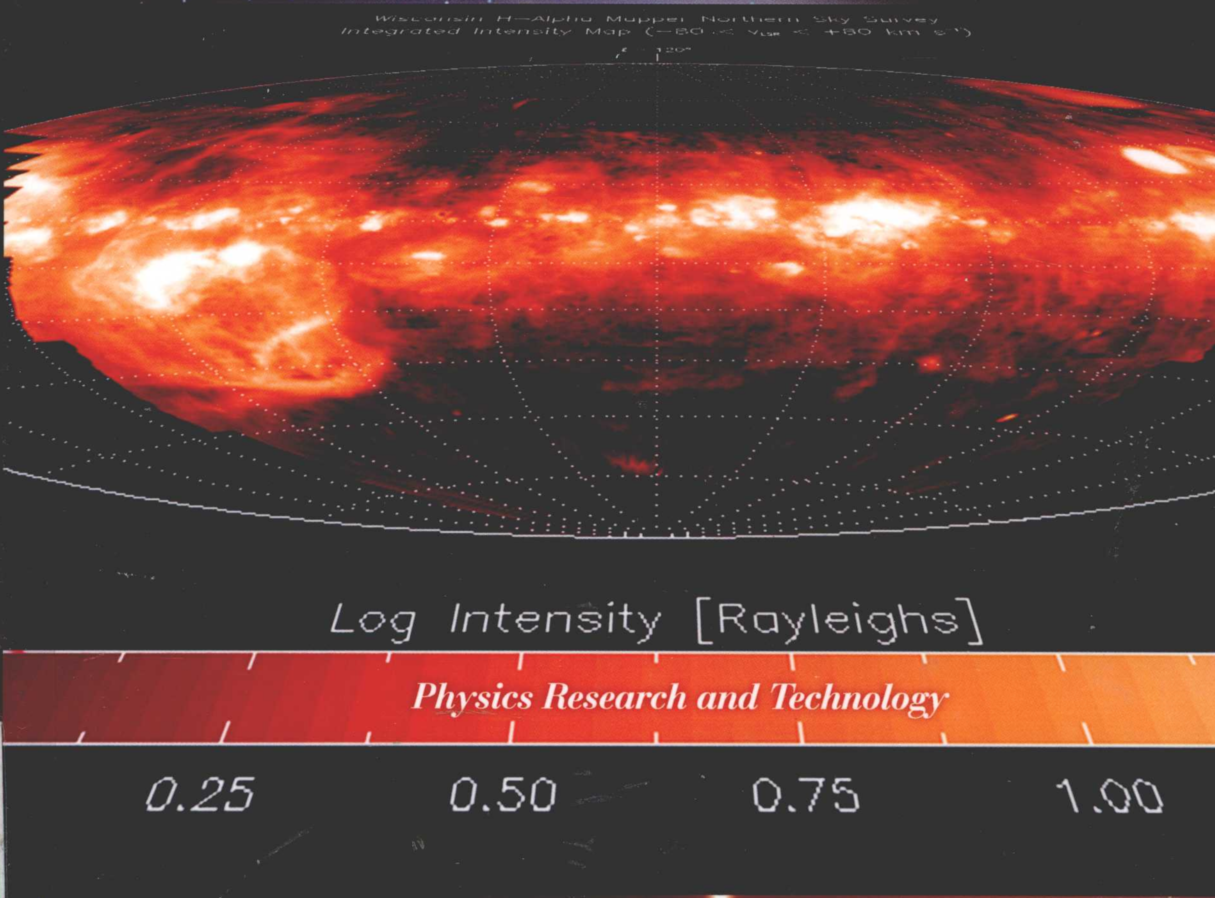
INTERSTELLAR MEDIUM

NEW RESEARCH



Physics Research and Technology

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PHYSICS RESEARCH AND TECHNOLOGY

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AND
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PREFACE

The interstellar medium (or ISM) is the matter that exists in the space between the star systems in a galaxy. This matter includes gas in ionic, atomic, and molecular form, dust, and cosmic rays. It fills interstellar space and blends smoothly into the surrounding intergalactic space. This book presents topical research in the study of interstellar medium, including explosive processes in the interstellar medium as sources of ultrahigh-energy cosmic rays; heteronuclear diatomics in diffuse and translucent clouds; far ultraviolet observations of the Large Magellanic Cloud; MDH simulations of Parker instability undergoing cosmic-ray diffusion; deuterium in the interstellar medium and interaction of planetary nebulae, Eta-Carinae and supernova remnants with the interstellar medium.

