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# ENCYCLOPÆDIA BRITANNICA

Volume 21

SORDELLO TO TEXTBOOKS



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"LET KNOWLEDGE GROW FROM MORE TO MORE  
AND THUS BE HUMAN LIFE ENRICHED."



# ENCYCLOPÆDIA BRITANNICA

## Volume 21 SORDELLO TO TEXTBOOKS

**SORDELLO**, a 13th-century Italian troubadour, born at Goito (c. 1200), who is praised by Dante in the *De vulgari eloquentia*, and in the *Purgatorio* is made the type of patriotic pride. He is the hero of a well-known poem by Robert Browning. The real Sordello was the most famous of the Italian troubadours. About 1220 he appeared at Florence in a tavern brawl; and in 1226, while at the court of Richard of Bonifazio at Verona, he abducted his master's wife, Cunizza, at the instigation of her brother, Ezzelino da Romano. The scandal resulted in his flight (1229) to Provence, where he seems to have been for some time. He entered the service of Charles of Anjou, and probably accompanied him (1265) on his Naples expedition; in 1266 he was a prisoner in Naples. The last documentary mention of him is in 1269, and he is supposed to have died in Provence. His didactic poem, *L'Ensenhamen d'onor*, and his love songs and satirical pieces have little in common with Dante's presentation, but the invective against negligent princes which Dante puts into his mouth in the 7th canto of the *Purgatorio* is more adequately paralleled in his *Serventese* (1237) on the death of his patron Blacatz, where he invites all Christian princes to feed on the heart of the hero.

**SORDINO**, **SORDONI**, **SORDUN**, musical terms somewhat promiscuously applied (1) to contrivances for damping or muting wind, string and percussion instruments (*Sordini*); (2) to a family of obsolete wind instruments resembling the bassoon, blown by means of a double reed (*Sordoni* or *Sordun*); (3) to a stringed instrument. To these is added the *Surdellina* or *Sordellina*, a kind of musette invented (see BAGPIPE) in Naples in the 17th century.

**SOREL, AGNÈS** (c. 1422–1450), mistress of Charles VII of France, was

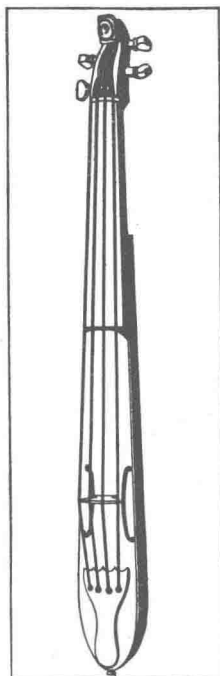
born of the lesser nobility at Fromenteau in Touraine. She was attached to the service of Isabella of Lorraine, wife of René of Anjou, then to that of the queen of France, Mary of Anjou. From 1444 she was the acknowledged mistress of the king, the first to hold that semi-official position. Her ascendancy dated from the festivals at Nancy in 1444, the first brilliant court of Charles VII. Charles's gift to her of the castle of Beauté-sur-Marne led to her being nicknamed "Reine de Beauté." Her death from dysentery, shortly after the birth of her fourth child in 1450, was attributed, apparently without foundation, to poison.

See G. du Fresne de Beaucourt, *Histoire de Charles VII*, vol. iii (Paris, 1881–91); P. Champion, *Agnès Sorel, la dame de Beauté* (Paris, 1931).

**SOREL, ALBERT** (1842–1906), French historian, was born at Honfleur on Aug. 13, 1842. He was of a characteristically Norman type and remained all his life a lover of his native province. He studied law in Paris and entered the foreign office (1866). In 1870 he was chosen as secretary by M. de Chaudordy, who had been sent to Tours as a delegate in charge of the diplomatic side of the problem of national defense; in these affairs he proved himself a most valuable collaborator. After the war of 1870–71, when Boutmy founded the *École libre des sciences politiques*, Sorel was appointed to teach diplomatic history (1872), a post in which he achieved great success.

Some of Sorel's courses have formed books: *Le Traité de Paris du 20 novembre 1815* (1873); *Histoire diplomatique de la guerre franco-allemande* (1875); also the *Précis du droit des gens* which he published (1877) in collaboration with his colleague Théodore Funck-Brentano. In 1875 Sorel left the foreign office and became general secretary of the *Présidence du sénat*.

His duties left him sufficient leisure for the great work of his life, *L'Europe et la révolution française* (8 vols., 1885–1904). His object was to do over again the work already done by Sybel, but from a less restricted point of view and with a clearer and more calm understanding of the chessboard of Europe. He spent almost 30 years in the preparation of this history; the analysis of the documents, mostly unpublished, on French diplomacy during the first years of the Revolution, which he published in the *Revue historique* (vol. v–vii, xi–xiii), shows with what scrupulous care he read the innumerable dispatches which passed under his notice. Sorel was elected a member of the Académie française (1894). He died in Paris on June 29, 1906.



BY COURTESY OF MESSRS. HOOPER AND JACKSON

**SORDINO**, AN OBSOLETE STRING INSTRUMENT

Sorel's other works include: *La Question d'Orient au XVIII<sup>e</sup> siècle, les origines de la triple alliance* (1878); *Montesquieu* (1887) and *Mme. de Staël* (1891) in the *Grands écrivains* series; *Bona-parte et Hoche en 1797* (1896) and *Recueil des instructions données aux ambassadeurs*, vol. i only (1884). Most of his essays and articles contributed to various reviews and to the *Temps* have been collected.

**SOREL, CHARLES**, SIEUR DE SOUVIGNY (1602–1674), French writer, author of *Histoire comique de Francion* (1623), a picaresque novel of the road in Burgundy and Paris, remarkable chiefly for its sense of life and vigour of language, which went through 30 editions before the end of the century. He also wrote *Le Berger extravagant* (1627), *Polyandre* (1648) and many shorter satires and scholarly and religious works, including one of the first histories of French literature, *La Bibliothèque Française* (1664).

Sorel was born in Paris. Guy Patin in his letters described him as a fat little man with a sharp nose, leading a quiet bachelor's life with his sister, planning more works than his health would allow him to write. His chief glory is to have inspired more than one scene of Molière. He died on March 7, 1674.

**BIBLIOGRAPHY.**—Biography by E. Roy (1891); *Francion*, ed. by E. Roy, 3 vol. (1928); G. Reynier, *Le Roman réaliste au 17<sup>e</sup> siècle* (1914).

(W. G. ME.)

**SORGHUM**, a cereal, forage and sirup crop plant grown in many countries and known botanically as *Sorghum vulgare*. Sorghum probably originated in Africa. The types grown for grain (grain sorghum) are called by various names, including durra, Egyptian corn, great millet or Indian millet. On the sub-continent of India it is known as jowar, cholam or jonna, and in the West Indies as petit mil or Guinea corn. In China and Manchuria it is called kaoliang. It is a strong grass, growing to a height of from 2 to 8 ft. or even 16 ft. The stalks and leaves are coated with a white waxy bloom. The pith in the stalks of different varieties may be juicy or rather dry. The juice may be sweet or nonsweet. The leaves are sheathing, solitary and about 2 in. broad and 2½ ft. in length; the panicles or flower clusters are loose, contracted or dense. Self-pollination of the flowers is common but considerable cross-pollination occurs. The grains may be either free or retained in the hulls after threshing. Many varieties are awned. The seeds are ellipsoid, rounded or flattened and of varied sizes somewhat smaller than wheat grains. The seeds may be white, yellow, red or brown. The hulls are mostly straw-coloured, red, brown or black.

Sorghum is the leading cereal grain in Africa and is important also in the United States, India, Pakistan, north China and Manchuria. It is grown to some extent in the U.S.S.R., Iran, Arabia, Argentina, Australia and southern Europe, as well as in other regions. It is best adapted to warm conditions and is very resistant to drought and heat. Hundreds of varieties are grown. The grain is similar in composition to that of maize except in being higher in protein and lower in fat. It replaces maize as a feed grain in hot, dry regions. For food it usually is ground into a meal and made into porridge, bread or cakes. Natives of south Africa refer to the product as "mealies." Whole grains sometimes are popped or puffed. The grain also is used in making starch, dextrose, paste and alcoholic beverages. The stalks provide fodder and building materials. The sweet sorghums (sorgos) are grown chiefly in the United States and south Africa for forage or for sirup manufacture. The sweet stalks are chewed by peoples of

various countries. The broomcorn plant, belonging also to the species *S. vulgare*, is similar to other sorghums in adaptation and many plant characteristics.

(J. H. MN.)

**SORIA**, a province of Spain, formed in 1833 out of Old Castile. Pop. (1950 census) 164,575; area, 3,977 sq.mi. Soria is a bleak region, bounded on three sides by mountains. A range of sierras culminating in the peaks of Urbion (7,310 ft.) and Cebolera (7,026 ft.) on the north and the great Sierra del Moncayo (7,588 ft.) on the east separate the valley of the Duero (Douro) from the Ebro. Almost the whole of the province belongs to the region watered by the Duero and its affluents. There are forests of pine, oak and beech and large tracts of pasture land. The climate is cold and dry, and the scenery austere.

**SORIA**, the capital of the Spanish province of Soria; on the Duero (Douro) river. Pop. (1950) 16,328. The churches of Santa Domingo and San Nicolas, the collegiate church of San Pedro, the cloisters of the convent of San Juan and several other ecclesiastical buildings are fine specimens of Romanesque work of the 12th and 13th centuries. Near the Duero are the ruins of the old citadel, and the 13th century walls remain.

**SORIANO**, a department in southwest Uruguay bordering on the Uruguay river. Area 3,414 sq.mi. Pop. (1954 est.) 110,939. Wheat, flax, corn, barley are grown, and poultry, swine and bees are raised. The northern region of Soriano, a sheep and cattle area, is famous for its rich pastures. The departmental capital is Mercedes (q.v.), an agricultural, livestock and communication centre on the Negro river. Dolores, a port on the San Salvador river with a population of about 19,000 (1954 est.), becomes active during the harvest.

(M. I. V.)

**SORIN, EDWARD FREDERICK** (1814–1893), French-U.S. Roman Catholic priest, educator, founder and first president of the University of Notre Dame, was born at Ahuillé, near Laval, France, on Feb. 6, 1814. Ordained priest in 1838, he joined in 1840 the Congregation of Holy Cross, a group of priests and brothers organized at Le Mans by Abbé Basil Antoine Moreau.

