Automatic Target Recognition XII

Vol. 4726

PROCEEDINGS OF SPIE
SPIE—The International Society for Optical Engineering

Automatic Target Recognition XII

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Chair/Editor

2–4 April 2002
Orlando, USA

Sponsored and Published by
SPIE—The International Society for Optical Engineering

Volume 4726

SPIE is an international technical society dedicated to advancing engineering and scientific applications of optical, photonic, imaging, electronic, and optoelectronic technologies.

Please use the following format to cite material from this book:

Author(s), "Title of paper," in *Automatic Target Recognition XII*, Firooz A. Sadjadi, Editor, Proceedings of SPIE Vol. 4726, page numbers (2002).

ISSN 0277-786X
ISBN 0-8194-4476-6

Published by
SPIE—The International Society for Optical Engineering
P.O. Box 10, Bellingham, Washington 98227-0010 USA
Telephone 1 360/676-3290 (Pacific Time) • Fax 1 360/647-1445
http://www.spie.org/

Printed in the United States of America.

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Introduction

This book is a collection of papers on automatic target recognition (ATR) technology that were presented at the 12th conference on Automatic Target Recognition (ATRXII), held in April 2002.

The goal of this conference is, and has been for the past 12 years, to provide an unrestricted medium for presentations and discussions of ATR-related issues. As evidenced by the success of these yearly meetings, the interest in ATR technology, both in its methodologies and specific applications, has been on a steady rise for the past decade.

With the rapid introduction of the unmanned aerial vehicles (UAV) and the more advanced avionics onboard of manned air platforms, the need for ATR systems has become more urgent than ever before. This urgency has also created a need for new methodologies to use new sensory outputs in faster and more adaptive ways. The papers contained in these conference proceedings reflect the most current trends and represent the latest ongoing research and development in the field.

I thank the conference committee members, session chairs, authors, and other participants who once again by their efforts and their enthusiasm made this year's meeting a fun and educational gathering for all of us.

Firooz Sadjadi

Contents

Generalization of Group Velocity and Group Acceleration

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Group velocity is typically defined as the derivative of the dispersion relation. However, we show that definition is not appropriate in general and is only applicable to wave equations with constant coefficients. We generalize the concept of group velocity and also show that in general one also has group acceleration. Explicit expressions are derived and examples are given.

1. INTRODUCTION

The classical method for solving a wave equation with constant coefficients,

$$\sum_{n=0}^{N} a_n \frac{\partial^n u}{\partial t^n} = \sum_{m=0}^{M} b_m \frac{\partial^m u}{\partial x^m} \tag{1}$$

is to substitute $e^{ikx-i\omega t}$ into it to obtain a relation between k and ω,

$$\sum_{n=0}^{N} a_n(-i\omega)^n = \sum_{m=0}^{M} b_m(ik)^m \tag{2}$$

One then solves for ω in terms of k to obtain the dispersion relation

$$\omega = W(k) \tag{3}$$

and typically there will be more than one solution. Each solution to Eq. (3) is called a mode and the solution to the wave equation for each mode is [5, 8],

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int S(k)\, e^{ikx-iW(k)t}\, dk \tag{4}$$

where $S(k)$ is the initial spatial spectrum which is obtained from the initial pulse by way of

$$S(k) = \frac{1}{\sqrt{2\pi}} \int u(x,0)\, e^{-ikx}\, dx \tag{5}$$

From the dispersion relation one obtains the group velocity defined by

$$v_g(k) = \frac{dW(k)}{dk} \tag{6}$$

Group velocity is interpreted in a variety of ways and we will discuss this at length in subsequent sections. But we emphasize here that while indeed $v_g(k)$ is called the group velocity in the literature, in the opinion of the author, it is not an discriptive terminology for what it represents becouse it is definitely not the velocity of a pulse or wave packet. As we will see in the next section the velocity of a group of waves is the average of the group velocity.

Examples of wave equations with constant coefficients are many, the classical wave equation

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \tag{7}$$

[1]Work supported by the Air Force Information Institute Research Program (Rome, NY) and the NSA HBCU/MI program.

being the best known and studied. The dispersion relation and group velocity are respectively

$$\omega = \pm ck \qquad v_g(k) = \pm c \tag{8}$$

which shows that all wave numbers travel with the same velocity and therefore there is no dispersion. Another example is the beam equation[7, 8],