Chapter 1 - The chapter discusses the possibility of particle acceleration up to high energies in relativistic waves generated by various explosive processes in the interstellar medium. Waves are assumed to be a type of high-amplitude Langmuir waves propagating in plasma in the presence of a weak magnetic field (high-frequency upper hybrid branch of oscillations) and magneto-sonic shocks (branch of fast magnetic sound). The authors propose to use the surfatron mechanism of acceleration (surfing) of charged particles trapped in the front of relativistic waves as a generator of high-energy cosmic rays (CRs). Conditions under which surfing in the shock waves under consideration can be made are studied thoroughly. Ultrahigh-energy CRs (up to 10^{20} eV) are shown to be obtained due to the surfing in relativistic plane and spherical waves generated, for instance, by relativistic jets or spherical formations that expand fast (fireballs).

Chapter 2 - Diffuse and translucent molecular clouds are very special environments in the Interstellar Medium (ISM). Recent advances in observational techniques of modern optical and ultraviolet spectroscopy led to detection of many features of atomic and molecular origin in spectra of such clouds. Molecular spectra of heteronuclear diatomic molecules, ie. OH, CH CH^+, CN, NH, CO play important role in understanding chemistry and physical conditions in environments where they exist. Review of astronomical observations of molecules together with history is reported. Recent results concerning visual and ultraviolet observations; appearance of molecular features in spectra of early type OB-stars are presented and discussed. Appearance of vibrational-rotationl spectra with observed correct transitions based on high-quality spectra, oscillator strengths and column densities are also presented. Relations between fundamental column densities of heteronuclear diatomics (based on recommended oscillator strengths) and relations with interstellar extinction and intensities of diffuse interstellar bands (DIBs) are also presented and discussed.

Chapter 3 - Author offers and develops the theory of a new class of space wing electro ship. A biplane wing and an electric field between the wings characterize this space ship. The interstellar and interplanetary mediums contain charged protons and other charged particles. The winged space ship can produce the lift, thrust and drag forces. The density of the space medium is small (10^0 - 10^5 charged particles/cm^3) but the high ship speed allows creating enough force for maneuvers, turning, acceleration and braking of ship especially at near relativistic speeds. Author shows the ratio of lift force/drag of the space wing electro ship may reach 100 and maneuver of wing space is big advantageous compared to maneuver using conventional rocket methods. In addition the biplane wing easily may be converted into a very efficient engine (brake) using external space matter and achieve something close to simple photon propulsion. That means the proposed wing-brake-engine is the most efficient and technologically realistic space drive available at the present time. The offered wing design allows collecting of particles from a very large space area. The method also allows decreasing the drag of a ship body.

Chapter 4 - In this chapter, the authors present their recent results on the far ultraviolet (FUV) observations of the Large Magellanic Cloud (LMC). They have used *Far Ultraviolet Spectroscopic Explorer (FUSE)* data to study the distribution of dust and the O VI abundances in the interstellar medium (ISM) of the LMC. O VI is an important constituent of the ISM that traces the interface of the hot (> 106 *K*) and warm (< 104 *K*) gas. The authors present a survey of O VI absorption in the LMC. This is the most extensive survey till date and explores the ISM on a very small scale (» 0 *pc*) in some regions of the LMC. They have studied the correlation of O VI column densities with H® and X-ray surface brightnesses. O VI absorption in the 30 Doradus region of the LMC shows good correlation with H® and X-ray surface brightnesses. They also find that the O VI abundance in the superbubbles of the LMC is higher compared to non-superbubble lines of sight. Observations of the diffuse UV emission track the transfer of radiation from the star to the ISM with the absorbed radiation reemitted as thermal emission in the infrared. The diffuse UV emission, thus, provides an insight to the properties of interstellar dust especially its distribution. They use all the *FUSE* observations in the LMC to extract diffuse FUV emission and correlate these observations with *Ultraviolet Imaging Telescope (UIT)* data to estimate the fraction of total light emitted as FUV diffuse emission. The study is the first of its kind and proves to be an important input to future interstellar dust modeling studies.

Chapter 5 - Parker instability arises from the presence of magnetic fields in a plasma in a gravitational field such as the interstellar medium (ISM), wherein the magnetic buoyant pressure expels the gas and causes the gas to move along the field lines. The subsequent gravitational collapse of the plasma gas is thought to be responsible for the formation of giant molecular clouds in the Galaxy. The process of mixing in the ISM near the Galactic plane is investigated. The initial ISM is assumed to consist of two fluids: plasma gas and cosmicray particles, in hydrostatic equilibrium, coupled with a uniform, azimuthally-aligned magnetic field. The evolution of the instability is explored in two models: an isothermal exponential-declining density model and a two-layered, hyperbolic tangent temperature model. After a small perturbation, the unstable gas aggregates at the bottom of the magnetic loops and forms dense blobs. The growth rate of the instability decreases as the coupling between the cosmic rays and the plasma becomes stronger (meaning a smaller CR diffusion coefficient). The mixing is enhanced by the cosmic-ray diffusion, while the shape of the condensed gas depends sensitively on the initial equilibrium conditions. The hyperbolic tangent temperature model produces a more concentrated and round shape of clumps at the foot points of rising magnetic arches, like the observed giant molecular cloud, whereas the exponential density model gives rise to a filamentary morphology of the clumpy structure. When considering a minimum perpendicular or cross field diffusion of cosmic rays, which is often substantially smaller than the parallel coefficient _k, around 2%–4% of _k, the flow speed is significantly increased such that the magnetic loops extend to a greater altitude. The authors speculate that the galactic wind flow perpendicular to the galactic disk may be facilitated by Parker instabilities through the cross field diffusion of cosmic rays.