At the invitation of Bishop Celestine Hailandière of Vincennes, Ind., Sorin and six brothers went to Vincennes in 1841, at first settling at St. Peter's in Daviess county. In 1842 Hailandière offered Sorin land near South Bend, in St. Joseph county, where Father Stephen Theodore Badin had formerly conducted the mission Sainte Marie des Lacs. Sorin arrived there on Nov. 26, and in 1844 he obtained from the general assembly of Indiana a charter for the University of Notre Dame. He was president of the university until 1865, was provincial superior of his community in the United States until 1868, and was from then until his death, on Oct. 31, 1893, the superior general.

In 1843 Sorin established at Bertrand, Mich., near Notre Dame, a community of French Sisters of Holy Cross, and in 1854 secured for them the site adjacent to Notre Dame on which St. Mary's college was founded. He was instrumental in bringing to the Sisters of Holy Cross Mother Angela (Eliza Maria Gillespie), who guided the community for nearly 30 years. In 1865 Sorin began publishing *Ave Maria* magazine.

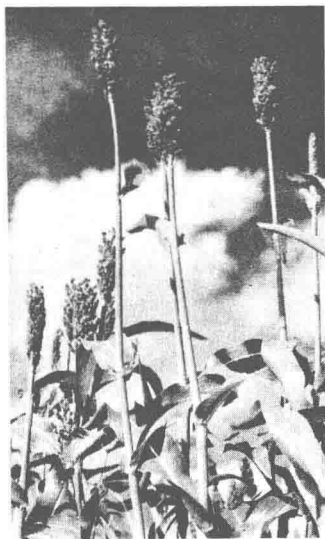
(T. T. MCA.)

**SORITES**. In traditional logic (see LOGIC), a chain of successive categorical syllogisms in the first figure may be so related that the conclusion of each except the last is the minor premiss of the next. If then the intermediate conclusions are suppressed, so that there are stated only the two premisses of the first syllogism, the major premisses of the remaining syllogisms, and the final conclusion, the resulting argument is a valid inference from the stated premisses, which may be considered independently of its analysis into syllogisms, and which is called a *sorites*. The distinction between the so-called *Aristotelian* sorites and the *Goclenian* sorites (Rudolph Goclenius, 1598) concerns only the order in which the premisses are stated.

(AO. C.)

**SOROCA**, a town of the Moldavian S.S.R., U.S.S.R., on the right bank of the Dniester, 81 mi. N.N.W. of Kishinev. Corn, wool, fruit, wine and cattle are exported. Soroca was the old Genoese colony of Olchiona and still has the ruins of a 13th-century Genoese castle. The Moldavians erected a fortress in the 15th century.

Soroca changed hands many times between Poland, Russia and



W. H. HODGE

SORGHUM (*S. VULGARE*)



Turkey; in 1940 Rumania ceded it to the U.S.S.R. but regained it the next year. After 1944 it was in the Moldavian Soviet Socialist Republic.

**SOROLLA Y BASTIDA, JOAQUÍN** (1863–1923), Spanish painter, whose style was a conservative variant of Impressionism, was born at Valencia on Feb. 27, 1863, and studied at the academy there, in Italy and in Paris, where he especially interested himself in the works of J. Bastien-Lepage and A. von Menzel; he was also influenced by the north European realists, particularly A. Zorn. His early paintings were of history and social realism (one of the latter kind, "Otra Margarita," 1892, being his earliest success), but he later became well known for brightly lit scenes with Valencian peasants and fisherfolk and children playing in the surf, his style after about 1903–04 becoming impressionistic and summary, with heavily impasted pigments. Between 1910 and 1920 he painted portraits of Spanish writers and a "Panorama of the Forty-nine Provinces of Spain" for the Hispanic Society of America. Sorolla died in Madrid on Aug. 10, 1923.

**SORORATE.** This term was introduced by Sir James Frazer to designate all marriages with a wife's sister, whether in the lifetime of the first wife or after her death. In his view, generally accepted today, it is complementary to the custom of the levirate (*q.v.*). The concept already appears in E. B. Tylor's "On a Method of Investigating the Development of Institutions," where levirate and sororate correctly figure as correlates of the postulate that matrimony is a bond between families rather than individuals. A. R. Radcliffe-Brown subsumes both institutions under the principle of the social equivalence of brothers and sisters, respectively. These relationship terms are to be understood in a classificatory sense, *i.e.*, a more remote relative of the same sex may serve as secondary mate instead of a blood sibling. Though related in principle, the levirate and sororate are not invariably associated, but they usually are, and appear to be the commonest of preferential secondary marriages. Either may be permissive rather than obligatory. The Maricopa (Arizona) insist on a widow's marrying a husband's relative, whereas the replacement of a deceased wife by a kinswoman of hers is customary, but not compulsory.

Though successive and simultaneous marriage of two or more sisters falls under the same principle, some tribes, *e.g.*, the Kazak, favour one, while tabooing the other practice. Hence it has become necessary to distinguish between sororal polygyny and sororate. The typical rule for the former is that the husband of the eldest girl in a family marries her juniors as they come of age; L. H. Morgan found this usage in at least 40 North American tribes; and even recently Navaho men often were simultaneously married to two sisters, occasionally to three. Australian aborigines recognized the same pre-emptive claim, but in many tribes the husband contented himself with the two oldest girls, conveying his claims on their younger sisters to his junior brother. With remarkable unanimity aborigines explain sororal polygyny on the ground that sisters are unlikely to quarrel as cowives.

The effect of both the sororate and sororal polygyny is to have children extend the term "mother" to the maternal aunt, but this terminological trait is more probably directly correlated with unilinear descent.

See also MARRIAGE.

**BIBLIOGRAPHY.**—A. R. Radcliffe Brown, *The Social Organisation of Australian Tribes* (1931); R. H. Lowie, *Social Organization* (1948); George P. Murdock, *Social Structure* (1949). (R. H. Lo.)

**SORORITY**, a social, professional or honorary organization of women, usually a secret society whose name consists of a series of Greek letters and which is connected with a college or university. See FRATERNITY AND SORORITY.

**SORREL**, the common name applied to various species of *Rumex* (family Polygonaceae), especially to *R. acetosa*, the garden sorrel; *R. acetosella*, sheep sorrel; *R. paucifolius*, mountain sorrel; and *R. hastatulus*, heartwing sorrel. The leaves of garden sorrel are used in soups, salads and sauces and as a potherb. French sorrel, *R. scutatus*, is a hardy perennial, distributed throughout Europe, but not native in Great Britain.

The species of the genus *Oxalis* (*q.v.*), (family Oxalidaceae),

are commonly called wood sorrel or lady's sorrel, and the sourwood (*Oxydendrum arboreum*) of the heath family is known as the sorrel tree. It is an attractive deciduous tree, native to the southeastern United States; its leaves turn a brilliant scarlet in the autumn. (J. M. Bl.)

**SORRENTO**, a city of Campania, Italy (ancient *Surrentum*). Pop. (1957 est.) 11,399 (commune). Sorrento stands on cliffs about 160 ft. high, between the Bay of Naples and the Bay of Salerno. It is a summer and winter resort, its northerly aspect rendering it comparatively cool.

At Sorrento, Bernardo Tasso wrote his romantic poem *Amadigi*. His son, the poet Torquato Tasso (1544–95) was born there.

The most important temples of the ancient city were those of Athena and of the Sirens, the latter the only one in the Greek world in historic times. The place was famous for its wine, its fish and its red Campanian vases. It was protected by deep gorges, except for a distance of 300 yd. on the southwest, defended by walls, the line of which is followed by those of the modern town. The arrangement of the modern streets also preserves that of the ancient town.

On the east the most important ancient ruin is the reservoir of the subterranean aqueducts, which had 27 chambers. There are also remains of villas, including that of Pollius Felix, the friend of Statius.

**SORSOGON**, the name of a municipality, provincial capital and the southernmost province of Luzon, Philippines. Area of province 793 sq.mi.; pop. (1960) 348,708. The province is bordered on the south by San Bernardino strait and comprises largely volcanic cones with broad rich level areas between; the highest peak is Mt. Bulusan (5,118 ft.), a recently active cone. Rainfall varies from 100 in. to more than 160 in. with no dry season and maximum fall in the winter.

Rice, sweet potatoes and cassava are the main food crops produced, but the province is best known as an abacá (Manila hemp) centre. Before World War II slightly over 40% of the cultivated area was devoted to abacá, but by the early 1960s this area was halved. Coconuts continue to occupy a significant proportion of the farmed area.

Sorsogon (1960 pop. 35,548) is a trading town, processing station for abacá and copra; and port on the northeastern shore of Sorsogon bay. (R. E. He.)

**SOSIGENES**, Greek astronomer and mathematician, probably of Alexandria, flourished in the 1st century B.C. According to Pliny's *Natural History*, he was employed by Julius Caesar in the reform of the Roman calendar (46 B.C.), and wrote three treatises. From another passage of Pliny it is inferred that Sosigenes maintained the doctrine of the motion of Mercury around the sun, which is referred to by Cicero and was also held by the Egyptians.

Sosigenes was the tutor of Alexander of Aphrodisias. He wrote *Revolving Spheres*, from which important extracts are preserved in Simplicius' commentary on Aristotle's *De caelo*.

**SOSITHEUS** (c. 280 B.C.), Greek tragic poet, of Alexandria Troas, a member of the Alexandrian "pleiad." He must have resided at some time in Athens, since Diogenes Laërtius tells us (vii, 5, 4) that he attacked the Stoic Cleanthes on the stage and was hissed off by the audience. Suídas calls him a Syracusan. According to an epigram of Dioscorides in the Greek anthology (*Anth. Pal.* vii, 707) he restored the satyric drama in its original form. Part of his pastoral play, *Daphnis* or *Lityerses*, is extant.

See O. Crusius *s.v.* Lityerses in Roscher's *Lexikon der griechischen und römischen Mythologie*. The fragment in Nauck's *Tragicorum graecorum fragmenta* apparently contains the beginning of the drama.

**SOSNOWIEC**, a town of Poland, in Katowice province. Pop. (1960) 131,600. It owes its importance to its position in the centre of Dabrowa coal field, near Bedzin, Dabrowa and Katowice. The towns of this region are almost continuous, extending from Kielce into the provinces of Kraków and Slask. Sosnowiec is also a railway junction. Situated on the Warsaw-Vienna railway, it is a junction for the Kielce and Radom, Kraków and Lvov and Katowice and Breslau lines. Electric power stations were established. Iron foundries and textile factories, as well as coal mines,



employ large numbers of workmen.