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^4 u}{\partial x^4} \tag{9}$$

and one has

$$\omega = \pm ck^2 \qquad v_g(k) = \pm 2ck \tag{10}$$

However, what do we do if the wave equation has variable coefficients? For example suppose we consider

$$\frac{1}{c^2(x)}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \tag{11}$$

where $c(x)$ is position dependent, what is the group velocity then? Or, consider the general string equation[7]

$$\varepsilon(x)\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[T(x)\frac{\partial u}{\partial x}\right] \tag{12}$$

where $\varepsilon(x)$ and $T(x)$ are position dependent density and tension. What is the group velocity? Another, example is the Schrodinger equation

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi \tag{13}$$

where $V(x)$ is the potential, and the same question may be asked. More generally what is the meaning of group velocity for wave equations of the form

$$\sum_{n=0}^{N} a_n(t)\frac{\partial^n u}{\partial t^n} = \sum_{m=0}^{M} b_m(x)\frac{\partial^m u}{\partial x^m} \tag{14}$$

Our aim is to address this question.

2. What is group velocity?

In discussions of group velocity a statement such as this is typically made: group velocity is the velocity with which a pulse propagates, and, arguments are then made to make Eq. (3) plausible. Typically, these arguments are made independent of any particular wave equation and the implication is that the arguments apply in general. We will argue that it is not the case that group velocity is the velocity with which a pulse propagates. In fact the velocity with which a pulse propagates will be seen to be the average of the group velocity, where the averaging is done with the initial spectrum and therefore the velocity of the pulse for a given group velocity varies dramatically with the initial spectrum! Second, we will argue that the concept of group velocity as usually discussed applies to pulses governed by linear differential equations with constant coefficients although sometimes it does apply in other situations. We first examine exact results for equations with constant coefficients and that will lead us to a way to generalize the concepts.

Motion of a pulse governed by wave equations with constant coefficients

It is a remarkable fact that for any wave equation with constant coefficients the velocity of the center of mass moves with a constant velocity. Explicitly, suppose we have an initial pulse $u(x,0)$ and we evolve it using any equation of the form given by Eq. (1) to obtain $u(x,t)$. We then ask for the center of mass motion for a node defined by

$$\langle x \rangle_t = \int x \, |u(x,t)|^2 \, dx \tag{15}$$

One can show that, exactly, [4]

$$\langle x \rangle_t = \langle x \rangle_0 + Vt \tag{16}$$

where

$$V = \int v_g(k) \, |S(k)|^2 \, dk \tag{17}$$

and where $S(k)$ is the initial spatial spectrum which obtained from the initial pulse,

$$S(k) = \frac{1}{\sqrt{2\pi}} \int u(x,0) \, e^{-ikx} \, dx \tag{18}$$

We emphasize that Eq. (15) is exact and applies to any wave equation with constant coefficients. We see that the velocity of the pulse is the average of the group velocity, the averaging being done with $|S(k)|^2$ and therefore the velocity of the center of mass varies dramatically because we can choose $|S(k)|^2$ at will. In addition we emphasize that there are no higher terms, that is, there is no acceleration! Why is it that $v_g(k)$ is sometimes called the group velocity? The reason is as follows. In many derivations that appear in the literature a narrowband pulse is assumed and that can be thought of as a pulse with a spectrum that is very peaked at some value of k say k_0. Then indeed $V \sim v_g(k_0)$, as can be seen from Eq. (15). However we emphasize that no such approximation is needed as Eq. (17) is exact.

3. Approach

It is our aim to generalize the concept of group velocity for wave equations with non-constant coefficients. The approach we will take is as follows. From the wave equation we will obtain the center of mass motion. The center of mass motion will not in general be of constant velocity but nontheless one will be able to expand it in terms of a Taylor series,

$$\langle x \rangle_t = \langle x \rangle_0 + V(0)t + \frac{1}{2}a(0)t^2 \cdots \tag{19}$$

From the derivation we will be able to see how the velocity, $V(0)$, depends on wave numbers or other physical quantities and therefore we will be able to define group velocity for the general case. Also, it will allow us to define group acceleration. The approach we will take is the one developed in quantum mechanics for the Schrodinger equation but we will be dealing with a general linear wave equation with non-constant coefficients.

Mathematical Preliminaries and Notation

We define the time dependent spatial spectrum, $S(k,t)$, by

$$S(k,t) = \frac{1}{\sqrt{2\pi}} \int u(x,t) \, e^{-ikx} \, dx \tag{20}$$

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int S(k,t)\, e^{ikx}\, dk \tag{21}$$

The position operator is denoted by $\mathcal{X}$ and in the k representation it is

$$\mathcal{X} = i\,\frac{\partial}{\partial k} \tag{22}$$

Also, the wave number operator is denoted by $\mathcal{K}$ and in the x representation it is

$$\mathcal{K} = -i\,\frac{\partial}{\partial x} \tag{23}$$

In general the moments of a pulse are defined by

$$\langle\, x^n \,\rangle_t = \int x^n\, |u(x,t)|^2\, dx \tag{24}$$

and they can be calculated from the spectrum by way of

$$\langle\, x^n \,\rangle_t = \int S^*(k,t)\, \mathcal{X}^n\, S(k,t)\, dk \tag{25}$$

This of course is nothing more than the momentum representation as usually defined in quantum mechanics.