Chapter 6 - The image of planetary nebulae (PN), supernova remnant (SNR) and Eta-Carinae is made by three different physical processes. The first process is the expansion of the shell that can be modeled by the canonical laws of motion in the spherical case and by the momentum conservation when gradients of density are present in the interstellar medium. The quality of the simulations is introduced along one direction as well along many directions. The second process is the diffusion of particles that radiate from the advancing layer. The 3D diffusion from a sphere, the 1D diffusion with drift and 1D random walk are analyzed. The third process is the composition of the image through an integral operation along the line of sight. The developed framework is applied to three PN which are A39, the Ring nebula and the etched hourglass nebula MyCn 18, the hybrid object Eta-Carinae, and to two SNR which are SN 1993J and SN 1006. In all the considered cases a careful comparison between the observed and theoretical profiles in intensity is done.

Chapter 7 - Deuterium is only created in the Big Bang nucleosynthesis while all other processes destroy it. Because of this unique property, deuterium has for a long time served as the key to determining important cosmological parameters, before modern observations of cosmic microwave background were available. Moreover, given the net destruction of deuterium with time, its remaining abundance in the interstellar medium is a direct probe of how much gas was processed through stars, and can thus serve as an important tool in the galactic chemical evolution modeling as well. However, in the past decade, observations of deuterium in the interstellar medium have revealed large variations of its gas-phase abundance, which called in question their understanding of deuterium, galactic chemical evolution, and even modern cosmology. As a solution to this crisis, it was suggested that deuterium might be depleted onto dust grains, however not all observation and models agree with this potential explanation. Presented in this chapter will be a review of the current state of affairs related to this persisting and crucial problem with deuterium abundance in the interstellar medium, whose solution will bare important consequences for galactic chemical evolution models, their understanding of the nucleosynthesis and dust formation in the interstellar medium, as well as for the larger cosmological context and models of galaxy formation.

Chapter 8 - The name magnetosphere was used for the first time by US physicist Thomas Gold [Gold, 1959]. He wrote "The region above the ionosphere in which the magnetic field of the earth has a dominant control over the motions of gas and fast charged particles is known to extend out to a distance of the order of 10 earth radii; it may appropriately be called the magnetosphere".

CONTENTS

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Chapter 1

EXPLOSIVE PROCESSES IN THE INTERSTELLAR MEDIUM AS SOURCES OF ULTRAHIGH-ENERGY COSMIC RAYS

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ABSTRACT

The paper discusses the possibility of particle acceleration up to high energies in relativistic waves generated by various explosive processes in the interstellar medium. Waves are assumed to be a type of high-amplitude Langmuir waves propagating in plasma in the presence of a weak magnetic field (high-frequency upper hybrid branch of oscillations) and magneto-sonic shocks (branch of fast magnetic sound). We propose to use the surfatron mechanism of acceleration (surfing) of charged particles trapped in the front of relativistic waves as a generator of high-energy cosmic rays (CRs). Conditions under which surfing in the shock waves under consideration can be made are studied thoroughly. Ultrahigh-energy CRs (up to 10^{20} eV) are shown to be obtained due to the surfing in relativistic plane and spherical waves generated, for instance, by relativistic jets or spherical formations that expand fast (fireballs).

Keywords: acceleration of particles - cosmic rays - plasmas - shock waves.

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INTRODUCTION

Explosive processes belong to rather frequent phenomena occurring in space. Among examples, we can mention explosions of supernovae, flaring X-ray stars (bursters), active processes in galactic nuclei, quasars, etc. Typical explosive processes, such as solar flares, are constantly observed on the Sun, i.e., on the nearest star to the Earth. Numerous explosive phenomena are characterized by a huge energy release, with part of the energy being transferred to the kinetic energy of moving substance accelerated up to high velocities. The universal features of the phenomena accompanying explosions in space are of urgent interest for researchers.

