Dynamical Systems with Applications using MAPLE

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Preface

This book provides an introduction to the theory of dynamical systems with the aid of the Maple algebraic manipulation package. It is written for both senior undergraduates and first-year graduate students. The first half of the book deals with continuous systems using ordinary differential equations (Chapters 1–12) and the second half is devoted to the study of discrete dynamical systems (Chapters 13–20). (The author has gone for breadth of coverage rather than fine detail and theorems with proof are kept at a minimum.) The material is not clouded by functional analytic and group theoretical definitions, and so is intelligible to readers with a general mathematical background. Some of the topics covered are scarcely covered elsewhere. Most of the material in Chapters 9–12, 16, 17, 19, and 20 is at postgraduate level and has been influenced by the author's own research interests. It has been found that these chapters are especially useful as reference material for senior undergraduate project work. The book has a very hands-on approach and takes the reader from the basic theory right through to recently published research material.

An efficient tutorial guide to the Maple symbolic computation system has been included in Chapter 0. Students should be able to complete tutorials one and two in under two hours depending upon their past experience. The author suggests that the reader should save the relevant example programs listed throughout the book in separate files. These programs can then be edited accordingly when attempting the exercises at the end of each chapter. The Maple commands, programs and output can also be viewed in color over the Web at either http://www.birkhauser.com/cgi-win/ISBN/0-8176-4150-5

or Maple's applications site,

http://www.maplesoft.com/apps/.

Throughout the book, Maple is viewed as a tool for solving systems or producing eye-catching graphics. The author has used Maple V release 5.1 and Maple 6 in the preparation of the material. However, the Maple programs have been kept as simple as possible and should also run under later versions of the package.

The first few chapters of the book cover some theory of ordinary differential equations and applications to models in the real world are given. The theory of differential equations applied to chemical kinetics and electric circuits is introduced in some detail. Chapter 1 ends with the existence and uniqueness theorem for the solutions of certain types of differential equation. The theory behind the construction of phase plane portraits for two-dimensional systems is dealt with in Chapters 2 and 3, and applications to modeling the populations of interacting species are discussed in Chapter 4. Limit cycles, or isolated periodic solutions, are introduced in Chapter 5. Since we live in a periodic world, these are the most common type of solution found when modeling nonlinear dynamical systems. They appear extensively when modeling both the technological and natural sciences. Hamiltonian (conservative) systems and stability are discussed in Chapter 6, and Chapter 7 is concerned with how planar systems vary depending upon a parameter. Bifurcation, multistability, and bistability are discussed.

The reader is first introduced to the concept of chaos in Chapters 8 and 9, where three-dimensional systems and Poincaré maps are investigated. These higher-dimensional systems can exhibit strange attractors and chaotic dynamics. Once again the theory can be applied to chemical kinetics and electric circuits; a simplified model for the weather is also briefly discussed. Both local and global bifurcations are investigated in Chapter 10. The main results and statement of the famous second part of David Hilbert's sixteenth problem are listed in Chapter 11. In order to understand these results, Poincaré compactification is introduced. The study of continuous systems ends with one of the authors specialities—limit cycles of Liénard systems. There is some detail on Liénard systems in particular in the first half of the book, but they do have a ubiquity for systems in the plane.

Chapters 13–20 deal with discrete dynamical systems. Chapter 13 starts with a general introduction to recurrence relations and iteration. Applications to population modeling and harvesting and culling policies is then investigated. Chaos in discrete systems is investigated and bifurcation diagrams are plotted in Chapter 14. The concept of universality is discussed for the first time. Complex iterative maps are introduced in Chapter 15. Julia sets and the now famous Mandelbrot set are plotted. As a simple introduction to optics, electromagnetic waves and Maxwell's equations are studied at the beginning of Chapter 16. A brief history of nonlinear bistable optical resonators is discussed and the simple fiber ring resonator is analyzed in particular. Chapters 16 and 17 are devoted to the study of these optical resonators and topics such as bifurcation, bistability, chaos, chaotic attractors, instabilities, linear stability analysis, multistability, and nonlinearity, which have already been dealt with in earlier chapters, are reviewed. Some simple fractals may be constructed using pencil and paper in Chapter 18, and the idea of fractal dimension is introduced. Fractals may be thought of as identical motifs repeated on ever reduced scales. Unfortunately, most of the fractals appearing in nature are not homogeneous but are more heterogeneous, hence the need for the multifractal theory given in Chapter 19. The final chapter is devoted to the new and exciting theory behind chaos control. For most systems, the maxim used by engineers in the past has been "stability good, chaos bad," but more and more nowadays this is being replaced with "stability good, chaos better." There are exciting and new applications to cardiology, laser technology, and space research, for example.