We will need certain commutation relations between position and wave number. The basic commutation relation is [2]

$$[\mathcal{X}, \mathcal{K}] = i \tag{26}$$

Other important relations that we will need are [1, 6, 3]

$$[\mathcal{X}^n, \mathcal{K}] = in\mathcal{X}^{n-1} \tag{27}$$

$$[\mathcal{X}, \mathcal{K}^n] = in\mathcal{K}^{n-1} \tag{28}$$

Also for any function of x, $f(x)$, we have

$$[\mathcal{K}^n, f(x)] = \sum_{l=1}^{n} \binom{n}{l} \left(\mathcal{K}^{\ell} f(x)\right) \mathcal{K}^{n-l} \tag{29}$$

Functions of Operators

Suppose we have a Hermitian operator $H(\mathcal{X}, \mathcal{K})$ made up of the operators $\mathcal{X}$ and $\mathcal{K}$. Since it is Hermitian the eigenvalue problem

$$H(\mathcal{X}, \mathcal{K}) u_\gamma(x) = \gamma u_\gamma(x) \tag{30}$$

produces a complete set of functions that satisfy

$$\int u^*_{\gamma'}(x) u_\gamma(x) dx \;=\; \delta(\gamma - \gamma') \tag{31}$$

$$\int u^*_{\gamma'}(x') u_\gamma(x) d\gamma \;=\; \delta(x - x') \tag{32}$$

[2]Commutators will be denoted by the usual notation, $[A, B] = AB - BA$.

Of particular interest is the construction of functions of operators. For a function $F(x)$ one can construct the operator $F(H(\mathcal{X},\mathcal{K}))$ by replacing x with $H(\mathcal{X},\mathcal{K})$. The operator $F(H(\mathcal{X},\mathcal{K}))$ has the same eigenfunctions as $H(\mathcal{X},\mathcal{K})$ but with eigenvalue $F(\gamma)$. That is,

$$F(H)u_\gamma(x) = F(\gamma)u_\gamma(x) \tag{33}$$

An arbitrary operator can be written in the following explicit form

$$H(\mathcal{X},\mathcal{K}) = \int\int \gamma u_\gamma(x)u^*(\gamma, x+x')e^{ix'\mathcal{K}}d\gamma dx' \tag{34}$$

and also,

$$F(H(\mathcal{X},\mathcal{K})) = \int\int F(\gamma)u_\gamma(x)u^*(\gamma, x+x')e^{ix'\mathcal{K}}d\gamma dx' \tag{35}$$

These are useful forms because one can write an arbitrary operator explicitly in terms of the position and wave number operators.[4]

4. Solution to the wave equation

We consider wave equations of the form

$$\sum_{n=0}^{N} a_n(t)\frac{\partial^n u}{\partial t^n} = \sum_{m=0}^{M} b_m(x)\frac{\partial^m u}{\partial x^m} \tag{36}$$

Our aim is to obtain a symbolic solution to Eq. (36) and then calculate the expectation value of position to pick out the velocity term. We write the wave equation as

$$T(t)u = H(x)u \tag{37}$$

with[3]

$$T(t) = \sum_{n=0}^{N} a_n(t)\frac{\partial^n}{\partial t^n} \qquad H(x) = \sum_{m=0}^{M} b_m(x)\frac{\partial^m}{\partial x^m} \tag{38}$$

Following the usual seperation of variables procedure we write

$$u = A(t)B(x) \tag{39}$$

to obtain

$$\frac{T(t)A}{A} = \frac{H(x)B}{B} = \gamma \tag{40}$$

which results in two eigenvalue equations

$$T(t)A_\gamma(t) = \gamma A_\gamma(t) \tag{41}$$

$$H(x)B_\gamma(x) = \gamma B_\gamma(x) \tag{42}$$

[3]We assume both T and H are Hermitian and assume that the operators have the properties needed to manipulate as we do. In a future publication we will consider the mathematical issues involved.