As a rule, the explosions under consideration occur in a medium that, in typical situations, can be related to weakly magnetized plasma. When fast traveling disturbances of the substance ejected into the interstellar medium upon the explosion propagate in space, shocks are produced. For example, intense relativistic shocks are excited as a result of the fast mass motion in the vicinity of black holes. In particular, this occurs in the processes of relativistic jet ejection, of the collision of neutron stars in collapsing stellar clusters, and of the spherical expansion of fireballs into the interstellar medium, as well as in a number of other cases. Solar flares are the motion of mass ejections in the heliosphere give rise to formation of interplanetary shocks. Shocks generated in space are the most probable sources of high-energy particles accelerated, as a rule, in the vicinity of shock fronts.

The issue of the particle acceleration mechanism arises. The mechanism proposed by Krymsky [1]-[4] as applied to shocks excited by supernova explosions cannot allow the particle energy to exceed 10^{15} eV [2], [3]. Moreover, the estimates made in the most general form show that, in supernova remnants, it is impossible to accelerate particles to energies higher than 10^{17} eV [5]. This is associated with a relatively low rate of supernova shell expansion. Relativistic shocks are to be considered to obtain cosmic rays (CRs) of extreme energies measured on the Earth (about 10^{20} eV). Velocity U of their motion is close to the speed of light c, and the corresponding Lorentz factor is $\Gamma = (1 - U^2/c^2)^{-1/2} >> 1$. Unfortunately, the possibility to apply the Krymsky mechanism to explain the particle acceleration by relativistic shocks seems rather doubtful [6]-[8]. Therefore, we have to apply alternative methods of particle acceleration. Among them, there is the conversion acceleration mechanism [6]. Another possible mechanism is the surfatron acceleration of particles (surfing [9]-[21]) trapped in high-amplitude waves propagating in space plasma.

Here, we estimate a possibility to attain ultrahigh energies (up to 10^{20}eV) of cosmic rays through the surfing mechanism. To this effect, we consider relativistic wave disturbances produced by penetration of the substance accelerated up to relativistic velocities in explosive processes into the interstellar medium.

1. Surfatron Acceleration Mechanism (Surfing) and its Characteristics

1.1. Essence of Surfing

Surfing occurs in weakly magnetized plasma. In this mechanism, particles are trapped and accelerated in a potential wave with a steep forefront. Traveling positive potential jump is capable of accelerating ions, and negative jump is capable of accelerating electrons. We consider the acceleration mechanism offered in the general case for a one-dimensional non-linear potential wave traveling in plasma with velocity U at angle θ_{Bn} to the magnetic field vector. As analysis shows, a portion of particles of plasma having finite temperature, when inleaking onto potential jump cannot overcome it and is reflected from the jump. Under certain conditions, the Lorentz force acting ahead of the potential jump can turn these reflected particles back towards the front. Thus, particles can appear trapped by the wave and can be accelerated by force $qUB_{0\perp}\Gamma/c$ to high energies (here q is the charge of particles, $B_{0\perp} = B_0 \sin \theta_{Bn}$ is the magnetic field component transversal to the wave motion direction which in the frame associated with the static plasma ahead of the front has value B_0). A remarkable peculiarity of surfing is in that both trapping and acceleration of particles are ensured by the same electromagnetic fields existing in the vicinity of the potential jump, and through surfing it is possible to accelerate both electrons and ions to ultimate energies with equal efficiency.

1.2. Types of Potential Traveling Disturbances in which Surfing is Possible

The most known and widespread wave formations containing a potential jump in magnetized plasma are: high-amplitude Langmuir waves [22], [23] propagating in plasma in the presence of a weak magnetic field (high-frequency upper hybrid branch of oscillations [24]) and magneto-sonic shocks (MSSs) [9] (branch of fast magnetic sound). Since a periodic plasma wave contains both positive and negative potential jumps, it can accelerate both ions and electrons. MSS is characterized by the positive potential jump, therefore, only ions can be accelerated at the MSS front. We note that in the collisionless space plasma longitudinal plasma waves and MSSs are easily excited at abrupt variations of weakly magnetized plasma parameters and damp relatively weakly.

We consider possible versions of exciting waves of the considered types by example of the near-Sun plasma. In the magnetospheric plasma, stationary magneto-sonic shocks are formed when solar wind interacts with magnetic fields of planets. The Earth's bow shock can be an example. The bulk of waves is excited in the solar atmosphere (chromosphere, corona). These waves propagate away from the Sun, thus, the most powerful waves such as plasma ones and MSSs (e.g., interplanetary shocks) emerge at chromospheric flares and other similar explosive processes on the Sun.

High-amplitude plasma (Langmuir) waves can form in various non-linear processes in plasma, but, generally, their formation occurs either due to the transformation of strong electromagnetic waves in plasma or during the development of instabilities in plasma as fast beams of charged particles move in it. In the waves under consideration, trapped are suprathermal particles from the tail of the plasma particle distribution function. A detailed consideration shows [18], [19] that at such a means of involving particles in the acceleration process their quantity is enough to ensure the observed concentration of CRs in the Galaxy.