This book is informed by the research interests of the author which are currently nonlinear ordinary differential equations, nonlinear optics and multifractals. Some references include recently published research articles.

The prerequisites for studying dynamical systems using this book are undergraduate courses in linear algebra, real and complex analysis, calculus and ordinary differential equations; a knowledge of a computer language such as Fortran or Pascal would be beneficial but not essential.

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Stephen Lynch

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Lee. *Mathematical Computation with Maple V: Ideas and Applications*, 0-8176-3724-9, $47.95, 1993

Contents

0
A Tutorial Introduction to Maple

Aims and Objectives

- To provide a tutorial guide to the Maple package.
- To give practical experience in using the package.
- To promote self-help using the on-line help facilities.

On completion of this chapter, the reader should be able to

- use Maple as a mathematical tool;
- produce simple Maple programs;
- access some Maple commands and programs over the Web.

It is assumed that the reader is familiar with either the *Windows* or *Unix* environment.

Commands listed in Sections 0.1 and 0.2 have been chosen to allow the reader to become familiar with Maple in a few hours. These tutorial sheets have been used with great success over a number of years with both mathematics and engineering undergraduate students. Experience has shown that the Maple worksheets can be completed in under two hours, after which students are able to adapt the commands to tackle their own problems. This method of teaching works well with computer laboratory class sizes of no more than 20 students to one staff member. Section 0.3 gives a brief introduction to programming with Maple.

If any problems result, there are several options. For example, there is an excellent help browser in Maple, the 10 most common errors are listed in Section 0.4, and Maple commands and programs with the respective output from this text can be found on the Web at

http://www.birkhauser.com/cgi-win/ISBN/0-8176-4150-5

or

http://www.maplesoft.com/apps/.

The Maple worksheets on the Web may be edited and copied.

Remember to save your Maple files at regular intervals. You could label your first file as *tut1.mws*, for example.

0.1 Tutorial One: The Basics (One Hour)

There is no need to copy the comments; they are there to help you.

Click on the Maple icon and copy the command after the > prompt.

Maple Commands	**Comments**
`> # This is a comment`	`# Helps when writing` `# programs.`
`> 1+2-3;`	`# Simple addition and` `# subtraction.`
`> 2*3/7;`	`# Multiplication and` `# division.`
`> 2*6+3^2-4/2;`	
`> (5+3)*(4-2);`	
`> sqrt(100);`	`# The square root.`
`> n1:=10:`	`# The colon suppresses` `# the output.`
`> lprint('n1:=',n1):`	`` # Use the ` character `` `# for quotes.`
`> n1^(-1);`	`# Negative powers.`
`> sin(Pi/3);`	`# Use capital P for Pi.`
`> y:=sin(x)+3*x^2;`	`# Equations and` `# assignments.`
`> evalf(sin(Pi/3));`	`# Evaluate as a floating` `# point number.`

```
> diff(y,x);                         # Differentiate y with
                                     # respect to x.

> y:='y':                            # Set y back equal to y.

> diff(x^3*y^2,x$1,y$2);             # Partial differentiation.

> int(cos(x),x);                     # Integration with
                                     # respect to x.

> int(x/(x^3-1),x=0..1);             # Definite integrals.

> int(1/x,x=1..infinity);            # Improper integrals.

> convert(1/((s+1)*(s+2)),parfrac,s);
                                     # Split into partial
                                     # fractions.

> expand(sin(x+y));                  # Expansion.

> factor(x^2-y^2);                   # Factorization.

> limit((cos(x)-1)/x,x=0);           # The limit as x goes
                                     # to zero.

> z1:=3+2*I;z2:=2-I;                 # Complex numbers. Use
                                     # I NOT i.

> z3:=z1+z2;
> z4:=z1*z2/z3;

> modz1:=abs(z1);                    # Modulus of a complex
                                     # number.

> evalc(exp(I*z1));                  # Evaluate as a complex
                                     # number.

> solve({x+2*y=1,x-y=3},{x,y});      # Solve two simultaneous
                                     # equations.

> fsolve(x*cos(x)=0,x=7..9);         # Find a root in a given
                                     # interval.

> S:=sum(i^2,i=1..n);                # A finite sum.

> ?linalg                            # Open a help page.

> with(linalg):                      # Load the linear
                                     # algebra package.

> A:=matrix([[1,2],[3,4]]);          # Defining 2 by 2
> B:=matrix([[1,0],[-1,3]]);         # matrices.

> evalm(B^(-1));                     # Matrix inverse.

> C:=evalm(A+2*B);                   # Evaluate the new
```

```
                                     # matrix.
> AB:=evalm(A &* B);                 # Matrix multiplication.
> A1:=matrix([[1,0,4],[0,2,0],[3,1,-3]]);
> det(A1);                           # The determinant.
> eigenvals(A1);                     # Gives the eigenvalues
                                     # of A1.
> ?eigenvects                        # Shows how the eigen-
                                     # vectors are displayed.
> eigenvects(A1);                    # Gives the eigenvectors
                                     # of A1.
> # Use of the help browser - one option.
> ?interp                            # Open a help page for
                                     # interpolation.
>??interp                            # List the syntax for
                                     # this command.
>???interp                           # List some examples.
> # End of Tutorial One.
```