Sosnowiec was seized by Germany in World War II and was returned to Poland in 1945.

**SOSTENUTO**, musical term signifying that the passage so marked is to be played in a "sustained" manner.

**SOTER, SAINT**, pope from about 166 to about 175. He wrote to the church of Corinth and sent it aid. His letter is mentioned in the reply given by Dionysius, bishop of Corinth, and A. Harnack thought that it could be identified with the second so-called epistle of Clement. St. Soter's feast day is April 22.

**SOTHERN, EDWARD HUGH** (1859–1933), U.S. actor, was born at New Orleans, La., on Dec. 6, 1859, the son of Edward Askew Sothern, noted English comedian. His first stage appearance was in a small part with his father's company at the Park theatre in New York city in 1879. He toured England in 1882–83, became leading comedian in John McCullough's company in 1883, and under Daniel Frohman was leading man at the Lyceum theatre in New York city. He married Virginia Harned in 1896, and in 1899 formed his own company with her as his leading lady.

In 1900 he appeared in the title role of *Hamlet*, in 1901 in that of *Richard Lovelace* and in 1902–03 as Villon in *If I Were King*, three of his greatest roles. In 1904 Sothern played opposite Julia Marlowe (*q.v.*) for the first time in *Romeo and Juliet* at Chicago, Ill. Thereafter, except for two years, 1907–09, they appeared together on the stage almost continuously until their retirement. They were married in 1911. Besides *Romeo and Juliet* they cooperated in *Much Ado About Nothing*, *Taming of the Shrew*, *Merchant of Venice*, *Twelfth Night*, *Macbeth*, *Jeanne D'Arc*, *John the Baptist*, *When Knighthood Was in Flower* and *The Sunken Bell*. Although noted chiefly as a Shakespearean actor, Sothern had a repertory of over 125 diverse parts. He won wide popularity as a dashing, romantic hero in melodramas such as *The Prisoner of Zenda*. Sothern wrote an autobiography, *The Melancholy Tale of Me* (1916).

**SOTHIC PERIOD**, in ancient Egyptian chronology, the period in which the year of 365 days circled in succession through all the seasons. The tropical year, determined as it was in Egypt by the heliacal rising of Sirius (Sothis), was almost exactly the Julian year of precisely 365½ days (differing from the true solar year, which was 11 minutes less than this). The sothic period was thus 1,461 years. See **CALENDAR**; **EGYPT**: *History: Ancient Civilization and Culture*.

**SOTHO**, a powerful nation of Bantu-speaking peoples which inhabits the British colony of Basutoland (*q.v.*) in South Africa. It is made up of a large number of different tribes, which were welded together early in the 19th century by the great chief Moshesh.

**SOTO, FERDINANDO DE**: see **DE SOTO, HERNANDO OR FERNANDO**.

**SOTTO VOCE** (It.), lit. "under the voice," that is, an undertone. Term applied both to music and speech.

**SOUBISE, BENJAMIN DE ROHAN, DUC DE** (?1589–1642), Huguenot leader, younger brother of Henri de Rohan, inherited his title through his mother Catherine de Parthenay. He served his apprenticeship as a soldier under Prince Maurice of Orange-Nassau in the Low Countries. In the religious wars from 1621 onwards his elder brother chiefly commanded on land and in the south, Soubise in the west and along the sea-coast. Soubise's chief exploit was a singularly bold and well-conducted attack (in 1625) on the Royalist fleet in the river Blavet (which included the cutting of a boom in the face of superior numbers) and the occupation of Oléron. He commanded at Rochelle during the famous siege. When surrender became inevitable he fled to England, which he had previously visited in quest of succour. He died in 1642 in London.

**SOUBISE, CHARLES DE ROHAN** (1715–1787), peer and marshal of France, the grandson of the princesse de Soubise known to history as one of the mistresses of Louis XIV. He accompanied Louis XV. in the campaign of 1744–48. Soon after the beginning of the Seven Years' War, through the influence of Mme. de Pompadour, he was put in command of a corps of 24,000 men, and was defeated at Rossbach (1757). He continued in the

service until the peace of 1763. He died in Paris on July 4, 1787.

**SOUHAM, JOSEPH, COUNT** (1760–1837), French soldier, was born at Lubersac on April 30, 1760, and became a general of division in 1793. He was disgraced with Moreau and Pichegru for alleged participation in the conspiracy of Cadoudal. He regained his rank in 1809, took a notable part in Gouvion St. Cyr's operations in Catalonia, and won the title of count. In 1812 Masséna, in declining the command of Marmont's army recommended Souham for the post. The latter was thus pitted against Wellington, and by his skilful manoeuvres regained the ground lost at Salamanca. At the fall of the First Empire he deserted the emperor, and was well received by Louis XVIII., who gave him high commands. He retired in 1832, and died on April 28, 1837.

**SOULOUQUE, FAUSTIN ÉLIE** (1789?–1867), Negro emperor of Haiti, was born a slave about 1789 while Haiti was under French rule. He participated in the successful Haitian revolt against the French in 1803 and thereafter continued as an officer in the Haitian army. He was made president of Haiti in 1847 because the mulatto leaders who had dominated the government under several figurehead Negro presidents thought that his illiteracy and ignorance would make him easy to control. He soon turned against his would-be advisers and ruled as a cruel and corrupt despot, proclaiming himself Emperor Faustin I in 1849 and creating a numerous nobility. Many of the mulatto leaders were killed or exiled.

Soulouque made several costly and unsuccessful attempts to conquer the Dominican Republic, until the United States and France and Great Britain in 1851 demanded that he desist. He later renewed his attacks and in 1855 was defeated by the Dominican army.

In 1859 he was ousted after the chief of his general staff, realizing that the emperor suspected his loyalty, led a revolt. Soulouque escaped and went into exile. He died in 1867.

See also **HAITI**: *History*.

(D. G. Mo.)

**SOULT, NICOLAS JEAN DE DIEU**, Duke of Dalmatia (1769–1851), marshal of France, was born at Saint-Amans-la-Bastide (now in department of the Tarn) on March 29, 1769, the son of a notary. He was intended for the bar, but on his father's death in 1785 he enlisted as a private in the French infantry, and rose rapidly in the army. He laid the foundations of his military fame by his conduct in Masséna's great Swiss campaign (1799), and especially at the battle of Zürich. He acted as Masséna's principal lieutenant through the protracted siege of Genoa, and after many successful actions he was wounded and taken prisoner at Monte Cretto on April 13, 1800. The victory of Marengo restoring his freedom, he received the command of the southern part of the kingdom of Naples, and in 1802 he was appointed one of the four generals commanding the consular guard. Despising Napoleon, Soult affected devotion, being appointed in 1803 to the command at Boulogne and in 1804 to be one of the first marshals of France. He commanded a corps at Ulm, and at Austerlitz (*q.v.*) he led the decisive attack. After the peace of Tilsit he was created (1808) duke of Dalmatia. In the following year he was given a command in Spain after the battle of Gamonal and he pursued Sir John Moore to Corunna.

For the next four years Soult remained in Spain, and his military history is that of the Peninsular War (*q.v.*). In 1812 he was obliged, after Wellington's victory of Salamanca, to evacuate Andalusia, and was soon after recalled from Spain at the request of Joseph Bonaparte, with whom he had always disagreed. In March 1813 he assumed the command of the IV. corps of the *Grande Armée*, but he was soon sent to the south of France to repair the damage done by the defeat of Vittoria. His campaign there is the finest proof of his genius as a general, although he was repeatedly defeated by Wellington, for his soldiers were raw conscripts, facing Wellington's veterans.

Marshal Soult's political career was less creditable, and it has been said of him that he had character only in front of the enemy. After the first abdication of Napoleon he declared himself a Royalist, received the order of St. Louis, and acted as minister for war (Dec. 3, 1814–March 11, 1815). When Napoleon returned from Elba Soult declared himself a Bonapartist, was

made a peer of France and acted as major-general (chief of staff) to the emperor in the Waterloo campaign.

At the second Restoration he was exiled, but was recalled in 1819 and in 1820 again made a marshal of France and in 1827 a peer. After the revolution of 1830 he made out that he was a partisan of Louis Philippe, who revived for him the title of marshal-general. He was minister for war, 1830-34 and 1840-44, and ambassador extraordinary to London for the coronation of Queen Victoria in 1838. In 1848, when Louis Philippe was overthrown, Soult again declared himself a republican. He died at his castle of Soultberg, near his birthplace, on Nov. 26, 1851. Soult published a memoir justifying his adhesion to Napoleon during the Hundred Days, and his notes and journals were arranged by his son Napoleon Hector (1801-57), who published the first part (*Mémoires du maréchal-général Soult*) in 1854. Le Noble's *Mémoires sur les opérations des Français en Galicie* are supposed to have been written from Soult papers.

See A. Sallé, *Vie politique du maréchal Soult* (1834); A. de Grozelier, *Le Maréchal Soult* (1851); A. Combes, *Histoire anecdotique du maréchal Soult* (1869).

**SOUND.** This article is divided into the following sections:

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3. Reflection and Refraction of Sound
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Sound Ranging in Air  
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### I. WHAT IS SOUND?

When a person opens his mouth and utters speech he is said to emit sound. Anyone in the vicinity with normal ears (or in default thereof a suitable hearing aid) is said to hear the sound. This common experience is a physical phenomenon of the greatest significance to human beings, who may be considered to be immersed in a world of sound having notable influence on most of their everyday activities.

The importance of this particular sensation in human life is reflected in the large number of words descriptive of the different kinds of sounds which come to our attention. Inanimate nature produces the thunder of the storm, the roar and pounding of the surf, the whistling of the wind, the whispering of the trees, the patter of rain, the rippling and gurgling of running water, the humming of wires, the creaking of snow. Even more rich is the vocabulary descriptive of the sounds of living things: the barking and snarling of dogs, mewing of cats, crowing of fowls, roaring of lions, hissing of snakes, the lowing and bellowing of cattle, blating of goats, chirping of birds and insects, screaming of gulls, crying of infants, etc.

Finally, man, not content with all the racket around him over which he has little or no control, has contrived to produce sounds of almost infinite variety causing both pleasure and pain to countless millions. The boom of cannon, the crack of the pistol, the rattle of musketry and machine-gun fire, the whine of the shell and the blast of its explosion are unpleasantly familiar. The whirl of machinery and the ticking of clocks have more agreeable connotations though they might be distracting to some people. But the melody and harmony of music are generally admitted to contribute aesthetic enjoyment.

