THIRD EDITION

Student Solutions Manual for

MATHEMATICAL METHODS FOR PHYSICS AND ENGINEERING

K. F. RILEY
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CAMBRIDGE

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Mathematical Methods for Physics and Engineering

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Student Solutions Manual for *Mathematical Methods for Physics and Engineering*, third edition

Mathematical Methods for Physics and Engineering, third edition, is a highly acclaimed undergraduate textbook that teaches all the mathematics needed for an undergraduate course in any of the physical sciences. As well as lucid descriptions of the topics and many worked examples, it contains over 800 exercises. New stand-alone chapters give a systematic account of the 'special functions' of physical science, cover an extended range of practical applications of complex variables, and give an introduction to quantum operators.

This solutions manual accompanies the third edition of *Mathematical Methods for Physics and Engineering*. It contains complete worked solutions to over 400 exercises in the main textbook, the odd-numbered exercises that are provided with hints and answers. The even-numbered exercises have no hints, answers or worked solutions and are intended for unaided homework problems; full solutions are available to instructors on a password-protected website, www.cambridge.org/9780521679718.

KEN RILEY read mathematics at the University of Cambridge and proceeded to a Ph.D. there in theoretical and experimental nuclear physics. He became a research associate in elementary particle physics at Brookhaven, and then, having taken up a lectureship at the Cavendish Laboratory, Cambridge, continued this research at the Rutherford Laboratory and Stanford; in particular he was involved in the experimental discovery of a number of the early baryonic resonances. As well as having been Senior Tutor at Clare College, where he has taught physics and mathematics for over 40 years, he has served on many committees concerned with the teaching and examining of these subjects at all levels of tertiary and undergraduate education. He is also one of the authors of *200 Puzzling Physics Problems*.

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Preface

The second edition of *Mathematical Methods for Physics and Engineering* carried more than twice as many exercises, based on its various chapters, as did the first. In the Preface we discussed the general question of how such exercises should be treated but, in the end, decided to provide hints and outline answers to all problems, as in the first edition. This decision was an uneasy one as, on the one hand, it did not allow the exercises to be set as totally unaided homework that could be used for assessment purposes, but, on the other, it did not give a full explanation of how to tackle a problem when a student needed explicit guidance or a model answer.

In order to allow both of these educationally desirable goals to be achieved, we have, in the third edition, completely changed the way this matter is handled. All of the exercises from the second edition, plus a number of additional ones testing the newly added material, have been included in penultimate subsections of the appropriate, sometimes reorganised, chapters. Hints and outline answers are given, as previously, in the final subsections, *but only to the odd-numbered exercises*. This leaves all even-numbered exercises free to be set as unaided homework, as described below.

For the four hundred plus *odd-numbered* exercises, complete solutions are available, to both students and their teachers, in the form of *this* manual; these are in addition to the hints and outline answers given in the main text. For each exercise, the original question is reproduced and then followed by a fully worked solution. For those original exercises that make internal reference to the text or to other (even-numbered) exercises not included in this solutions manual, the questions have been reworded, usually by including additional information, so that the questions can stand alone. Some further minor rewording has been included to improve the page layout.

In many cases the solution given is even fuller than one that might be expected

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of a good student who has understood the material. This is because we have aimed to make the solutions instructional as well as utilitarian. To this end, we have included comments that are intended to show how the plan for the solution is formulated and have provided the justifications for particular intermediate steps (something not always done, even by the best of students). We have also tried to write each individual substituted formula in the form that best indicates how it was obtained, before simplifying it at the next or a subsequent stage. Where several lines of algebraic manipulation or calculus are needed to obtain a final result, they are normally included in full; this should enable the student to determine whether an incorrect answer is due to a misunderstanding of principles or to a technical error.

The remaining four hundred or so *even-numbered* exercises have no hints or answers (outlined or detailed) available for general access. They can therefore be used by instructors as a basis for setting unaided homework. Full solutions to these exercises, in the same general format as those appearing in this manual (though they may contain references to the main text or to other exercises), are available without charge to accredited teachers as downloadable pdf files on the password-protected website http://www.cambridge.org/9780521679718. Teachers wishing to have access to the website should contact solutions@cambridge.org for registration details.

