane; and (2) the fracture tip is a cusp-shaped tip as proposed by Barenblatt [12]. This assumption of a cusp-shaped tip removes the stress singularity at the fracture tip that would otherwise be predicted by the elasticity analysis.

Following Geertsma-de Klerk, the fracture is approximated as a channel of opening width w. The pressure distribution for the flow of a viscous fluid (Newtonian fluid) inside the fracture can be written as

$$p_{\mathrm{w}} - p = \frac{12\mu QL}{h}\int_{f_{\mathrm{Lw}}}^{f_{\mathrm{L}}} \frac{df_{\mathrm{L}}}{w^3} \tag{1-4}$$

where $f_{\mathrm{L}} = x/L$, $f_{\mathrm{Lw}} = r_{\mathrm{w}}/L$, h is the fracture height, L the total length of the fracture, p the local fluid pressure, p_{w} the fluid pressure at wellbore, Q the fluid injection rate into one wing of the fracture, r_{w} the wellbore radius, w the local fracture width, and μ the frac-fluid viscosity.

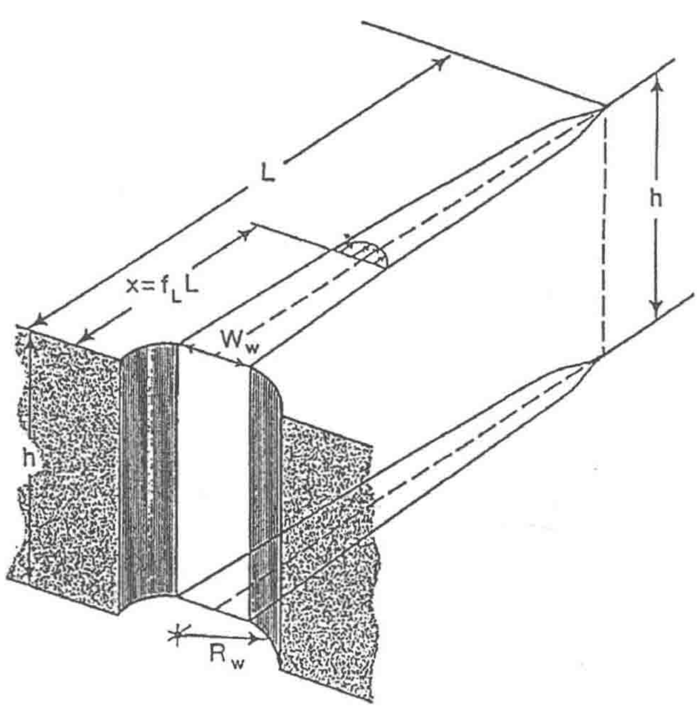

Figure 1-5 The KGD constant height fracture model.

The above equation has two unknowns, p and w. England and Green's solution [13] for a plane fracture in an infinite elastic medium provides another relationship between p and w as

$$w = \frac{4(1-\nu)L}{\pi G}\left[\int_{f_L}^{1}\frac{f_2 \mathrm{d}f_2}{\sqrt{f_2^2 - f_L^2}}\int_0^{f_2}\frac{p(f_1)\mathrm{d}f_1}{\sqrt{f_2^2 - f_1^2}} - \frac{\pi}{2}\sigma_{\min}\sqrt{1-f_L^2}\right] \tag{1-5}$$

where G and ν are the shear modulus and Poisson's ratio of the rock, respectively, f_1 and f_2 the fraction of fracture extent ($=x/L$), and $\sigma_{\min}$ the minimum in situ stress.

The time history of fracture width $w(t)$ and fluid pressure $p(t)$ can be obtained by solving Eqs (1-4) and (1-5) with proper boundary conditions. The following smooth fracture tip condition proposed by Barenblatt [12] is used by Geertsma and de Klerk:

$$\left(\frac{\mathrm{d}w}{\mathrm{d}f_L}\right)_{f_L=1} = 0 \tag{1-6}$$

The same condition is also used by Khristianovic and Zheltov [14] in their study of hydraulic fracturing. Note that Eq. (1-5) is derived by applying the elasticity theory. The proper boundary condition at the fracture tip should be $f_L = 1$, $w = 0$, not as in Eq. (1-6). Therefore, there is a mathematical inconsistency at the fracture tip. Geertsma and de Klerk argue that since the tip is a local singularity of the fracture, its effect on the overall fracture geometry should be small and their solution is a good approximation for the fracture opening width and the overall fracture length. We shall return to this fracture tip problem in the discussion of circular fractures next in this chapter.