What is this sound which forms so large a part of man's waking experience? The purpose of this article is to give the physicist's answer to this question; it will be shown that there is far more to sound than "meets the ear" and that the sounds which are not heard are in some respects the most important ones for modern physics and its technical applications.

Sound is a physical phenomenon and the branch of physics which describes it is called acoustics (from the Greek word meaning "hearing"). Physics is an abstract science and its method of description employs mathematical analysis freely. Without such analysis a physicist considers a deep understanding of a physical phenomena unattainable. On the other hand, the general reader understandably prefers to be told the results of scientific description in the language of everyday speech. This article introduces the subject, therefore, with a general nonmathematical survey of sound. This is followed by a more analytical account.

**1. Motion and Sound.**—It is obvious to even the most casual observer that a thorough analysis of all that is involved in the emission of any one of the sounds mentioned above must be complex. To make clear what really goes on, for example, in the head and larynx when one speaks would undoubtedly require the combined efforts of a physiologist, a psychologist and a physicist (with, of course, the aid of a mathematician). Nevertheless, it is generally agreed that the ultimate result is some motion of the air in front of the mouth. It is also generally accepted that when the normal person hears a sound there is some motion of the air at the entrance to his ear.

A careful examination of all sound-producing phenomena always shows the existence of motion of some medium. This is rather satisfying to physicists, who have long sought to explain most natural phenomena in terms of motion, because they believe they understand what the latter is. To those whose ideas of motion are confined to such solid things as automobiles and airplanes, the association of motion with sound may provide some difficulty. Realizing that in open, still air it is possible to hear a cricket chirp at a distance of half a mile, the sceptic asks how it is possible that such a small insect can move the mass of air in a hemisphere with a radius of half a mile (over 1,000,000 tons) so that the air in the

vicinity of one's ear may move sufficiently to lead to hearing. The fallacy of this reasoning lies in the assumption that the sound producer must move *all* the surrounding air *all at once* in order to produce the sensation of sound. It ignores the fact that air is a compressible elastic fluid, *i.e.*, can be squeezed, so that it is possible for motion to take place in one part without appearing simultaneously everywhere else. One of the interesting properties of such a medium, however, is that if a "squeeze" is produced in one small region, it does not stay, as it were, "frozen" there. Once squeezed the medium tends to expand again and in so doing compresses (*i.e.*, moves) an adjoining portion of the medium, which in turn repeats the process, with the result that portions of the medium far from the source ultimately get squeezed (of course at a time later than the original disturbance). This kind of motion, communicated in time from one part of a medium to another, is called wave motion. We say that sound travels in air as a compressional or squeeze wave.

**2. Wave Motion; Velocity and Intensity.**—The standard pictorial illustration of wave motion is provided by the ripples produced on the surface of water when a stone is dropped into it. One of the striking features of this phenomenon is the fact that the water itself does not move outward from the centre of the disturbance, whereas the distortion of the surface which constitutes the ripple does precisely this. The distance traveled by the ripple per unit time is called the wave velocity. All waves studied by physicists have a finite wave velocity, although its magnitude varies greatly from type to type. Thus ripples on the surface of water may move only a metre or so per second. Light in free space, on the other hand, travels at the enormous speed of  $3 \times 10^8$  m. per second. What can we say of the velocity of sound?

The speed of sound in air increases (as we shall see later) with the temperature, but at room temperature it is about 344 m. (1,125 ft.) per second. Its relatively small value compared with that of light is responsible for the common experience in which the sound of a distant boat whistle is heard some time after the puff of steam is seen.

Not all sound waves travel with the speed of sound in air. Thus sound in fresh water has a velocity of about 1,500 m. per second at room temperature. More details on this are presented in the analytical discussion in section II.3 below.

In everyday language we distinguish between loud and soft sounds. This suggests attention to the magnitude of the motions of air involved in sound propagation. In normal conversational speech the pressure of the air in front of the mouth of the speaker is changed at the most by only about one-millionth of the standard atmospheric pressure. At the same time the accompanying motion that produces the compression leads to an air-flow velocity with a maximum value of only two one-hundredths of a centimetre per second. That such small changes in the air when reproduced at the ear cause the sensation of hearing should strengthen respect for the sensitivity of that sense organ. Since sound involves propagated motion and motion implies energy, we can look upon sound in air as equivalent to the transmission of mechanical energy through the air. It then turns out that an average rate of transfer of only  $10^{-16}$  (1 divided by 10 raised to the 16th power) watt per square centimetre of acoustic power is sufficient to produce the sensation of hearing in a normal youthful ear. The average power transmission in a sound wave per unit area is called its intensity. The modern unit for this quantity is the decibel (abbreviated to db.). Strictly this measures the intensity of a sound relative to a standard intensity. (See section II.8 below.) For convenience the latter is often chosen as that corresponding to minimum audibility (maximum excess pressure  $2 \times 10^{-4}$  dynes/cm.<sup>2</sup>). On this basis the sound of conversational speech has an intensity of about 50 db. at a distance of a few feet and that of traffic at a busy intersection 70 db.; that of a boiler factory can attain the value 110 db.

Everyone has noticed that the farther off he is from a source of sound in the open, as, for example, an airplane, the less distinctly he hears it. In technical terms, the intensity of sound in an unenclosed space decreases with the distance from the source. This is really merely a matter of geometry. The airplane may be

thought of as emitting the same amount of energy every second in every direction, but as the distance increases this energy passes through surfaces of larger and larger area, and hence the average flow per unit area is diminished. In fact, if the sound disturbance travels with the same velocity in every direction, it will at any given instant reach the surface of a *sphere* with the source at the centre and with radius equal to the product of the velocity of sound in the medium and the elapsed time. Hence the intensity will decrease in the same ratio as the surface of the sphere increases with its radius; *i.e.*, with the square of the distance from the source. (See section II.9 below.) The situation is unfortunately more complicated for a source of sound on the ground or under water (near the surface), and even more involved when the source is enclosed in a room. Nature is rarely simple! And the only way to begin to understand its ways is to pick rather idealized situations.

Even the inverse-square law just mentioned does not tell the whole truth about the change of sound intensity with distance from the source in an unbounded three-dimensional medium. It is observed actually that the drop is greater than is predicted by this law. Why is this? Apparently some of the energy represented by the sound disturbance gets "lost" in the process of propagation. We say it is absorbed and changed into heat. This absorption of sound is particularly noticeable in a viscous liquid like glycerin. In air it is accentuated by reflection and refraction and scattering (see sections I.3 and I.4 following).

**3. Reflection and Refraction of Sound.**—If sound is a wave motion it must manifest other properties of waves besides velocity and intensity. We know that light waves are reflected from surfaces separating media of different properties, and refracted (*i.e.*, bent from the original direction of propagation) in crossing such surfaces obliquely. There are sound phenomena with precisely these characteristics. A familiar example is the echo produced when a loud, sharp sound is emitted near a high wall or cliff. This is most readily interpreted as caused by the reflection of the original sound from the solid surface, verified by the experimental observation that the time interval elapsing between the initial sound and the echo is the time taken for the sound to reach the cliff and the reflected disturbance to return.

Important also is the concept of acoustic image. Like its optical counterpart this is an imaginary source of sound which, if it were located exactly as far behind the reflecting surface as the initial source is in front of it and if the surface were then removed, would give the observed echo at the proper time and with the proper intensity.

It is much easier for a speaker talking with a certain strength of voice to reach an auditor at a given distance if the two are in a closed room rather than out in the open air. Curiously enough this also is an important illustration of the reflection of sound. For the sound from the speaker reaches the auditor not merely in the direct line between the two but also by many paths involving reflection by the walls, floor and ceiling of the room. Consequently much of the energy which would be dissipated in all directions in space out of doors is, so to speak, trapped in the room and helps to build up the sound intensity at every point. In a not-too-large room the echoes from the various reflecting surfaces merge to form a continuous sound or reverberation, which lasts longer than the direct sound and can impair the distinctness of hearing if it is too long. We then say that the room needs acoustic treatment to cut down the effect of wall reflection. This is done by the introduction of suitable sound-absorbing materials (see section V.3 below).

The refraction of sound is less obvious than refraction of light but produces well-recognized effects. Since sound travels faster in warm air than cold and since the atmosphere almost always manifests a vertical temperature gradient, it is to be expected that sound will rarely travel through it in straight lines from the source. Actually it is observed that in the presence of a negative temperature gradient (the normal condition) the sound rays (the significance of a ray of sound will be explained fully in section I.6) are bent upward and hence the range of hearing in the horizontal direction is materially reduced. On the other hand when the temperature gradient is positive, with the temperature increasing upward (so-



called temperature inversion, a rather infrequent phenomenon in most localities, but one which occurs occasionally in the early daylight hours after a clear, wind-free night), the upward-moving rays tend to be bent downward, assuring longer range of transmission. Just as the refraction of light through air of varying temperature leads to the phenomenon known as the mirage, so it is also possible to have acoustic mirages under similar conditions, and sound may appear to come from a quite different direction than it would through a homogeneous medium. All these effects of refraction caused by temperature are further complicated by refraction caused by the motion of the medium; *i.e.*, winds. It is well known that sound in air travels better with the wind than against it, and the effective propagation velocity is materially changed by vector addition with the wind velocity.

The refraction of sound in air by wind and temperature plays a very significant and somewhat embarrassing role in gun ranging, the acoustical method of locating guns by the sounds produced by their firing (section V.5 below). The reflection and refraction of sound in water have taken on great practical importance in the echo-ranging method of detecting underwater objects in which a beam of sound is projected in water and a portion of it reflected by a solid object (*e.g.*, a submarine) which it strikes. The detection of the reflected sound leads to an estimate of range and bearing of the object being sought after. But refraction or bending of the sound paths caused by temperature gradients in the water introduces obvious complications (section V.5 below).

**4. Diffraction and Scattering.**—One of the most striking properties of sound is its ability to bend around obstacles. It is because of this that one can hear around a corner. This is an illustration of the characteristic of all waves known as diffraction. Even light displays it but obviously not to such a pronounced degree as sound, since we cannot see around a corner (without a periscope). The reason for this difference is discussed in section I.5 below.

Diffraction accounts for the inability of objects of ordinary size to form sharp shadows of audible sound. Here again further discussion is necessary since not all sounds bend easily around obstacles (section II.14 below).