1.3. Surfing in Non-Linear Plasma (Langmuir) Waves

First, we study particle surfing in a non-linear Langmuir wave propagating across a weak magnetic field B. We consider electrons as particles subjected to acceleration. In magnetized plasma with the n_0 concentration and particle temperature $T << m_e c^2$ for the longitudinal plasma wave we consider a most typical case for space plasma, when $\omega_{pe}^2 >> \omega_{ce}^2$, where $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ is the electron plasma frequency (e and m_e being the electron charge and rest mass, respectively) and $\omega_{ce} = eB_0/m_e c$. In this case we can exclude the magnetic field effect on plasma dispersion and assume that the magnetic field does not affect the structure of non-linear Langmuir waves. Further, we consider that the Γ parameter for the waves under consideration does not exceed 10^5, which allows us, based on [23], to use formula $\omega \approx \omega_{pe}$ for the frequency of a non-linear Langmuir wave.

1.3.1. Problem statement and input equations

Let us analyze in detail the process of electron surfing in a non-linear Langmuir wave within a simple model. We consider a one-dimensional wave running strictly across the magnetic field in the negative direction of the Ox axis whose electric field has a saw-tooth form (Figure 1). The electric field is chosen saw-tooth due to the following reasons. First, as appears from the theory [23], the field of a relativistic non-linear Langmuir waves has a saw-tooth form. Second, a field very similar to a saw-tooth form is obtained in numeric computations where wave fields excited by laser or beam are considered. Third, as seen below, such a choice simplifies solving the problem, which allows one to obtain the solution for the equation of motion of a wave-trapped electron in an analytical view.

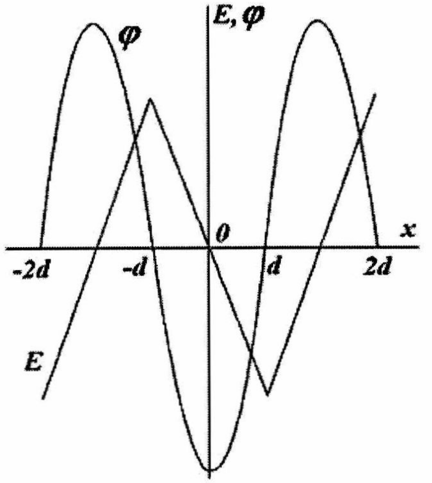

Figure 1. Potential and electric field profile in a non-linear Langmuir wave.

For the steady-state wave under consideration, the analysis of particle motion is convenient to carry out in a wave frame. We accept that the wave traveling velocity $U = \omega/k_0$ in the laboratory reference frame does not exceed the speed of light. Thus, in our assumptions, in the wave frame field components $E_y=UB/c$ and B are uniform in space, the x-component of the electric field and potential depend only on x, and, over the $-d < x < d$ interval we are interested in, these dependences have the view (Figure 1): $E(x) = E_A x/d$, $\varphi(x) = \varphi_A(1 - x^2/d^2)$, where $d = \pi/2k$, E_A and $\varphi_A = E_A d/2$ are the amplitudes of the electric field and potential, respectively. As per transition formulas, $k = k_0/\Gamma$, $\varphi_A = \Gamma\varphi_0$, where k_0 and φ_0 are the wave vector and potential amplitude in the laboratory frame, respectively ($\Gamma = (1 - U^2/c^2)^{-1/2}$).

Let a small beam of electrons be injected at initial instant $t = 0$ onto the wave potential well bottom, where $x = 0$, $E(0) = 0$, $\varphi(0) = \varphi_A$. We are interested only in those electrons whose motion occurs over the $-d < x < d$ interval. Let us consider the behavior of the trapped electrons which will move in the $x0y$ plane as per equations of motion in the given electromagnetic fields and accepted assumptions within the wave frame

$$dP_x(t)/dt = -eE_A x(t)/d + eV_y(t)B/c,$$

$$dP_y(t)/dt = eUB/c - eV_x(t)B/c,$$

where V_x, $P_x = \gamma m_e V_x$, and V_y, $P_y = \gamma m_e V_y$, respectively, x- and y- are the components of velocity and momentum, $\gamma(t) = 1/(1 - V_x^2/c^2 - V_y^2/c^2)^{1/2}$. We consider, without loss of generality, that $V_z = dz/dt = 0$, $P_z = 0$. We introduce dimensionless variables: $\tau = \omega_{ce} t$, $v = V_x/c$, $w = V_y/c$, $p_x = P_x/m_e c$, $p_y = P_y/m_e c$, $\chi = w_{ce} x/c$, $\nu = w_{ce} y/c$, where $\omega_{ce} = eB/m_e c$ is non-relativistic cyclotron frequency of electrons. In the dimensionless variables, the equations of electron motion will accept the view:

