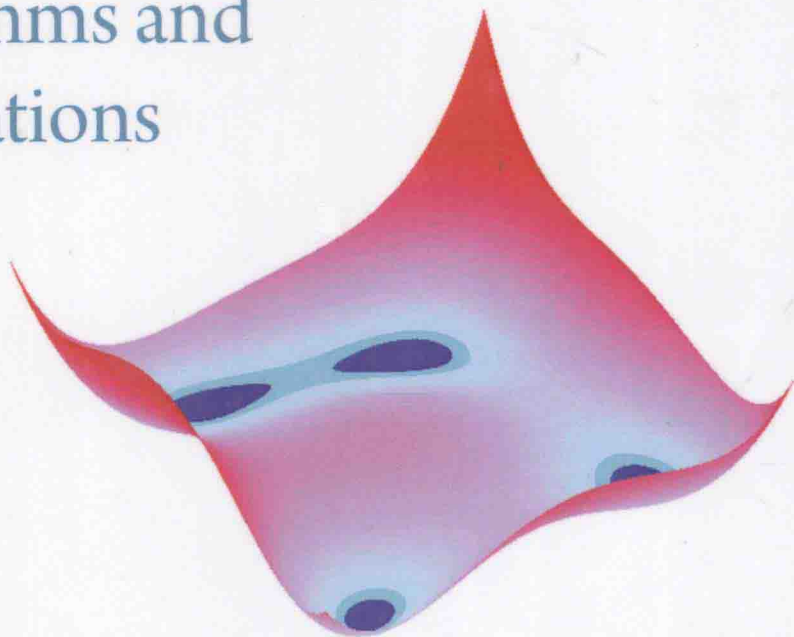


OPTIMIZATION

Algorithms and Applications



Rajesh Kumar Arora



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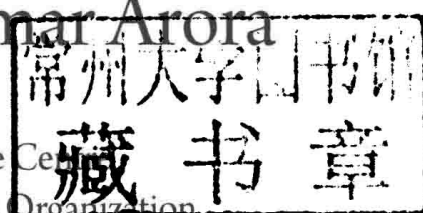
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Applications

Dedicated to my mother

Preface

There are numerous books on the subject of optimization, attributable to a number of reasons. First, the subject itself is mathematically rigorous and there are a number of solution methods that need to be examined and understood. No single solution method can be applied to all types of optimization problems. Thus a clear understanding of the problem, as well as solution techniques, is required to obtain a proper and meaningful solution to the optimization problem. With the progression of time, optimization problems have also become complex. It is necessary not only to obtain the global optimum solution, but to find local optima as well. Today's problems are also of the multiobjective type, where conflicting objective functions are to be handled. There is also a need to simultaneously handle objective functions and constraints of different disciplines, resulting in multidisciplinary design optimization (MDO) problems that are handled using different architectures. Gradient-based methods were popular until the 1990s. At present, a large number of complex problems are solved using guided random search methods such as genetic algorithm, simulated annealing, and particle swarm optimization (PSO) techniques. Even hybrid algorithms, that use a combination of gradient-based and stochastic methods, are also very popular. Different authors have addressed these issues separately, resulting in a number of books in this area.

So how does this book differ from the others? The solution techniques are detailed in such a way that more emphasis is given to the concepts and rigorous mathematical details and proofs are avoided. It is observed that a method can be understood better if different parameters in the algorithm are plotted or printed over different iterations while solving a problem. This can be accomplished by writing a software code for the method or the algorithm. It is often difficult for a newcomer to write a software code if the algorithm such as, say, Broyden–Fletcher–Goldfarb–Shanno (BFGS) or PSO is given to him or her. In this book, a step-by-step approach is followed in developing the software code from the algorithm. The codes are then applied to solve some standard functions taken from the literature. This creates understanding and confidence in handling different solution methods. The software codes are then suitably modified to solve some real-world problems. A few books on optimization have also followed this approach. However, the software code in these books is hard to correlate with the corresponding algorithms mentioned in the book and readers are forced to use them as black box optimization tools. The codes presented in this book are user friendly in the sense that they can be easily understood. A number of practical problems are solved using these codes.

The codes are written in the MATLAB[®] environment and the use of ready-made optimization routines available in MATLAB is avoided. The algorithms are developed right from computing the gradient or Hessian of a function to a complex algorithm such as for solving a constraint optimization problem. MATLAB is a software package for technical computing that performs both computing and visualization with ease. It has a number of built-in functions that can be used by an individual's application. The main advantage of MATLAB is the ease with which readers can translate their ideas into an application.

The book covers both gradient and stochastic methods as solution techniques for unconstrained and constrained optimization problems. A separate chapter (Chapter 5) is devoted to stochastic methods, where genetic algorithm, PSO, simulated annealing, ant colony optimization, and tabu search methods are discussed. With simple modifications of the basic PSO code, one can also solve nonconvex multiobjective optimization problems. This is probably the first optimization book in which MDO architectures are introduced (Chapter 9). Software codes are also developed for the simplex method and affine-scaling interior point method for solving linear programming problems. Gomory's cutting plane method, branch-and-bound method, and Balas' algorithm are also discussed in the chapter on integer programming (Chapter 10). A number of real-world problems are solved using the MATLAB codes given in the book. Some applications that are included in this book are solving a complex trajectory design problem of a robot (Chapter 3), multiobjective shape optimization problem of a reentry body (Chapter 7), portfolio optimization problem (Chapter 4), and so forth.

I thank my organization, Vikram Sarabhai Space Centre (a lead center of Indian Space Research Organisation [ISRO]), for giving permission to publish this book. The book has been reviewed internally by Dr. Mohankumar D., Head, Computer Division. I thank him for his suggestions and corrections. I thank Mr. Pandian, S., Deputy Director, VSSC for supporting the idea to write this book. I am ever grateful to Prof. M Seetharama Bhat from IISc, Bangalore and Dr. Adimurthy, V. for their support during the last ten years. I thank my colleagues Dr. Jayakumar K., Mr. Priyankar, B., Mr. Sajan Daniel and Mr. Amit Sachdeva for many hours of discussions on book-related aspects.

I am grateful to Taylor & Francis Group for agreeing to publish this book and agreeing to most of my suggestions. Much credit should be given to Ms. Aastha Sharma, Editor, for her prompt actions and follow-up with the reviewers. Thanks are also due to three anonymous reviewers for their critical remarks, corrections, and suggestions. I thank Mr. Sarfraz Khan, assistant to Ms. Aastha Sharma, for providing online support in signing the contract. I also thank Mr. David Fausel for coordinating and reviewing the style and art files of the book. My sincere thanks to Mr. Ed Curtis and Ms. Amor Nanas for language corrections, copyediting, and other production related works. The cover page is designed by Mr. Kevin Craig.

I thank the MATLAB book program for supporting the idea of this book on optimization with MATLAB codes. They have also agreed to give wide publicity to this book on their website, for which I am grateful.

I thank my wife, Manju, and children, Abhinav and Aditi, for their patience during the last two years. In fact my whole family—father, brothers, sister, and in-laws—are eagerly waiting for the launch of this book.

This is the first edition of this book. Some errors and omissions are expected. The MATLAB codes are validated with a number of test problems taken from the literature. It is still possible that some pathways in the codes would not have been exercised during this validation. As a result, no claim is made that these codes are bug-free. I request readers to report corrections and suggestions on this book at rk_arora@vssc.gov.in or arora_rajesh@rediffmail.com.

The MATLAB codes mentioned in this book can be downloaded from the weblink <http://www.crcpress.com/product/isbn/9781498721127>.

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