An interesting example of sound diffraction is provided by the human head. Our hearing of sound is definitely conditioned by the fact that our ears are imbedded in a roughly spherical, more or less solid sphere. The ticking of a watch sounds different in front of the head than it does at the same distance directly behind. This is due to diffraction. Theory and experiment alike indicate that because of diffraction a source of sound produces a greater intensity at the head if it lies on an extension of the line joining the ears than if it is at the same distance directly in front of the head. This effect has significant bearing on the testing of microphones.

When the obstacle diffracting the sound wave is relatively small, *e.g.*, a fog droplet, the wave is said to be scattered, since the bending makes the sound turn in all directions from the original one. Examples of this are presented later, in section II.14.

**5. Harmonic Sound Waves; Pitch, Frequency and Wave Length.**—In the preceding discussion of simple acoustical phenomena, little has been said about one striking difference between sounds from different sources. This is the difference in pitch. Instinctively when we hear two successive sounds we classify one as higher or lower than the other. This characteristic is obviously quite different from loudness, previously described in terms of the intensity of the sound. What physical effect is associated with pitch? Here it is necessary to invoke actual experiments. For rough qualitative considerations it will suffice to call attention to the ordinary air siren, which produces sound by interrupting jets of air at different rates by means of a rotating disk with orifices in its periphery (section III.11 below). The more rapidly the interruption takes place, *i.e.*, the greater the number of separate puffs of air per unit time, the higher the pitch. But a source of sound in which the disturbance repeats itself regularly like the puffs of air in the siren is said to produce a harmonic sound wave characterized by a definite frequency. This is defined as the number of times per second the disturbance in the sound wave at any point repeats itself.

Actually the relation between pitch and frequency in a complicated sound wave like that associated with the human voice or ordinary musical instruments is not at all simple. Hence it is advantageous at this point to confine attention to a pure harmonic wave like that emitted by a carefully made tuning fork. This is said to produce a pure tone of definite single frequency.

In a harmonic sound wave in air, then, the "squeeze" responsible for the sound varies periodically at every point. The time for one complete cycle of pressure change at any point is called the period of the wave. The period in seconds is equal to the reciprocal of the frequency, where the latter is represented by the number of complete cycles per second.

One of the most important facts about harmonic sound waves is that we do not hear sounds of all frequencies equally well. If the hand is waved back and forth periodically with a frequency less than 15 cycles per second no sound is heard. An object vibrating above this frequency will produce an audible sensation if the intensity is sufficiently great. However, when the frequency is increased to above 20,000 cycles per second, audibility vanishes for most people. In fact, with advancing age the frequency threshold for hearing diminishes decidedly. Of course the sound intensity plays a role here and the matter is not simple. Nevertheless the significant thing is that it is possible to have sounds of such high frequency that they are inaudible no matter how great their intensity. These sounds are called ultrasonic and they play an important role in the modern study of acoustics. (*See ULTRASONICS.*) As far as the physics of sound is concerned, the sounds which are not heard attract greater interest than those which are. One reason is that ultrasonic waves tend to travel in beams like light, while low-frequency sound waves tend to spread in every direction from the source. Obviously such sounds need special devices for their production and detection. Much of the progress of acoustics after about 1925 was due to the development of such equipment, which made possible extensive investigations with sounds of frequencies up to 100,000,000 cycles per second (*see* section V.6 below).

To come back to harmonic waves in general, they have another useful characteristic—the wave length, which is the distance between successive points in a spreading wave at which the disturbance is exactly the same and doing the same thing (*i.e.*, getting either larger or smaller). At two such points the phase of the wave is said to be effectively the same (section II.4 below). Since the disturbance in a wave travels a distance of one wave length in one period, the product of wave length and frequency equals the wave velocity. This important relation is true for waves of all kinds. It means that high-frequency acoustic waves in a given medium have smaller wave length than low-frequency sounds. Thus if we are talking about water, a wave of frequency 1,000 cycles (1 kc.) has a wave length of approximately 1.5 m., whereas for a frequency of 1,000,000 cycles (1 mc.) the corresponding wave length is only 1.5 mm.

Many of the properties of sound discussed above are definitely dependent in their behaviour and magnitude on the frequency. Very few sounds arising in practice are pure tones characterized by a single frequency. Most actual sounds insofar as they are periodic can be considered as more or less complicated combinations of component harmonic sounds, each of definite frequency. One or more frequencies will often predominate and help give the sound its observed quality. In general a high-frequency sound wave is bent by diffraction less than a low-frequency wave is, and hence enables an obstacle to cast a sharper acoustic shadow. (There is a corresponding phenomenon in the case of visible light, where the frequency is very much higher than that of any sound wave yet produced.) On the other hand when sound is scattered by objects whose dimensions are small compared with the wave length, the high-frequency components are scattered much more effectively than the low. This is quite analogous to the scattering of sunlight by the small dust and molecular particles of the atmosphere; the short wave length or high-frequency light (*i.e.*, the blue) is scattered much more than the long wave length (red) and so the sky appears blue.

**6. Wave Fronts and Rays.**—We have already commented on

the fact that an audible sound wave tends to spread out in all directions from the source. On the other hand it is possible to produce a beam of sound if the frequency is high enough. To describe this situation more precisely the concept of wave front is introduced. This is a surface of such a kind that at a particular instant the disturbance characterizing the sound wave is the same at every point.

An example is the airplane propeller radiating sound into the surrounding air. At any instant of time the state of "squeeze" of the air in this sound wave will be the same on the surface of a sphere with centre at the propeller and with radius equal to the time taken by the squeeze to travel at the speed of sound from the propeller to the sphere. We shall call this sphere a wave front of the sound from the propeller and can think of the propagation of the sound as the motion of such a wave front carrying with it a definite state of squeeze, and moving with the speed of sound. The expanding circular ripples on water produced by a dropped stone form an excellent two-dimensional analogy; in the present example, however, the wave fronts are spherical. On the other hand it is possible to confine a sound wave to a tube of constant cross section, as for example a speaking tube. Though the propagation of the disturbance here is indeed a somewhat complicated affair because of the reflections from the walls of the tube, it is approximately correct to assume that at any instant at any place the squeeze is the same over the plane at that place perpendicular to the direction of propagation. In other words, the transmission proceeds by the motion of a *plane* wave front.

Plane and spherical wave fronts are the most important types encountered in sound propagation. Of the two, plane waves are the more important for at great distances from the source the effective part of a spherical wave is very nearly plane.

Lines perpendicular to a wave front are known as rays. These indicate the direction in which the wave front and hence the sound is moving at any place at any instant. For many purposes it is more convenient to describe sound transmission in terms of rays just as in the analogous case of light. However, it must be remembered that the wave length of even ultrasound is long compared with that of visible light and consequently ray acoustics is in general by no means as accurate a representation as ray optics (*see* section II.5 below). It is more difficult to produce a beam of sound than a beam of light, but it can be done if the wave length is sufficiently small compared with the size of the sound source. The mechanism by which this takes place is called interference and is explained in section III.10 below. Such sound beams are very useful in underwater detection.

## II. ANALYSIS OF SOUND PROPAGATION

**1. Wave Functions.**—It is now necessary to investigate more carefully the qualitative considerations of the foregoing and introduce some quantitative description. The mathematical treatment of sound waves in general is very complicated, so we start by idealizing the problem. It is simplest to begin with the kind of wave one can produce in a long, perfectly flexible string by flicking one end. The kink so produced as a disturbance in the original straight position of the string travels along it in an easily visualized fashion. (*See* fig. 1.)

Here we have a wave traveling in one direction only. Since the disturbance is at right angles to the direction of propagation it is called a transverse wave. In this respect it is different from a sound wave. In the latter the disturbance, being a pressure squeeze, is in the same direction as the wave transmission and the wave is called longitudinal. Each particle of the fluid in this case is displaced from its equilibrium position in the direction of the wave propagation. However, the mathematical analysis in its more fundamental aspects is the same for both kinds of waves.

If we were to take a snapshot picture of the kinked string at a given instant, say  $t = t_0$ , it might look like A in fig. 1. Here the disturbance, which is measured by the distance each part of the

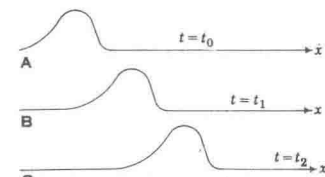


FIG. 1.—TRANSVERSE WAVE IN A STRING

string has been displaced from its original undisturbed position, can be represented by a mathematical function  $f(x)$ , if we denote distance along the string by  $x$ . At the later time,  $t = t_1$ , the disturbance has moved a distance  $V(t_1 - t_0)$  along the string, where  $V$  is the velocity of the wave, so the same function  $f(x)$  reappears, but displaced (B in fig. 1). And so likewise for  $t = t_2$ , as indicated in C. The problem then is to represent mathematically a function of  $x$  which moves along the positive  $x$  axis with velocity  $V$ . It turns out that this is

$$\xi = f(x - Vt) \quad (\text{II.1-1})$$

The Greek letter  $\xi$  is used to denote the displacement from equilibrium representing the disturbance. Moreover, if the wave travels in the negative  $x$  direction the corresponding wave function, as it is called, is

$$\xi = g(x + Vt) \quad (\text{II.1-2})$$

It will be noted that there is nothing essentially periodic about the wave functions (1) and (2). However, as was stressed in section I.5, periodic waves are the most interesting kind, and hence we proceed at once to specialize the above to waves characterized by frequency and wave length. The simplest of all periodic waves is the harmonic, in which (1) takes the form

$$\xi = A \sin(\omega t - kx) = -A \sin k(x - \omega/k \cdot t) \quad (\text{II.1-3})$$

This is evidently in the proper form for a wave function if we interpret  $\omega/k$  to be equal to the velocity  $V$ . But what are  $\omega$  and  $k$ ? If we fix our attention on a particular point  $x = x_0$  of the string traversed by such a wave and take a motion picture of the way the displacement there varies with time we find it is sinusoidal with period  $P = 2\pi/\omega$  or frequency  $\nu = \omega/2\pi$ . Hence this fixes the frequency of the wave. On the other hand, if we take a snapshot picture of the string at the instant  $t = t_0$ , we find that the displacement varies sinusoidally with  $x$  and the distance between successive points of maximum displacement in the same direction is  $2\pi/k$ . But we have already defined this as the wave length  $\lambda$  of the wave. Hence we have

$$\nu = \omega/2\pi, \lambda = 2\pi/k \quad (\text{II.1-4})$$

Since  $\omega/k = V$ , we also obtain the important relation already commented on in section I.4 above

$$\nu\lambda = V \quad (\text{II.1-5})$$

This relation holds for any harmonic wave. It is worth noting that if we used the cosine function in place of the sine function in (3) the physical meaning would not be altered. The quantity  $\omega$  is often referred to as the angular frequency of the wave.