Exit the Maple worksheet by clicking on the **File** and **Exit** buttons, but remember to save your work.

0.2 Tutorial Two: Plots and Differential Equations (One Hour)

There is no need to copy the comments, they are there to help you.

Click on the Maple icon and copy the command after the > prompt.

Maple Commands **Comments**

```
> ?plot                              # Open a help page.
> with(plots):                       # Load the plots package.
> plot(cos(2*x),x=0..4*Pi);          # Plot a trigonometric
                                     # function.
> plot(x*(x^2-1),x=-3..3,y=-10..10,
> title=`A cubic polynomial`);       # Plot a cubic polynomial
                                     # and add a title.
> plot(tan(x),x=-2*Pi..2*Pi,y=-10..10,
```

```
> discont=true);                        # Plot a function with
                                        # discontinuities.

> plot({x*cos(x),x-2},x=-5..5);         # Plot two curves on one
                                        # graph.

> c1:=plot(sin(x),x=-2*Pi..2*Pi,
> linestyle=1):
> c2:=plot(2*sin(2*x-Pi/2),x=-2*Pi..2*Pi,
> linestyle=3):
> display({c1,c2});

> points:=[[n,sin(n)]$n=1..10]:         # Plot points and lines
> pointplot(points,style=point,         # joining the points on
> symbol=circle);                       # two separate graphs.
> pointplot(points,style=line);

> implicitplot(y^2+y=x^3-x,x=-2..3,
> y=-3..3);                             # Implicit plots.

> animate(sin(x*t),x=-4*Pi..4*Pi,t=0..1,
> color=red);                           # 2-D animation.

> plot3d(sin(x)*exp(-y),x=0..Pi,y=0..3,
> axes=boxed);                          # 3-D plots. You can
                                        # rotate the figure
                                        # with the left mouse
                                        # button.

> cylinderplot(z+3*cos(2*theta),
> theta=0..Pi,z=0..3);

> animate3d(t*y^2/2-x^2/2+x^4/4,x=-2..2,
> y=-2..2,t=0..2);                      # 3-D animation.

> ?DEtools                              # Open a help page.

> with(DEtools):                        # Load the differential
                                        # equations package.

> dsolve(diff(y(x),x)=x,y(x));          # Solve a differential
                                        # equation.

> dsolve({diff(v(t),t)+2*t=0,v(1)=5},
> v(t));                                # Solve an initial value
                                        # problem.

> dsolve(diff(x(t),t$2)+8*diff(x(t),t)
> +25*x(t)=0,x(t));                     # Solve second-order
                                        # differential equations.

> dsolve(diff(x(t),t$2)+8*diff(x(t),t)
> +25*x(t)=t*exp(t),x(t));
```