As noted above, the original questions are reproduced in full, or in a suitably modified stand-alone form, at the start of each exercise. Reference to the main text is not needed provided that standard formulae are known (and a set of tables is available for a few of the statistical and numerical exercises). This means that, although it is not its prime purpose, this manual could be used as a test or quiz book by a student who has learned, or thinks that he or she has learned, the material covered in the main text.

In all new publications, errors and typographical mistakes are virtually unavoidable, and we would be grateful to any reader who brings instances to our attention. Finally, we are extremely grateful to Dave Green for his considerable and continuing advice concerning typesetting in LaTeX.

Ken Riley, Michael Hobson,
Cambridge, 2006

Contents

1

Preliminary algebra

Polynomial equations

> ***1.1*** *It can be shown that the polynomial*
>
> $$g(x) = 4x^3 + 3x^2 - 6x - 1$$
>
> *has turning points at* $x = -1$ *and* $x = \frac{1}{2}$ *and three real roots altogether. Continue an investigation of its properties as follows.*
>
> (a) *Make a table of values of* $g(x)$ *for integer values of* x *between* -2 *and* 2. *Use it and the information given above to draw a graph and so determine the roots of* $g(x) = 0$ *as accurately as possible.*
> (b) *Find one accurate root of* $g(x) = 0$ *by inspection and hence determine precise values for the other two roots.*
> (c) *Show that* $f(x) = 4x^3 + 3x^2 - 6x - k = 0$ *has only one real root unless* $-5 \leq k \leq \frac{7}{4}$.

(a) Straightforward evaluation of $g(x)$ at integer values of x gives the following table:

x	-2	-1	0	1	2
$g(x)$	-9	4	-1	0	31

(b) It is apparent from the table alone that $x = 1$ is an exact root of $g(x) = 0$ and so $g(x)$ can be factorised as $g(x) = (x-1)h(x) = (x-1)(b_2x^2 + b_1x + b_0)$. Equating the coefficients of x^3, x^2, x and the constant term gives $4 = b_2$, $b_1 - b_2 = 3$, $b_0 - b_1 = -6$ and $-b_0 = -1$, respectively, which are consistent if $b_1 = 7$. To find the two remaining roots we set $h(x) = 0$:

$$4x^2 + 7x + 1 = 0.$$

The roots of this quadratic equation are given by the standard formula as

$$\alpha_{1,2} = \frac{-7 \pm \sqrt{49-16}}{8}.$$

(c) When $k = 1$ (i.e. the original equation) the values of $g(x)$ at its turning points, $x = -1$ and $x = \frac{1}{2}$, are 4 and $-\frac{11}{4}$, respectively. Thus $g(x)$ can have up to 4 subtracted from it or up to $\frac{11}{4}$ added to it and still satisfy the condition for three (or, at the limit, two) distinct roots of $g(x) = 0$. It follows that for k outside the range $-5 \le k \le \frac{7}{4}$, $f(x)$ $[= g(x) + 1 - k]$ has only one real root.

1.3 *Investigate the properties of the polynomial equation*

$$f(x) = x^7 + 5x^6 + x^4 - x^3 + x^2 - 2 = 0,$$

by proceeding as follows.

(a) *By writing the fifth-degree polynomial appearing in the expression for* $f'(x)$ *in the form* $7x^5 + 30x^4 + a(x-b)^2 + c$, *show that there is in fact only one positive root of* $f(x) = 0$.
(b) *By evaluating* $f(1)$, $f(0)$ *and* $f(-1)$, *and by inspecting the form of* $f(x)$ *for negative values of* x, *determine what you can about the positions of the real roots of* $f(x) = 0$.

(a) We start by finding the derivative of $f(x)$ and note that, because f contains no linear term, f' can be written as the product of x and a fifth-degree polynomial:

$$\begin{aligned} f(x) &= x^7 + 5x^6 + x^4 - x^3 + x^2 - 2 = 0, \\ f'(x) &= x(7x^5 + 30x^4 + 4x^2 - 3x + 2) \\ &= x[\,7x^5 + 30x^4 + 4(x - \tfrac{3}{8})^2 - 4(\tfrac{3}{8})^2 + 2\,] \\ &= x[\,7x^5 + 30x^4 + 4(x - \tfrac{3}{8})^2 + \tfrac{23}{16}\,]. \end{aligned}$$

Since, for positive x, every term in this last expression is necessarily positive, it follows that $f'(x)$ can have no zeros in the range $0 < x < \infty$. Consequently, $f(x)$ can have no turning points in that range and $f(x) = 0$ can have at most one root in the same range. However, $f(+\infty) = +\infty$ and $f(0) = -2 < 0$ and so $f(x) = 0$ has at least one root in $0 < x < \infty$. Consequently it has exactly one root in the range.

