



Intermediate Algebra *for College Students*

Fourth Edition

Annotated
Instructor's
Edition



Annotated Instructor's Edition



Intermediate Algebra

for College Students

Fourth Edition

Jerome E. Kaufmann



PWS-KENT Publishing Company

Boston



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Intermediate Algebra

for College Students



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P r e f a c e

When I prepared this fourth edition, I attempted to preserve the features that made the previous editions successful; at the same time I wanted to incorporate a number of improvements that the reviewers suggested.

This text was written for college students who need an algebra course that bridges the gap between elementary algebra and the more advanced courses in precalculus mathematics. It covers topics that are usually classified as intermediate algebra topics.

The basic concepts of intermediate algebra are presented in a simple, straightforward manner. Algebraic ideas are developed in a logical sequence, but in an easy-to-read manner without excessive formalism. Concepts are frequently motivated by examples and they are continuously reinforced by additional examples. *In this fourth edition I added several examples to better illustrate the concept under discussion. Users of the previous editions were very helpful in suggesting such examples.

The examples demonstrate a large variety of situations, but I still leave many things for the students to think about in the problem sets. In the examples, I guide students to organize their work and to decide when a meaningful shortcut might be used.

A focal point of every revision is the problem sets. *Again in this edition I carefully analyzed the problem sets to make sure that variations of skill development exercises are contained in both the even- and odd-numbered problems. Users of the previous editions were very helpful in suggesting problems to be added, deleted, or changed.

* Specific references to this fourth edition

Some of the problem sets contain a category of problems called **Miscellaneous Problems**. These are either problems designed to give the students an opportunity to see what lies ahead in college algebra or problems designed to broaden the students' background in a particular topic. All of them could be omitted without breaking the continuity pattern of the text; however, I feel that they do add another flexibility feature. You may wish to turn to Problem Sets 5.6 and 7.5 to see examples of these problems.

Chapter Review Problem Sets provide a comprehensive vehicle for students to use to pull together the concepts of a chapter. **Cumulative Review Problem Sets** appear at the ends of Chapters 3, 5, 7, and 9.

In this edition, we have also included a few special problems at the end of most Chapter Review Problem Sets. These problems, titled **Thoughts into Words**, provide the students with an opportunity to express, in written form, their thoughts about various mathematical ideas.

There is a common thread throughout the book, which is *learn a skill, then use the skill to help solve equations and inequalities, and then use equations and inequalities to solve word problems*. This thread influenced some other decisions.

1. Numerous word problems are scattered throughout the text. Every effort was made to start with easy ones, then gradually increase the level of difficulty.
2. Many problem solving suggestions are offered throughout, with special discussions in several sections. The key is to work with various problem solving techniques; not to become overly concerned that all of the traditional types of problems are studied.
3. Newly acquired skills are used as soon as possible to solve equations and inequalities, which are, in turn, used to solve word problems. Therefore, the concept of solving equations and inequalities is introduced early and developed throughout the text. The concepts of factoring, solving equations, and solving word problems are tied together in Chapter 3.

As recommended by the American Mathematical Association of Two-Year Colleges, some basic geometric concepts used in subsequent courses were integrated in a problem solving setting as follows.

- Section 2.2. Complementary and supplementary angles; sum of the angles of a triangle equals 180°
- Section 2.4. Area and volume formulas
- Section 3.4. More on area and volume formulas, perimeter and circumference formulas
- Section 3.7. Pythagorean Theorem
- Section 6.2. More on Pythagorean Theorem, including work with isosceles right triangles and 30° – 60° right triangles

I tried to assign the calculator its rightful place in the study of mathematics, that is, as a tool, useful at times, unnecessary at other times. No special problems

were created just so that we could use the calculator. Instead some of the usual intermediate algebra problems, which lend themselves to the use of a calculator, were labeled as calculator problems.

Specific Comments about Some of the Chapters

1. Chapter 1 was written so that it can be covered quickly, and on an individual basis if desired, by those needing only a brief review of some basic algebraic concepts. *In this fourth edition, Sections 1.2 (Addition and Subtraction of Integers) and 1.3 (Multiplication and Division of Integers) were combined to further facilitate a review situation.
2. Chapter 2 presents an early introduction to the heart of an intermediate algebra course. By introducing problem solving and the solving of equations and inequalities early, they can be used as unifying themes throughout the text.
3. In Chapter 5 exponents and radicals are developed separately and then merged at the end of the chapter to unify rational exponents and roots. The general concept of n th root is discussed, but in the simplification problems we concentrate on square root and cube root.
4. Chapter 6 was organized to give students an opportunity to learn on a day-to-day basis different techniques for solving quadratic equations. *At the request of users of the previous editions, I moved the work on complex numbers into the first section of the chapter. The other sections were rewritten to include complex numbers throughout the development. The process of completing the square is treated as a viable equation solving process for certain types of quadratic equations. The emphasis on completing the square here pays dividends in Chapter 7 when graphing parabolas and circles. Section 6.5 offers some guidance as to when to use a particular technique for solving quadratic equations. Furthermore, the often overlooked relationships involving the sum and products of roots are discussed and used as an effective checking procedure.
5. Chapter 7 was written on the premise that intermediate algebra students should become *very familiar* with straight lines, parabolas, and circles with only limited exposure to ellipses and hyperbolas. I intentionally kept the function concept out of Chapter 7. My personal experience indicates that these students need more work with coordinate geometry concepts—specifically graphing techniques—before being introduced to the idea of a function. *Section 7.1 was rewritten to place emphasis on finding intercepts and using the concept of symmetry for graphing purposes.
6. Chapter 8 is devoted entirely to functions and the issue is not clouded by jumping back and forth between functions and relations that are not functions. It includes some work on the composition of functions and the use of quadratic functions in problem solving situations.