We assume that the eigenvalues are continuous; the discrete case can be obtained by a simple rewriting of the equations obtained. Very often the eigenvalue will be a function of parameters, say, η and ℓ and in those cases we write

$$T(t)A_\eta(t) = \alpha(\eta)A_\eta(t) \tag{43}$$

$$H(x)B_\ell(x) = \beta(\ell)B_\ell(x) \tag{44}$$

and for this situation we must have that

$$\alpha(\eta) = \beta(\ell) = \gamma \tag{45}$$

We now show that the solution, $u(x,t)$, can be written as

$$u(x,t) = A_H(t)u(x,0) \tag{46}$$

where $A_H(t)$ means that we have substituted the operator H for γ. First, we point out that symbolically

$$T(t)A_H(t) = HA_H(t) \tag{47}$$

To show that Eq. (46) is correct, operate on both sides with $T(t)$

$$T(t)u(x,t) = T(t)A_H(t)u(x,0) \tag{48}$$
$$= HA_H(t)u(x,0) \tag{49}$$
$$= Hu(x,t) \tag{50}$$

which is the governing wave equation, Eq. (37).

Now consider the expectation value of position

$$\langle x \rangle_t = \int x\,|u(x,t)|^2\,dx \tag{51}$$
$$= \int [A_H(t)u(x,0)]^*\,xA_H(t)u(x,0)\,dx \tag{52}$$
$$= \int u^*(x,0)\mathcal{X}(t)u(x,0)\,dx \tag{53}$$

where we have defined the time dependent position operator by

$$\mathcal{X}(t) = A^\dagger(t,H)\mathcal{X}(0)A_H(t) \tag{54}$$

and where $A^\dagger$ is the Hermitian adjoint. We expand $\mathcal{X}(t)$ as a power series up to the second order in t. We have

$$\frac{\partial\mathcal{X}(0)}{\partial t} = \frac{\partial A^\dagger(0,H)}{\partial t}\mathcal{X}(0)A(0,H) + A^\dagger(0,\beta)\mathcal{X}(0)\frac{\partial A(0,H)}{\partial t} \tag{55}$$

$$\frac{\partial^2\mathcal{X}(0)}{\partial t^2} = \frac{\partial A^{+2}(0,H)}{\partial t^2}\mathcal{X}(0)A(0,H) + A^\dagger(0,\beta)\mathcal{X}(0)\frac{\partial A^2(0,H)}{\partial t^2} \tag{56}$$
$$+2\frac{\partial A^\dagger(0,H)}{\partial t}\mathcal{X}(0)\frac{\partial A(0,H)}{\partial t} \tag{57}$$

and therefore

$$\mathcal{X}(t) = \mathcal{X}(0) + V(0)t + \frac{1}{2}a(0)t^2\cdots\cdots \tag{58}$$

with

$$V(0) = \frac{\partial A^{\dagger}(0,H)}{\partial t}\mathcal{X}(0)A(0,H) + A^{\dagger}(0,\beta)\mathcal{X}(0)\frac{\partial A(0,H)}{\partial t} \tag{59}$$

$$a(0) = \frac{\partial A^{+2}(0,H)}{\partial t^2}\mathcal{X}(0)A(0,H) + A^{\dagger}(0,\beta)\mathcal{X}(0)\frac{\partial A^2(0,H)}{\partial t^2} + 2\frac{\partial A^{\dagger}(0,H)}{\partial t}\mathcal{X}(0)\frac{\partial A(0,H)}{\partial t} \tag{60}$$

Special case 1

Of particular interest is the equation of the form

$$i\frac{\partial u}{\partial t} = Hu \tag{61}$$

with

$$H(x) = \sum_{m=0}^{M} b_m(x)\frac{\partial^m}{\partial x^m} = \sum_{m=0}^{M} i^m b_m(\mathcal{X})\mathcal{K}^m \tag{62}$$

Solving the eigenvalue problem

$$T(t)A_\gamma(t) = i\frac{\partial}{\partial t}A_\gamma(t) = \gamma A_\gamma(t) \tag{63}$$

gives

$$A_\gamma(t) = e^{-i\gamma t} \tag{64}$$

and therefore

$$u(x,t) = A_H(t)u(x,0) = e^{-iHt}u(x,0) \tag{65}$$

Now, as for the quantum case we have that [1, 6][4]

$$\mathcal{X}(t) = e^{itH}\mathcal{X}e^{-itH} \tag{66}$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}[\mathcal{X},H]_n\left(\frac{t}{i}\right)^n \tag{67}$$

$$= \mathcal{X} + [\mathcal{X},H]\frac{t}{i} + \frac{1}{2!}[[\mathcal{X},H],H]\left(\frac{t}{i}\right)^2\cdots \tag{68}$$

Therefore the velocity and acceleration operators are

$$v(0) = -i[\mathcal{X},H] \tag{69}$$

$$a(0) = -[[\mathcal{X},H],H] \tag{70}$$

[4]For notational simplicity when we write $\mathcal{X}$ we shall mean $\mathcal{X}(0)$.