(b) $f(1) = 5$, $f(0) = -2$ and $f(-1) = 5$, and so there is at least one root in each of the ranges $0 < x < 1$ and $-1 < x < 0$.

There is no simple systematic way to examine the form of a general polynomial function for the purpose of determining where its zeros lie, but it is sometimes

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helpful to group terms in the polynomial and determine how the sign of each group depends upon the range in which x lies. Here grouping successive pairs of terms yields some information as follows:

$$x^7 + 5x^6 \text{ is positive for } x > -5,$$
$$x^4 - x^3 \text{ is positive for } x > 1 \quad \text{and} \quad x < 0,$$
$$x^2 - 2 \text{ is positive for } x > \sqrt{2} \quad \text{and} \quad x < -\sqrt{2}.$$

Thus, all three terms are positive in the range(s) common to these, namely $-5 < x < -\sqrt{2}$ and $x > 1$. It follows that $f(x)$ is positive definite in these ranges and there can be no roots of $f(x) = 0$ within them. However, since $f(x)$ is negative for large negative x, there must be at least one root α with $\alpha < -5$.

***1.5** Construct the quadratic equations that have the following pairs of roots:* (a) $-6, -3$; (b) $0, 4$; (c) $2, 2$; (d) $3 + 2i, 3 - 2i$, *where* $i^2 = -1$.

Starting in each case from the 'product of factors' form of the quadratic equation, $(x - \alpha_1)(x - \alpha_2) = 0$, we obtain:

$$\text{(a)} \qquad (x + 6)(x + 3) = x^2 + 9x + 18 = 0;$$
$$\text{(b)} \qquad (x - 0)(x - 4) = x^2 - 4x = 0;$$
$$\text{(c)} \qquad (x - 2)(x - 2) = x^2 - 4x + 4 = 0;$$
$$\begin{aligned}\text{(d)} \quad (x - 3 - 2i)(x - 3 + 2i) &= x^2 + x(-3 - 2i - 3 + 2i) \\ &\quad + (9 - 6i + 6i - 4i^2) \\ &= x^2 - 6x + 13 = 0.\end{aligned}$$

Trigonometric identities

***1.7** Prove that*

$$\cos\frac{\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

by considering

(a) *the sum of the sines of* $\pi/3$ *and* $\pi/6$,
(b) *the sine of the sum of* $\pi/3$ *and* $\pi/4$.

(a) Using

$$\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right),$$

we have

$$\sin\frac{\pi}{3} + \sin\frac{\pi}{6} = 2\sin\frac{\pi}{4}\cos\frac{\pi}{12},$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2} = 2\frac{1}{\sqrt{2}}\cos\frac{\pi}{12},$$

$$\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

(b) Using, successively, the identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B,$$

$$\sin(\pi - \theta) = \sin\theta$$

$$\text{and}\quad \cos(\tfrac{1}{2}\pi - \theta) = \sin\theta,$$

we obtain

$$\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4},$$

$$\sin\frac{7\pi}{12} = \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} + \frac{1}{2}\frac{1}{\sqrt{2}},$$

$$\sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}},$$

$$\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

1.9 *Find the real solutions of*

(a) $3\sin\theta - 4\cos\theta = 2$,
(b) $4\sin\theta + 3\cos\theta = 6$,
(c) $12\sin\theta - 5\cos\theta = -6$.

We use the result that if

$$a\sin\theta + b\cos\theta = k$$

then

$$\theta = \sin^{-1}\left(\frac{k}{K}\right) - \phi,$$

where

$$K^2 = a^2 + b^2 \quad\text{and}\quad \phi = \tan^{-1}\frac{b}{a}.$$

Recalling that the inverse sine yields two values and that the individual signs of a and b have to be taken into account, we have

(a) $k = 2$, $K = \sqrt{3^2 + 4^2} = 5$, $\phi = \tan^{-1}(-4/3)$ and so

$$\theta = \sin^{-1}\tfrac{2}{5} - \tan^{-1}\tfrac{-4}{3} = 1.339 \text{ or } -2.626.$$

(b) $k = 6$, $K = \sqrt{4^2 + 3^2} = 5$. Since $k > K$ there is no solution for a real angle θ.