* Specific references to this fourth edition

7. Chapter 9 contains the various techniques for solving systems of linear equations. It was organized so that instructors can use as much of the chapter as needed for their particular course. *I rewrote the first three sections so that we now begin with the substitution method in Section 9.1. Then in Section 9.2 the elimination-by-addition method is presented using an approach that emphasizes equivalent systems and sets the stage for future work with matrices.
8. Chapter 10 presents a modern day version of the concepts of exponents and logarithms. The emphasis is on making the concepts and their applications understood. The calculator is used as a tool to help with the messy computational aspects. *In this fourth edition, I have rewritten several parts of this chapter in order to place more emphasis on the applications of exponents and logarithms.

Supplements

For Instructors:

The following supplements are available to adopters:

1. An *Annotated Instructors's Edition* that contains answers for all of the problems in the text.
2. An *Answer Book* that contains answers for all of the problems except the Chapter Review Problem Sets, which are in the back of the book.
3. A printed *Test Bank* that contains two multiple choice and one short-answer test for each chapter.
4. An *EXPTest* computerized test bank for IBM PCs and compatibles that features over one thousand questions written specifically to accompany this text by Joan and Stuart Thomas of the University of Oregon. This program allows users to view and edit all tests, to add to, delete from, and modify existing questions, and to print any number of student tests.
5. A computerized testing program for the MacIntosh that features test questions written by Joan and Stuart Thomas. Questions can be stored by objectives, and the user can scramble the order of questions.

For Students:

1. A *Partial Solutions Manual* (which students may purchase) that contains solutions for most of the problems numbered 1, 5, 9, 13, etc.
2. A *TrueBASIC Algebra* software package available for both IBM PCs and compatibles and the MacIntosh contains a disk and user manual for

* Specific references to this fourth edition

self-study, free exploration of topics, and solution of problems. This package also includes a record and playback feature that makes the program ideal for classroom demonstrations.

3. A set of *Expert Algebra Tutor* disks for IBM PCs and compatibles by Sergei Ovchinnikov of San Francisco State University. These disks are tutorial software page-referenced to specific sections of the text. They define the level of tutoring needed by evaluating the user's need for further remediation or advancement in the tutoring session.
4. A set of *Videotapes* that can be used by students in an independent developmental math lab setting to review the topics in the textbook.

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Again, very special thanks are due my wife, Arlene, who spends numerous hours typing and proofreading manuscripts, answer keys, solutions manuals, and test banks.

Jerome E. Kaufmann

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Appendixes**A1**

Appendix A Synthetic Division and the Factor Theorem

A2

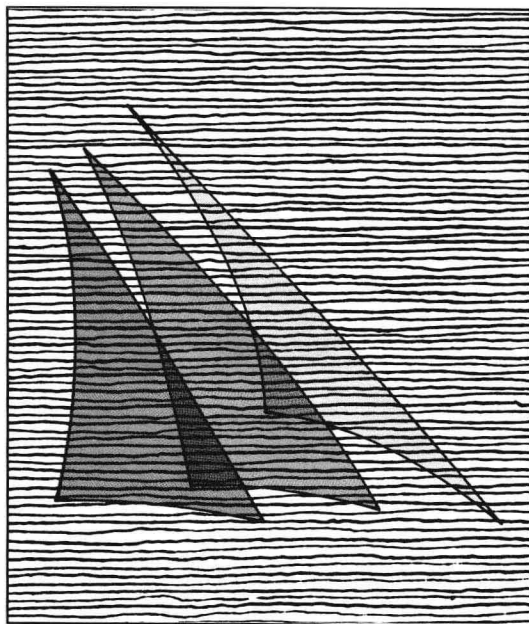
Appendix B Common Logarithms

A6

Appendix C Natural Logarithms

A16**Answers to Odd-Numbered Problems and all
Chapter Review Problems****A19****Index****I2**

Chapter 1



Algebra is often described as a **generalized arithmetic**. That description may not tell the whole story, but it does indicate an important idea: A good understanding of arithmetic pro-

Basic Concepts and Properties

- 1.1 Sets, Real Numbers, and Numerical Expressions
 - 1.2 Operations with Real Numbers
 - 1.3 Properties of Real Numbers and the Use of Exponents
 - 1.4 Algebraic Expressions
-

vides a sound basis for the study of algebra. Be sure that you thoroughly understand the basic concepts that we review in this first chapter.