**2. Differential Equation of Wave Motion and Its Deduction.**—If we differentiate  $\xi$  in equation (II.1-3) twice with respect to  $x$  (keeping  $t$  constant), and then twice with respect to  $t$  (keeping  $x$  constant) we can show, keeping in mind equation (II.1-5), that

$$\frac{\partial^2 \xi}{\partial t^2} = V^2 \frac{\partial^2 \xi}{\partial x^2} \quad (\text{II.2-1})$$

As a matter of fact the existence of this equation also follows from the general wave functions (II.1-1) and (II.1-2). It is called the general differential equation for wave motion in the  $x$  direction. Its general solution is the sum of wave functions (II.1-1) and (II.1-2) representing general waves progressing in both positive and negative  $x$  directions. The advantage of a partial differential equation like (1) from a physical point of view is that it contains the description of so much in so compact a form. The task of theoretical physics in every branch may be said to be the development of such equations, which by their very generality include within themselves a vast amount of information about physical phenomena. The whole theory of the propagation of sound may be considered to be implied in the above equation and its generalization to three dimensions (in which any point in

<sup>1</sup>For simplicity references to equations in the same section are given in terms of the last digit in the equation number. Reference to an equation in a different section employs the whole equation number.

space has the co-ordinates  $x, y, z$ ), viz.,

$$\frac{\partial^2 \xi}{\partial t^2} = V^2 \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} \right) \quad (\text{II.2-2})$$

The physicist therefore feels that he understands something about the propagation of sound waves if he can show that an equation like (1) or (2) is a logical consequence of the behaviour of a compressible fluid when it is squeezed.

The behaviour of an ideal fluid when disturbed from its state of rest is determined (see MECHANICS, FLUID) by three fundamental equations: (1) equation of continuity; (2) equation of motion; and (3) equation of state. The first says that no matter how the fluid moves, as long as it hangs together as a continuous fluid there must be conservation of mass. The second, or equation of motion of the ideal fluid, says that the total time rate of change of velocity of a particle of fluid must equal the force per unit mass acting on the particle, the latter for purely compressional disturbances being proportional to the negative of the rate of change of the excess pressure with distance. Finally the third, or equation of state, connects the change in pressure with the change in density. This says that the excess pressure  $p_e$  produced by any disturbance in the fluid (i.e., the difference between the actual pressure and the equilibrium pressure prevailing before the disturbance) is directly proportional to the excess density  $\rho_e$  (i.e., the difference between the actual density in the disturbed fluid and the equilibrium density). The coefficient of proportionality is usually written as  $c^2$ . It should be noted that this is an idealized static equation of state and takes no account of the fact that in an actual fluid the imposition of a given excess pressure needs time to produce the excess density indicated.

It is an interesting fact that the mathematical combination of the three equations just described leads to an equation of precisely the same mathematical form as (1). This should not be too surprising when it is recalled that when a fluid is disturbed mechanically the pressure is bound to change; and this leads to motion of the fluid as well as a squeezing or rarefying action (viz., change in density). This must take place in such a way that no mass is lost (continuity), and if the fluid is truly elastic the squeeze will not stay put but will inevitably move through the fluid as a wave. The result of the combination can be expressed most conveniently in terms of the excess pressure  $p_e$  and then takes the form

$$\frac{\partial^2 p_e}{\partial t^2} = c^2 \frac{\partial^2 p_e}{\partial x^2} \quad (\text{II.2-3})$$

which is of precisely the same mathematical form as equation (1) with  $p_e$  in place of  $\xi$  and  $c$  in place of  $V$ . Physically it therefore means that the acoustical disturbance represented by the excess pressure is propagated as a wave along the  $x$  axis with velocity  $c$ . Actually it is not difficult to show that the displacement  $\xi$  and the excess density  $\rho_e$  are propagated in the same way.

It must indeed be emphasized that in the mathematical deduction leading to (3) it has been assumed that the disturbance is not too extreme. Specifically this means that the ratio of the excess density to the equilibrium density (usually called the condensation) is very small compared with unity.

This also means that the gradient of the particle displacement in the fluid (i.e.,  $\partial \xi / \partial x$ ) is much smaller than unity in magnitude. Moreover the flow or particle velocity associated with the disturbance must be very small indeed compared with  $c$  (the wave velocity). These conditions prevail for all acoustic phenomena associated with speech, sound reproduction and even for most acoustic transmission, as in underwater sound signaling, etc. However, they no longer hold for sounds produced by explosions, or by the rush of gas in the jet of a jet plane. The same is true for sound which can cause cavitation (i.e., the appearance of bubbles) in liquids (see section V.6 below). High-intensity sound, to which may be attached the designation macrosonics, is of importance in the use of sound for processing. Under these conditions the acoustic wave equation no longer has the forms (1) or (3). In place of (1) we now must write

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{V^2 \frac{\partial^2 \xi}{\partial x^2}}{(1 + \frac{\partial \xi}{\partial x})^{\gamma+1}} \quad (\text{II.2-4})$$

in which the quantity  $\gamma$  in the exponent, if the medium is a gas, is the ratio  $C_p/C_v$ , where  $C_v$  = specific heat of gas at constant volume and  $C_p$  = specific heat of the gas at constant pressure. The equation can also hold for solid and liquid media, though here  $\gamma$  has a different meaning; in the simplest cases it has the value unity, but it can have higher values. The solution of (4) is by no means so simple as that of (1), to which it reduces for  $\partial \xi / \partial x \ll 1$ . For  $\gamma = 1$ , for example, the solution can be put into the form

$$\frac{\partial \xi}{\partial t} = f[x + \xi - (V + \frac{\partial \xi}{\partial t})t] \quad (\text{II.2-5})$$

where  $f$  is an arbitrary function. The physical meaning of this rather complicated expression is that the various parts of the wave hump or profile (see fig. 1) do not move forward with the same velocity. Rather, the top of the hump moves faster than the bottom and hence a disturbance which starts out with a symmetrical profile as in fig. 2(A) will develop an asymmetric form as in fig. 2(B) in which the front of the profile will be steeper than the rear. Eventually the front will become vertical and then lean

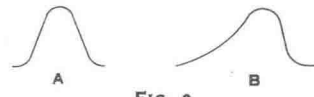


FIG. 2

over so that its top is farther to the right than its bottom. This state of affairs, however, cannot be of physical significance—in any actual case the wave “breaks,” as in water waves at the seashore. The only way a macrosonic wave can maintain its profile or wave form is through some kind of damping mechanism, as, for example, viscosity (see section II.18).

**3. Velocity of Sound Waves in Fluids.**—As was pointed out in section II.2, the velocity of sound in an ideal fluid is given by  $c$  in the expression

$$c = \sqrt{p_e / \rho_e} \quad (\text{II.3-1})$$

The evaluation of the radical for a liquid follows from the definition of the bulk modulus

$$B = \frac{p_e}{\rho_e / \rho} \quad (\text{II.3-2})$$

whence

$$c = \sqrt{B / \rho} = \sqrt{1 / \rho K} \quad (\text{II.3-3})$$

where  $K$  is the compressibility. Since liquids have a relatively small compressibility and it changes but little with the density, it is in general adequate to insert  $\rho_0$  (the mean density) for  $\rho$  in (3).

For gases the situation is rather different. Thus for an ideal gas the equation of state takes the form

$$p / \rho = RT \quad (\text{II.3-4})$$

in which  $R$  is the so-called gas constant per unit mass and  $T$  is the absolute temperature. We therefore obtain

$$p_e / \rho_e = RT = p / \rho \quad (\text{II.3-5})$$

if the temperature remains constant. Hence if the compressional disturbance in the gas takes place isothermally, the velocity has the form

$$c = \sqrt{p / \rho} \quad (\text{II.3-6})$$

This formula was first derived by Newton. It gives, however, a value definitely smaller than the measured one for the nearly ideal gases like oxygen and nitrogen. On the other hand, if we assume that the disturbance constituting the sound takes place adiabatically, we must supplement (4) by

$$p / \rho^\gamma = p_0 / \rho_0^\gamma \quad (\text{II.3-7})$$

in which  $\gamma = C_p/C_v$  is the ratio of the specific heat of the gas at constant pressure to that at constant volume. It follows then that

$$c = \sqrt{\gamma p / \rho} \quad (\text{II.3-8})$$

This result agrees so well with experiment that the measurement of sound velocity became a standard scheme for evaluating  $\gamma$ , and



it is generally assumed that sound disturbances in an ideal fluid take place adiabatically. It must not be assumed offhand that the same is true of a viscous fluid, for example, or one displaying conductivity of heat. For liquids (equation [3]) we must take the adiabatic compressibility in computing  $c$ , but the difference between adiabatic and isothermal compressibilities here is generally small.

For an ideal gas it follows from (4) and (7) that the velocity of sound is independent of the pressure. On the other hand  $c$  does depend on the temperature. In fact

$$c = \sqrt{\gamma RT} \quad (\text{II.3-9})$$

If the velocity at  $0^\circ \text{C.}$  is  $c_0$ , that at temperature  $t^\circ \text{C.}$  is clearly

$$c_t = c_0 \sqrt{1 + t/273} \quad (\text{II.3-10})$$

For dry air under standard conditions  $c_0 = 331.3 \text{ m. per second.}$  This agrees rather well with actual measurements made in the open air, but it must be remembered that the latter are extremely difficult to perform precisely since atmospheric conditions are rarely uniform, and sound transmission in air from the standpoint of precision is extremely complicated because of nonhomogeneity and wind disturbances.

For real gases (9) must be modified, since the equation of state is no longer in the form (4). It is now more convenient to revert to equation (3). Realizing that the adiabatic bulk modulus must be used for  $B$ , we write  $B_{\text{ad}} = \gamma B_{\text{iso}}$  and by definition

$$B_{\text{iso}} = \rho(\partial p / \partial \rho)_T \quad (\text{II.3-11})$$

The expression for the velocity  $c$  then becomes

$$c = \sqrt{\gamma(\partial p / \partial \rho)_T} \quad (\text{II.3-12})$$

For the ideal gas (4), this formula yields (8). One useful form of the equation of state of a real gas is

$$p/\rho = RT(1 + \beta\rho + C\rho^2 + \dots) \quad (\text{II.3-13})$$

The quantities  $\beta$  and  $C$  are known as the second and third virial coefficients per gram of gas respectively and are functions of the temperature. Substituting into (12) yields

$$c = \sqrt{\gamma RT(1 + 2\beta\rho + 3C\rho^2 + \dots)} \quad (\text{II.3-14})$$

In this form it is clear that the velocity of sound depends on the pressure as well as the temperature. The effect in air is small except at very low temperatures. Thus for air at  $90.1^\circ \text{K.}$  the velocity at 1 atm. pressure is about 188 m. per second (A. van Itterbeek and W. van Doninck, *Annales de Physique*, 19:88 [1944]), whereas at 0.2 atm. it is 191 m. per second. (Note that at this temperature  $\beta$  is negative.)