$$dp_x(\tau)/d\tau = w(\tau) - D^2\chi(\tau), \qquad (1.3.1)$$

$$dp_y(\tau)/d\tau = \beta - v(\tau). \qquad (1.3.2)$$

Here we use the following designations for the dimensionless parameters: $\beta = U/c$, $R = E_A/B$, $\psi_A = e\varphi_A/m_e c^2$, $D^2 = R^2/2\psi_A$. We take electron potential energy $\psi(\chi) = -e[\varphi(\chi) - \varphi_A]/m_e c^2 = 0$ at $\chi = 0$, hence, $\psi(\chi) = D^2\chi^2/2 = R\chi^2/2\chi_d$, where $\chi_d = 2\psi_A/R = R/D^2$ (we remind that $-\chi_d \le \chi \le \chi_d$). Equation (1.3.2) can be integrated once with result:

$$p_y(\tau) = p_{yo} + \beta\tau - \chi(\tau). \qquad (1.3.3)$$

In ratio (1.3.3) the accepted initial conditions are taken into account: $\chi(0) = \nu(0) = 0$, $v(0) = v_0$, $w(0) = w_0$, $p_x(0) = p_{xo}$, $p_y(0) = p_{y0}$ from which it follows that we confine ourselves to considering the behavior of the electrons which are at the well bottom at the initial instant. By equation (1.3.3), we can write the y-component of dimensionless velocity and electron total energy, respectively, in the view:

$$w(\tau) = p_y/[\gamma_v(\tau)(1 + p_y^2)^{1/2}], \qquad (1.3.4)$$

$$\gamma(\tau) = \gamma_v(\tau)(1 + p_y^2)^{1/2} \qquad (1.3.5)$$

where $\gamma_v(\tau) = 1/(1 - v^2)^{1/2}$, and the law for total energy conservation under the given initial conditions looks like this:

$$\gamma(\tau) + \psi(\chi) - \beta v(\tau) = \gamma_0, \quad (1.3.6)$$

where $\gamma_0 = 1 / (1 - v_0^2 - w_0^2)^{1/2}$. Since we assume that the ensemble of the initially trapped electrons is non-relativistic, then $\gamma_0 \approx 1$. The above equations completely describe the behavior of the trapped electrons at all the instants. We will search the solution for the equations of motion separately in two extreme cases: 1) $\psi_A << 1$, 2) $\psi_A >> 1$.

1.3.2. Potential low amplitude wave (ψ_A <<1)

In this case, assuming $R \geq 1$, we obtain a very large value for parameter D: $D = R/(2\psi_A)^{1/2} >> 1$. The well width is thus small: $\chi_d = R/D^2 << 1$. At the non-relativistic phase ($\beta\tau \leq 1$, $\gamma \approx 1$), the equations of motion have analytical solutions:

$$\chi(\tau) = [(v_0 - v_{d0})/\Omega] \sin \Omega\tau + w_0(1 - \cos \Omega\tau)/\Omega^2 + v_{d0}\tau, \quad (1.3.7)$$

$$v(\tau) = (v_0 - v_{d0})\cos \Omega\tau + [w_0 /\Omega]\sin \Omega\tau + v_{d0}, \quad (1.3.8)$$

$$w(\tau) = w_0 + \beta\tau - \chi(\tau), \quad (1.3.9)$$

$$v(\tau) = w_0 \tau D^2/\Omega^2 + \beta\tau^2 D^2/(2\Omega^2) + (v - v_0)/\Omega^2; \quad (1.3.10)$$

here $\Omega = (1+D^2)^{1/2} \approx D$, $v_{d0} = \beta/\Omega^2$. By equations (1.3.7)-(1.3.10), we obtain the law for conservation:

$$(D^2\chi - w)^2/\Omega^2 + (v - v_{d0})^2 = w_0^2/\Omega^2 + (v_0 - v_{d0})^2. \quad (1.3.11)$$