(c) $k = -6$, $K = \sqrt{12^2 + 5^2} = 13$, $\phi = \tan^{-1}(-5/12)$ and so

$$\theta = \sin^{-1}\tfrac{-6}{13} - \tan^{-1}\tfrac{-5}{12} = -0.0849 \text{ or } -2.267.$$

***1.11** Find all the solutions of*

$$\sin\theta + \sin 4\theta = \sin 2\theta + \sin 3\theta$$

that lie in the range $-\pi < \theta \le \pi$. What is the multiplicity of the solution $\theta = 0$?

Using

$$\sin(A+B) = \sin A\cos B + \cos A\sin B,$$

$$\text{and} \quad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

and recalling that $\cos(-\phi) = \cos(\phi)$, the equation can be written successively as

$$2\sin\frac{5\theta}{2}\cos\left(-\frac{3\theta}{2}\right) = 2\sin\frac{5\theta}{2}\cos\left(-\frac{\theta}{2}\right),$$

$$\sin\frac{5\theta}{2}\left(\cos\frac{3\theta}{2} - \cos\frac{\theta}{2}\right) = 0,$$

$$-2\sin\frac{5\theta}{2}\sin\theta\sin\frac{\theta}{2} = 0.$$

The first factor gives solutions for θ of $-4\pi/5$, $-2\pi/5$, 0, $2\pi/5$ and $4\pi/5$. The second factor gives rise to solutions 0 and π, whilst the only value making the third factor zero is $\theta = 0$. The solution $\theta = 0$ appears in each of the above sets and so has multiplicity 3.

Coordinate geometry

1.13 *Determine the forms of the conic sections described by the following equations:*

(a) $x^2 + y^2 + 6x + 8y = 0$;
(b) $9x^2 - 4y^2 - 54x - 16y + 29 = 0$;
(c) $2x^2 + 2y^2 + 5xy - 4x + y - 6 = 0$;
(d) $x^2 + y^2 + 2xy - 8x + 8y = 0$.

(a) $x^2 + y^2 + 6x + 8y = 0$. The coefficients of x^2 and y^2 are equal and there is no xy term; it follows that this must represent a circle. Rewriting the equation in standard circle form by 'completing the squares' in the terms that involve x and y, each variable treated separately, we obtain

$$(x+3)^2 + (y+4)^2 - (3^2 + 4^2) = 0.$$

The equation is therefore that of a circle of radius $\sqrt{3^2 + 4^2} = 5$ centred on $(-3, -4)$.

(b) $9x^2 - 4y^2 - 54x - 16y + 29 = 0$. This equation contains no xy term and so the centre of the curve will be at $(\,54/(2 \times 9), 16/[2 \times (-4)]\,) = (3, -2)$, and in standardised form the equation is

$$9(x-3)^2 - 4(y+2)^2 + 29 - 81 + 16 = 0,$$

or

$$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{9} = 1.$$

The minus sign between the terms on the LHS implies that this conic section is a hyperbola with asymptotes (the form for large x and y and obtained by ignoring the constant on the RHS) given by $3(x-3) = \pm 2(y+2)$, i.e. lines of slope $\pm\frac{3}{2}$ passing through its 'centre' at $(3, -2)$.

(c) $2x^2 + 2y^2 + 5xy - 4x + y - 6 = 0$. As an xy term is present the equation cannot represent an ellipse or hyperbola in standard form. Whether it represents two straight lines can be most easily investigated by taking the lines in the form $a_i x + b_i y + 1 = 0$, $(i = 1, 2)$ and comparing the product $(a_1 x + b_1 y + 1)(a_2 x + b_2 y + 1)$ with $-\frac{1}{6}(2x^2 + 2y^2 + 5xy - 4x + y - 6)$. The comparison produces five equations which the four constants a_i, b_i, $(i = 1, 2)$ must satisfy:

$$a_1 a_2 = \frac{2}{-6}, \quad b_1 b_2 = \frac{2}{-6}, \quad a_1 + a_2 = \frac{-4}{-6}, \quad b_1 + b_2 = \frac{1}{-6}$$

and

$$a_1 b_2 + b_1 a_2 = \frac{5}{-6}.$$

Combining the first and third equations gives $3a_1^2 - 2a_1 - 1 = 0$ leading to a_1 and a_2 having the values 1 and $-\frac{1}{3}$, in either order. Similarly, combining the second and fourth equations gives $6b_1^2 + b_1 - 2 = 0$ leading to b_1 and b_2 having the values $\frac{1}{2}$ and $-\frac{2}{3}$, again in either order.