By assuming that the dry zone in front of fracture tip is small and that the shape of wet portion in the fracture can be approximated by an ellipse, the following approximate solutions (no fluid leak off) are obtained by Geertsma and de Klerk:

Fracture length

$$L = 0.48\left[\frac{8GQ^3}{(1-\nu)\mu}\right]^{1/6} t^{2/3} \tag{1-7}$$

Maximum fracture opening width

$$w_o = 1.32\left[\frac{8(1-\nu)Q^3\mu}{G}\right]^{1/6} t^{1/3} \tag{1-8}$$

Wellbore pressure

$$p_w = \sigma_{\min} + 0.96\left[\frac{2G^3 Q\mu}{(1-\nu)^3 L^2}\right]^{1/4} \tag{1-9}$$

One sees that the fracture opening width increases in proportional to $t^{1/3}$ and the wellbore pressure decreases with the increase of fracture length and approaches to in situ stress for a large value of L. Since the fracture is assumed to be at a plane strain condition in the horizontal plane, the KGD model is best suited for fractures whose length/height ratio is near unity or less.

The PKN model

Figure 1-6 is a sketch of a PKN fracture. In addition to assuming a constant fracture height, the other two assumptions are (1) the fracture is at a state of plane strain in the vertical plane and the vertical fracture cross-section is elliptical and (2) the fracture toughness has no effect on the fracture geometry, that is, the K_{IC} of the rock medium is assumed to be zero.

Following Nordgren [11], the continuity equation for flow of an incompressible fluid inside the fracture can be written as

$$\frac{\partial q}{\partial x} + q_{\ell} + \frac{\partial A}{\partial t} = 0 \tag{1-10}$$

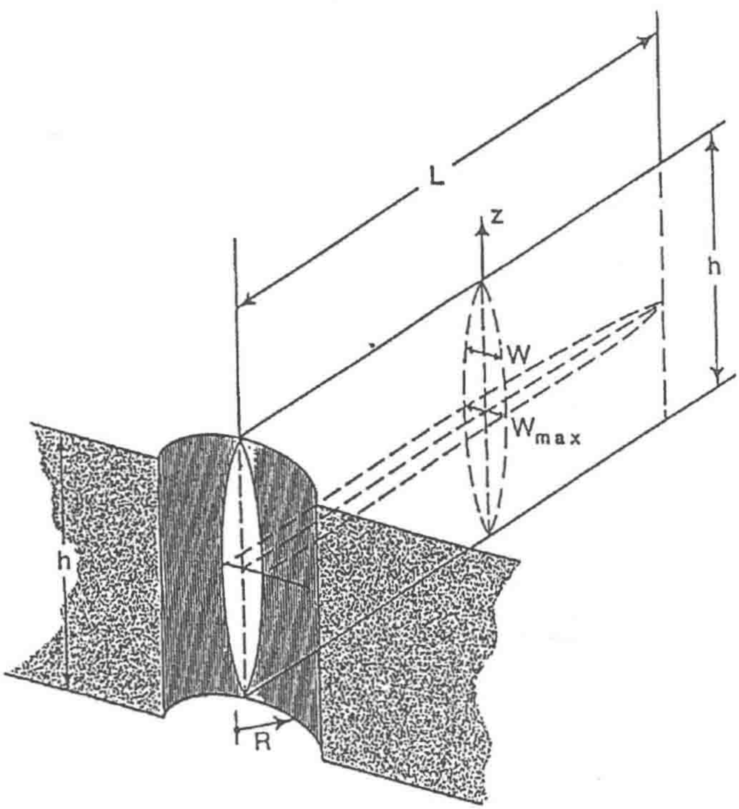

Figure 1-6 The PKN constant height fracture model.