From solutions (1.3.7)-(1.3.10) it follows that for $D >> 1$ in time $\tau \leq 1$ the electron trapped in the well makes a great number of oscillations, and the oscillation amplitude remains constant. The absolute value of the v velocity component does not change, the w velocity component grows proportional to $\beta\tau$ with time. Particle motion represents a drift along the Ox axis with velocity v_{d0} and continuous acceleration along the Oy axis on which oscillations with frequency Ω are superimposed. In particular, we pay attention to the fact that the electrons having at $\chi = 0$, $\tau = 0$ the values of components: $v_0 = v_{d0}$, $w_0 = 0$, later move towards the wave (on the Ox axis) strictly with constant velocity v_{d0}, and all the remaining particles move at the same velocity, too, but only on average (at oscillation period averaging). We also note that the drift velocity value v_{d0} may be obtained from equations (1.3.1) and (1.3.9), by setting the right member of equation (1.3.1) to zero. The conditions for the long-term confinement of the particles trapped in the well at the initial instant depend greatly on the value of R: at $R < 1$ all the particles over some time escape from the well, at $R \geq 1$ a very small portion is trapped, at $R >> 1$ almost all the particles are trapped. However, the value of R was not expedient to take too large since the accelerating field decreases for the given value of E_A. Indeed, at surfatron acceleration the particle gains energy in the electric field $E_y = \beta B = \beta E_A/R$, hence, to obtain large values of E_y at the given E_A, the value of R is obviously to be close to unity, and the wave velocity is to be close to the speed of light. Thus, we take the values of $R \geq 1$ at which a portion of particles is still confined in the $|\chi > |\chi_d$, region, and, hence, we do not consider large R necessary to confine particles out of this region.

Let us proceed to searching solutions for equations (1.3.1), (1.3.2) at the relativistic phase ($\beta\tau >> 1$). Taking into account that the absolute values of the v velocity component do not change at the non-relativistic phase, i.e., they remain the same as originally, and assuming that $v \ll 1$ at $\beta\tau \gg 1$, from equations (1.3.3)-(1.3.5) we obtain the following solutions, supposing $\gamma_v \approx 1$:

$$\gamma(\tau) \approx (1+\beta^2\tau^2)^{1/2},\ w(\tau) \approx \beta\tau/\gamma(\tau),\ p_y \approx \beta\tau,\ v(\tau) \approx \gamma(\tau). \quad (1.3.12)$$

Assuming, that the solution character for equations (1.3.1), (1.3.2) does not change at the relativistic phase, we obtain the solution for the remaining unknown value of $\chi(\tau)$ from equation (1.3.1) in the form of the two summand sum: $\chi(\tau) = \chi_d(\tau) + \xi(\tau)$, where χ_d is the coordinate of the force equilibrium point on the χ axis which moves with the drift velocity $v_d(\tau) = d\chi_d/d\tau$, $\xi(\tau)$ is the oscillating part of the solution. Equating the right member of equation (1.3.1) to zero, we obtain an expression for temporal variation of the local equilibrium point coordinate in zero approximation in the view $\chi_d(\tau) = \beta\tau/[\Omega^2\gamma(\tau)]$. Hence, the drift velocity is $v_d(\tau) = \beta/[\Omega^2\gamma(\tau)^3]$. Confining ourselves to this approximation, we obtain solutions for $\chi_d(\tau)$ and $v_d(\tau)$ in the view:

$$\chi_d(\tau) = v_{d0}\tau/\gamma(\tau),\ v_d(\tau) = v_{do}/\gamma(\tau)^3.$$

The equation for the oscillating part

$$\tau\, d^2\xi/d\tau^2 + d\xi/d\tau + D^2/\beta = 0$$

has the solution $\xi(\tau) = J_0(z)$, where $z = 2D(\tau/\beta)^{1/2}$, J_0 is the Bessel function of the first kind of zero order. Thus, we obtain common solutions for $\chi(\tau)$ and $v(\tau)$ in the view:

$$\chi(\tau) \approx v_{d0}\tau/\gamma + v_0\,[\beta^{1/4}/(\pi^{1/2}D^{1/2}\tau^{1/4})]\sin z, \quad (1.3.13)$$

$$v(\tau) \approx v_{d0}\tau/\gamma^3 + v_0\,[D^{1/2}/(\pi^{1/2}\beta^{1/4}\tau^{3/4})]\cos z. \quad (1.3.14)$$

Here, the expressions for $\chi(\tau)$, $v(\tau)$ are written by using the Bessel function representations at large arguments ($z \gg 1$). So, relations (1.3.7)-(1.3.14) represent the complete set of solutions describing the behavior of the electron in surfatron at $\psi_A \ll 1$.

From the analysis of these results, there follows an important and simple conclusion: at all the phases, the particle, continuously accelerated on the *Oy* axis, tends to move along the *Ox* axis in the vicinity of the point at which all the forces' *x*-component total (i.e. the right member of equation (1.3.1)) is equal to zero. Obviously, this conclusion is common enough to the surfatron mechanism for particle acceleration. As a consequence of this remarkable fact, in computations we revealed a particular acceleration mode of the electron which is in the equilibrium state from the very beginning. Such an electron, having initial velocity components $v_0 = v_{d0}$, $w_0 = 0$, moves on the *Ox* axis strictly at velocity v_{d0} at the non-relativistic phase. At a given parameter $R \approx 1$, the electron, having initial velocity $v_0 = v_{d0}$, is confined in the well longer than the electron having the initial velocity $v_0 = 0$. Hence, one may conclude that it is more optimal to inject particles at the velocity less than the wave velocity by value of v_{d0} into the wave. As it follows from the solutions obtained, at $\tau \to \infty$ the electron frequency period $\sim \tau^{1/2}$ grows and both the drift velocity $\sim \tau^{-3}$ and the oscillation amplitude $\sim \tau^{-1/4}$ decrease tending to zero.