Either of the two combinations ($a_1 = -\frac{1}{3}$, $b_1 = -\frac{2}{3}$, $a_2 = 1$, $b_2 = \frac{1}{2}$) and ($a_1 = 1$, $b_1 = \frac{1}{2}$, $a_2 = -\frac{1}{3}$, $b_2 = -\frac{2}{3}$) also satisfies the fifth equation [note that the two alternative pairings do not do so]. That a consistent set can be found shows that the equation does indeed represent a pair of straight lines, $x + 2y - 3 = 0$ and $2x + y + 2 = 0$.

(d) $x^2 + y^2 + 2xy - 8x + 8y = 0$. We note that the first three terms can be written as a perfect square and so the equation can be rewritten as

$$(x + y)^2 = 8(x - y).$$

The two lines given by $x + y = 0$ and $x - y = 0$ are orthogonal and so the equation is of the form $u^2 = 4av$, which, for Cartesian coordinates u, v, represents a parabola passing through the origin, symmetric about the v-axis ($u = 0$) and defined for $v \geq 0$. Thus the original equation is that of a parabola, symmetric about the line $x + y = 0$, passing through the origin and defined in the region $x \geq y$.

Partial fractions

***1.15** Resolve*

$$\text{(a)}\ \frac{2x + 1}{x^2 + 3x - 10}, \qquad \text{(b)}\ \frac{4}{x^2 - 3x}$$

into partial fractions using each of the following three methods:

(i) *Expressing the supposed expansion in a form in which all terms have the same denominator and then equating coefficients of the various powers of x.*
(ii) *Substituting specific numerical values for x and solving the resulting simultaneous equations.*
(iii) *Evaluation of the fraction at each of the roots of its denominator, imagining a factored denominator with the factor corresponding to the root omitted – often known as the 'cover-up' method.*

Verify that the decomposition obtained is independent of the method used.

(a) As the denominator factorises as $(x + 5)(x - 2)$, the partial fraction expansion must have the form

$$\frac{2x + 1}{x^2 + 3x - 10} = \frac{A}{x + 5} + \frac{B}{x - 2}.$$

(i)

$$\frac{A}{x+5} + \frac{B}{x-2} = \frac{x(A+B) + (5B-2A)}{(x+5)(x-2)}.$$

Solving $A + B = 2$ and $-2A + 5B = 1$ gives $A = \frac{9}{7}$ and $B = \frac{5}{7}$.

(ii) Setting x equal to 0 and 1, say, gives the pair of equations

$$\frac{1}{-10} = \frac{A}{5} + \frac{B}{-2}; \quad \frac{3}{-6} = \frac{A}{6} + \frac{B}{-1},$$

$$-1 = 2A - 5B; \quad -3 = A - 6B,$$

with solution $A = \frac{9}{7}$ and $B = \frac{5}{7}$.

(iii)

$$A = \frac{2(-5)+1}{-5-2} = \frac{9}{7}; \quad B = \frac{2(2)+1}{2+5} = \frac{5}{7}.$$

All three methods give the same decomposition.

(b) Here the factorisation of the denominator is simply $x(x-3)$ or, more formally, $(x-0)(x-3)$, and the expansion takes the form

$$\frac{4}{x^2-3x} = \frac{A}{x} + \frac{B}{x-3}.$$

(i)

$$\frac{A}{x} + \frac{B}{x-3} = \frac{x(A+B) - 3A}{(x-0)(x-3)}.$$

Solving $A + B = 0$ and $-3A = 4$ gives $A = -\frac{4}{3}$ and $B = \frac{4}{3}$.

(ii) Setting x equal to 1 and 2, say, gives the pair of equations

$$\frac{4}{-2} = \frac{A}{1} + \frac{B}{-2}; \quad \frac{4}{-2} = \frac{A}{2} + \frac{B}{-1},$$

$$-4 = 2A - B; \quad -4 = A - 2B,$$

with solution $A = -\frac{4}{3}$ and $B = \frac{4}{3}$.

(iii)

$$A = \frac{4}{0-3} = -\frac{4}{3}; \quad B = \frac{4}{3-0} = \frac{4}{3}.$$

Again, all three methods give the same decomposition.

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