The experimental determination of the velocity of sound can be used as a method of measuring  $\gamma$  as well as the virial coefficient  $\beta$ . It is also possible to calculate theoretically the velocity of sound in a mixture of two gases and then use the formula to determine the relative concentrations of the components by a sound velocity measurement. (H. B. Dixon and G. Greenwood, *Proc. Roy. Soc. Lon. (A)*, 109:561 [1925].)

As might be expected, the velocity of sound is much more sharply dependent on pressure in the neighbourhood of the critical point. Thus in carbon dioxide at  $31^\circ \text{C.}$ , for example, the value of  $c$  drops from about 260 m. per second at 20 atm. pressure to a minimum of 150 m. per second at 71 atm. and rises again steeply thereafter to 330 m. per second at 90 atm.

The formulas so far presented involve no dependence of the velocity of sound on the frequency. This corresponds to experience as far as the so-called permanent gases are concerned at ordinary pressure and low frequencies; they show no dispersion. However as the frequency-pressure ratio increases all gases ultimately exhibit dispersion, and indeed the velocity is found to increase with increasing  $\nu/p$ . Thus for hydrogen,  $c$  increases about 8% above its low-frequency standard-pressure value of 1,284 m. per second as  $\nu/p$  is raised from 1 mc. per atmosphere to 30 mc. per atmosphere. The reason for this will be discussed in section II.13.

The velocity of sound in gases also depends on the purity of the

gas. Small traces of foreign gases can produce significant changes in  $c$ . Thus for pure carbon dioxide (with only a trace of neon and argon) the velocity for  $\nu/p = 3.2 \times 10^4$  cycles per atmosphere corresponds to a value of  $\gamma$  in (8) of approximately 1.34. With the addition of 2.8% water vapour, however, this same value of  $\gamma$  is associated with  $\nu/p = 1.6 \times 10^6$  cycles per atmosphere. The whole dispersion curve is thus displaced. Alternatively put, the sound velocity at a given  $\nu/p$  value is much decreased by the addition of the water vapour. The velocity of sound is of course affected by the motion of the medium. Thus in the open air with the wind blowing the velocity of sound with respect to the ground will be greater with the wind than against it.

Coming now to the velocity of sound in liquids we can still use equation (3) but must necessarily employ the adiabatic bulk modulus or compressibility. The change in compressibility of a liquid with temperature is much more complicated than that of a gas and it is practically impossible to develop helpful general formulas. The case of water has been carefully studied and the following equation (G. W. Willard, *J. Acous. Soc. Amer.*, 19:235 [1947]) represents rather accurately the dependence of sound velocity in metres per second on temperature in degrees centigrade

$$c_t = 1,557 - 0.0245(74 - t)^2 \quad (\text{II.3-15})$$

This indicates that  $c_t$  increases with temperature up to a maximum value at  $t = 74^\circ \text{C.}$ , and thereafter decreases. For practically all other liquids which have been studied  $c$  decreases with temperature over the whole range in which the material can stay liquid. Salt solutions obey an equation like (15) with different values of the temperature for maximum  $c_t$ . The case of sea water is of particular interest because of the importance of underwater sound. Here matters are complicated by the fact that changes in salinity and pressure (with depth in the ocean) as well as temperature are effective in changing velocity. The following empirical formula (L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics*, p. 435 [1950]) holds fairly well over the range of variables usually encountered

$$c_t = 1.41 \times 10^6 + 4.21 \times 10^4 t - 3.7t^2 + 1.1 \times 10^3 S + 1.8 \times 10^{-2} d \quad (\text{II.3-16})$$

where  $t$  is in degrees centigrade,  $S$  is the salinity in parts per 1,000 and  $d$  is the depth in centimetres. The velocity is given in centimetres per second.

As is suggested by (16), sound velocity in solutions is markedly dependent on the concentration of the solute. For NaCl, for example, the velocity in a 10% solution at room temperature is 1,600 m. per second while for a 20% solution it rises to 1,720 m. per second. Not all solutions show a rise over the velocity in the pure solvent, however, and the problem is complicated.

As (16) also indicates, increase in pressure on a liquid raises the sound velocity more or less linearly. Thus for benzene P. Biquard found that the velocity is increased by about 17% in going from atmospheric pressure to 500 atm.

Unlike the situation in gases, no definite indication of sound dispersion in liquids has been found experimentally, though theoretical considerations suggest that it should exist for sufficiently high frequencies, probably of the order of several hundred megacycles.

**4. Mathematical Representation of Wave Fronts.**—In section I.6 it was seen that to describe adequately the propagation of acoustic waves it is necessary to introduce the concept of wave front as a surface at every point of which at a given instant the propagated disturbance is the same and doing the same thing (*i.e.*, either increasing or decreasing). We now give this analytical precision, confining our attention to harmonic waves. Previously (equation II.1-3) we dealt with harmonic wave functions of the form

$$\xi = A \sin(\omega t - kx)$$

corresponding to propagation along the positive  $x$  axis only. But we have already pointed out that it is the general tendency for sound waves in free space to spread in all directions and hence we must generalize our analysis. To simplify matters, when we consider wave propagation in three-dimensional space, we shall use



the excess pressure as the measure of the acoustical disturbance. Consider the function

$$p_e = A(x, y, z) \sin \omega[t - \psi(x, y, z)/c_0] \quad (\text{II.4-1})$$

in which  $A$  is the amplitude of the pressure wave and may be a function of position in space. The function  $\psi(x, y, z)$  is assumed to have the dimensions of distance only, and  $c_0$  is a constant having the dimensions of velocity.

It turns out by careful examination that (1) is a genuine harmonic wave function (*i.e.*, a solution of the general three-dimensional wave equation of the form of [II.2-2]) under certain conditions. The phase of the wave, *viz.*,  $\omega[t - \psi(x, y, z)/c_0]$ , has the constant value  $\omega t_0$  at the time  $t$  over the surface with the equation

$$\psi(x, y, z) = c_0(t - t_0) \quad (\text{II.4-2})$$

Hence this is the general equation for the wave front at time  $t$ . In the passage of time the wave front equation will alter just as if the wave front itself were moving through space, carrying with it the same value of the disturbance.

If

$$\psi(x, y, z) = \alpha x + \beta y + \gamma z \quad (\text{II.4-3})$$

equation (2) represents a plane wave front with the direction cosines of the normal to the front equal to  $\alpha, \beta, \gamma$ . In this case  $A(x, y, z)$  reduces to a constant.

If

$$\psi(x, y, z) = \sqrt{x^2 + y^2 + z^2} \quad (\text{II.4-4})$$

equation (2) represents a spherical wave front. It develops that (1) is then a harmonic pressure wave spreading out from the origin if  $A(x, y, z) = A_0/r$ , where

$$r = \sqrt{x^2 + y^2 + z^2} \quad (\text{II.4-5})$$

and  $A_0$  is constant. Plane and spherical waves are the two most important types for sound propagation. Both are obviously somewhat idealized; a spherical wave front demands a point source and there is no such thing as a precise point source of sound. A plane wave front demands an infinitely extended plane source, which likewise does not exist. Nevertheless actual wave fronts often approximate these ideal types sufficiently well to justify their use. In general the smaller the dimensions of a source compared with the wave length of the sound the more nearly the wave front approximates the spherical or hemispherical shape. The larger the dimensions of a plane source compared with the wave length, the more nearly is a plane wave front realized.

**5. Sound Rays; Refraction.**—For many purposes the propagation of light can be effectively studied in terms of rays—curves which are always perpendicular to the family of wave fronts associated with a particular propagation. The ray concept is also useful in acoustics but only if the wave length is sufficiently small or, more accurately, if the change in wave length (due to change in velocity from one point to another in the medium) over a distance of the order of one wave length is very small compared with the wave length itself.

Moreover, the ray must not bend too rapidly as the sound progresses. Finally, the change in amplitude over a like distance must be small compared with the initial amplitude. If these conditions are satisfied it is meaningful to write the differential equations of the ray paths in any medium in which the wave velocity  $c$  varies from point to point. Actually, in analogy with optics it is customary to define the ratio  $c_0/c$  as the index of refraction  $n$  of the medium. The physical meaning of  $c_0$  is the nonvarying velocity in some homogeneous medium bounding the given nonhomogeneous medium. Let the direction cosines of the ray path be  $\alpha, \beta, \gamma$ . These are in general functions of position, *i.e.*, of the co-ordinates  $x, y, z$ . If  $ds$  is the element of distance along the ray, the differential equations of the ray paths take the form

$$d(\alpha n)/ds = \partial n/\partial x, \quad d(\beta n)/ds = \partial n/\partial y, \quad d(\gamma n)/ds = \partial n/\partial z \quad (\text{II.5-1})$$

As soon as  $n$  is known as a function of  $x, y, z$ , equations (1) can be integrated to give the rays. As an illustration, suppose  $n =$

constant, so that there is no variation of sound velocity in the medium. Equations (1) then lead to

$$\alpha = K_1, \beta = K_2, \gamma = K_3 \quad (\text{II.5-2})$$

where  $K_1, K_2, K_3$  are constants, giving then the direction cosines of a straight line. The rays are straight lines, irrespective of the character of the wave fronts (*e.g.*, the latter may be plane or spherical).