1.3.3. High amplitude wave ($\psi_A \gg 1$)

In this extreme case, dimensionless parameters have values for optimal $R \geq 1$:

$D \ll 1$, $\Omega \approx 1$, $v_{d0} \approx 1$, and the well width becomes very great: $\chi_d \gg 1$. This means that at the motion initial phase, in the right member of equation (1.3.1), the second term is negligibly small. Indeed, the electron here starts moving in stationary and homogeneous fields B and $E_y = \beta B$. Assuming $\beta \approx 1$, $D^2 \chi \approx 0$, $p_{x0} \ll 1$, $p_{y0} \ll 1$, $\gamma_0 \approx 1$, solutions for equations (1.3.1), (1.3.2) may be written as a function of variable $p_y = p_y(\tau)$, defined from equation $p_y + p_y{}^3/6 = \tau$:

$$p_x(\tau) = p_y{}^2/2,\ \gamma(\tau) = 1 + p_y{}^2/2,\ w(\tau) = 2p_y/(2 + p_y{}^2),$$

$$v(\tau) = p_y{}^2/(2 + p_y{}^2),\ \chi(\tau) = p_y{}^3/6.$$

At the non-relativistic phase $p_y(\tau) \approx \tau < 1$ these solutions have a simple view:

$$p_x(\tau) = v(\tau) = \tau^2/2,\ w(\tau) = \tau,\ \gamma(\tau) = 1 + \tau^2/2,\ \chi(\tau) = \tau^3/6. \qquad (1.3.15)$$

At the relativistic phase $p_y = (6\tau)^{1/3} > 1$, and solutions become as follows:

$$p_x(\tau) = \gamma(\tau) = (6\tau)^{2/3}/2,\ \chi(\tau) = \tau, \qquad (1.3.16)$$

$$w(\tau) = 2(6\tau)^{-1/3},\ v(\tau) = (6\tau)^{2/3}/[2 + (6\tau)^{2/3}]. \qquad (1.3.17)$$

From the given solutions it follows that by $\tau \approx 2$, at which $p_y = 2^{1/2}$, the w velocity component will reach a maximum value of $w = 2^{-1/2}$, thus $v = 1/2$. Later, $v \to 1$, and the w component decreases with time by the law for $w \sim \tau^{-1/3}$. Solutions (1.3.16), (1.3.17) are true at $1 \ll \tau \ll \tau_q$, where τ_q is the time defined from the condition $w(\tau_q) = D^2 \chi(\tau_q)$: $\tau_q \sim D^{-3/2} \gg 1$. By this time, the particle will escape at a distance $\chi \approx \tau_q \approx D^{-3/2} \ll \chi_q = D^{-2}$, and the force equilibrium condition on the Ox axis for it will be reached. Further, as already noted, by the law for surfatron acceleration, the particle's motion along the Ox axis occurs so that the equilibrium condition maintain. Thus, at $\tau > \tau_q$ solutions (1.3.12)-(1.3.14), which have $\Omega \approx 1$, $\beta \approx 1$, become possible and, hence, the v velocity component starts to decrease, and the w component increases. At a time, the particle, having velocity components $w \approx 1$, $v \ll 1$, reaches a point χ_q and starts to oscillate in the vicinity of that point. Consequently, the character of the motion along the Ox axis and at $\psi_A \gg 1$ does not change: first, the particle with the drift velocity (over the $1 < \tau < \tau_q$ interval velocity $v_{d0} \approx 1$) moves to the asymptotic equilibrium point $\chi_q = 1/D^2$, and then starts to oscillate near that point with an amplitude attenuating with time.

So, we managed to find analytical solutions for the problem set from which it follows that the electrons trapped in a non-linear wave, being continuously accelerated along the wave front, oscillate along the Ox axis near a coordinate and drift to meet the wave motion towards a point where the electric field value is compared with the magnetic field value (in the wave frame). Further, when electrons are accelerated in surfatron, their oscillation amplitude decreases, and the accelerated particle actually moves with the wave. Thus, from the solutions obtained, phase positive stability or phase focusing at electron surfatron acceleration follows in explicit form.

In conclusion, we note that for the considered non-linear wave, which is periodic, surfing is possible simultaneously both for electrons trapped by the wave and for ions. Throughout the process, the obtained surfing regularities for electrons and ions will be absolutely equal.