One typical case of ray propagation occurs when the medium is stratified; *i.e.*, the change in velocity takes place in one direction only. Let us take this as the  $z$  axis. It will be sufficient to consider the ray path in the  $xz$  plane. In fig. 3 let  $AB$  be the element  $ds$  of the ray and let the tangents at  $A$  and  $B$ , respectively, make the angles  $\Theta$  and  $\Theta + d\Theta$  with the  $x$  axis. We now have

$$\begin{aligned} \partial n/\partial x &= \partial n/\partial y = 0, \\ \alpha &= \cos \Theta, \beta = 0, \gamma = \sin \Theta \end{aligned}$$

Equations (1) now yield

$$d(n \cos \Theta)/ds = 0, \quad d(n \sin \Theta)/ds = dn/dz \quad (\text{II.5-3})$$

The first equation says that along the ray

$$n \cos \Theta = \text{constant} = n_1 \cos \Theta_1 \quad (\text{II.5-4})$$

where  $\Theta_1$  is the direction of the ray at the place where the index is  $n_1$ . If we write  $\Phi = \pi/2 - \Theta$ , where  $\Phi$  is the angle the ray makes with  $z$ , (4) becomes

$$\sin \Phi/\sin \Phi_1 = n_1/n = c/c_1 \quad (\text{II.5-5})$$

This has the familiar form of Snell's law for the refraction of light rays. Here, however, it is of much more general form since the

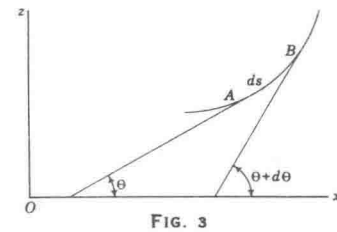


FIG. 3

variation of velocity is continuous and does not take place abruptly at a boundary plane as in the usual elementary optical case. The abrupt change involves a discontinuity in  $n$  and therefore demands special treatment, discussed in section II.7. It is interesting to observe that the second equation in (3) makes possible the evaluation of the curvature ( $d\Theta/ds$ ) of the ray at any point and hence ultimately the equation of the ray path. By straightforward analysis (3) leads to

$$d\Theta/ds = -dc/dz \cdot \cos \Theta_1/c_1 \quad (\text{II.5-6})$$

As an illustration, suppose

$$c = c_1 + az \quad (\text{II.5-7})$$

*i.e.*, the velocity is a linear function of the depth  $z$  ( $a$  is a constant), with  $c_1 =$  velocity at  $z = 0$ , which might, for example, be the surface of the sea. Then (6) says that the curvature is constant. This means that the path must be a circle with radius  $= |c_1/a \cos \Theta_1|$ . If  $a$  is positive, the velocity increases with depth and the curvature is negative and the path bends upward toward the surface  $z = 0$ . If  $a$  is negative the velocity decreases with depth (a more common situation in the ocean) and the circular path bends downward. These results can readily be followed graphically.

**6. Refraction in a Moving Medium.**—The propagation of sound through a moving medium (*e.g.*, the atmosphere with the wind blowing) is extremely complicated and its analysis will not be given here. However, the law of refraction for a stratified medium turns out to be a rather simple modification of Snell's law (II.5-5). If the velocity of the medium is confined to the  $x$  direction only and is equal to  $u$ , which is a function of  $z$  (*e.g.*, height above the ground, taken as the  $xy$  plane, in the case of transmission through the atmosphere), the law of refraction becomes

$$c/\cos \Theta - c_1/\cos \Theta_1 = u_1 - u \quad (\text{II.6-1})$$

Here  $u_1$  is the medium velocity at the point where the sound velocity is  $c_1$  and the ray direction is given by  $\Theta_1$ . Equation (1) reduces to (II.5-5) for  $u = u_1 = 0$ . It is clear from (1) and

also from relatively simple graphical constructions that the effect of the motion of the medium is to bend the wave front and ray in the direction in which the motion takes place. Thus if the wind velocity increases upward from the ground, sound wave fronts are lifted to windward and depressed to leeward. Other things being equal, this decreases the range to windward and increases it to leeward. These considerations were once of considerable importance in the acoustical detection of aircraft, but were superseded by radar. They remain of great importance, however, in sonic gun ranging (see section V.5 below).

**7. Huygens' Principle; Reflection and Refraction at an Interface.**—The propagation of an acoustical wave front through a medium can also be studied by means of a principle developed by Christiaan Huygens. According to this principle, in its elementary form, each point on a wave front (like  $F_1$  in fig. 4, for example) may be assumed to be the source of a hemispherical wavelet which moves outward from  $F_1$  in the direction of propagation; the new wave front after a short time is the mathematical envelope of all these wavelets; e.g.,  $F_2$  in the figure. This principle can be demonstrated by considering the solution of the general wave equation. A simple illustration of the principle is the establishment of the laws of reflection and refraction at a plane interface separating fluid media with different properties; e.g., different mean densities and sound velocities.

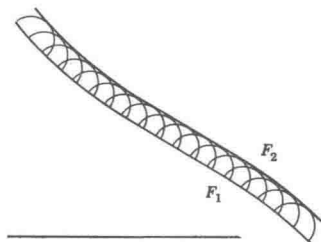


FIG. 4

Let  $AB$  in fig. 5 represent the trace in the plane of the paper of the plane interface between a fluid medium I in which the velocity of sound is  $c_1$  and the mean density  $\rho_1$ , and a medium II in which the corresponding quantities are  $c_2$  and  $\rho_2$  respectively. Let  $CD$  denote the trace of a plane wave front (the latter being perpendicular to the plane of the paper) incident on  $AB$  at angle  $\Theta_i$ . This means that the incident ray travels in the direction  $HC$

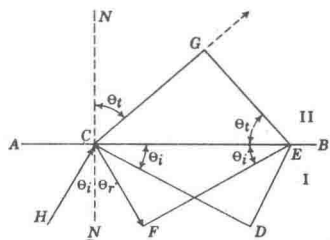


FIG. 5

perpendicular to  $CD$ . We can construct the reflected and refracted wave fronts by means of Huygens' principle. At the instant the disturbance at one end of the wave front has reached  $C$  on the interface, the disturbance at the other end is still at  $D$  in medium I. In the time  $t_1 = DE/c_1$  which it takes the disturbance at  $D$  to reach  $E$  on the interface, the disturbance will

travel out in all directions from  $C$ . In particular in I it will traverse a distance equal to  $DE$ .

If we draw with  $C$  as centre a semicircle with radius  $DE$  and construct the straight line from  $E$  tangent to this we obtain  $FE$  which from geometry makes the angle  $\Theta_r = \Theta_i$  with  $AB$ . That this is the reflected wave front can be confirmed by making a Huygens construction for other points on the incident wave front. We see that the reflected wave front makes the same angle with the interface as the incident wave front. This, combined with the readily established additional fact that both wave fronts are perpendicular to the same plane (namely the plane of the paper) constitutes the law of reflection. It can of course be equally well expressed in terms of the incident and reflected rays, and is the same law that holds for light wave fronts and rays.

Precisely the same kind of analysis shows that after transmission across the interface the incident wave front  $CD$  is changed to the refracted wave front  $GE$  making the angle  $\Theta_t$  with  $AB$ , such that

$$\sin \Theta_i / \sin \Theta_t = c_2 / c_1 = n_{21} \quad (\text{II.7-1})$$

This is Snell's law again, with  $n_{21}$  the index of refraction of medium II with respect to medium I. If the media are in motion with respect to each other, this result must be modified (cf. equation II.6-1). The above formula accounts well for the

passage of sound from air to water or vice versa.

**8. Energy Propagation in Sound Waves; Intensity.**—In section I.2 the intensity of sound was discussed, and its relation with energy was implied. We now look upon sound propagation as essentially a form of transmission of energy through a fluid. When a fluid is disturbed by squeezing it at any place, work is done, representing the expenditure of energy. The reappearance of the disturbance at a distant point by wave propagation corresponds to the transfer of the original energy from the point where it was put into the fluid to the distant point. The greater the amount of energy transported on the average per unit area of wave front per unit of time, the greater will be the intensity of the corresponding sound wave.

To be more precise, the average rate of flow of energy in the sound wave per unit time per unit area of wave front is simply calculated as the average of the product of the excess pressure  $p_e$  and the flow velocity  $\partial \xi / \partial t$  in the medium. Analysis shows that the intensity  $I$  of a plane wave computed in this way can be expressed in the following way

$$I = p_0^2 / 2\rho_0 c \quad (\text{II.8-1})$$

This important formula (G. W. Stewart and R. B. Lindsay, *Acoustics*, pp. 62 ff. [1930]) says that the intensity of a plane wave is equal to the square of the excess pressure amplitude  $p_0$  divided by twice the product of mean density and sound velocity. Significantly it is independent of frequency.

The standard absolute unit of intensity is the erg/sec. cm.<sup>2</sup> More practical is the watt/cm.<sup>2</sup> or, in the metre-kilogram-second system favoured by some engineers and physicists, the watt/metre<sup>2</sup>. However, as noted in section I, the decibel is now used as the standard measure of sound intensity. This is a relative measure. If two sound waves have absolute intensities  $I_1$  and  $I_2$  respectively they are said to differ in intensity level by  $D$  decibels (or db.), where

$$D = 10 \log_{10} I_2 / I_1 \quad (\text{II.8-2})$$

Hence if  $I_2 = 2I_1$ ,  $D = 3.01$  db., or doubling the intensity means an increase of intensity of a little more than 3 db. To use this method of measuring intensity requires the choice of a standard level. This is often taken as that corresponding to an average excess pressure of 1 dyne/cm.<sup>2</sup> in air (close to the normal level for conversational speech). The minimum audible sound lies approximately 70 db. below this level, while the threshold of feeling, when hearing becomes painful, lies about 70 db. above this level. These figures are approximate and actually depend on frequency.

From equation (1) it is clear that the intensity of sound at given pressure depends on the medium and in particular on the product  $\rho_0 c$ . This quantity is known as the specific acoustic resistance of the medium (for a plane wave). For water, for example, it is about 3,800 times its value for air at standard pressure. Consequently the same excess pressure in water produces a much smaller intensity than in air and it might seem that it is harder to produce an intense sound in water than in air. This, however, neglects considerations connected with the source of sound, which are discussed later (section III.9). It turns out that at given frequency and pressure a solid radiating source is actually more efficient in water (and in liquids generally) than in air.

**9. Intensity of Spherical Sound Waves.**—Though equation (II.8-1) was cited as applying to a plane wave, a similar formula applies to a spherical wave in the corresponding medium. Here, to be sure, the pressure and particle velocity expressions are different. Whereas for a plane wave the excess pressure and flow velocity (sometimes called particle velocity) amplitudes are constant in a nonabsorbing medium, in the case of a spherical wave they both fall off with the distance from the source of the radiation (section II.4 above). The analytical expressions are complicated and will not be reproduced here. When the multiplication and averaging processes are carried out as in section II.8 above, we obtain for the intensity in the spherical wave at distance  $r$  from the source

$$I = p_0^2 / 2\rho_0 c r^2 \quad (\text{II.9-1})$$

Both (II.8-1) and (II.9-1) can clearly be put in the same form if