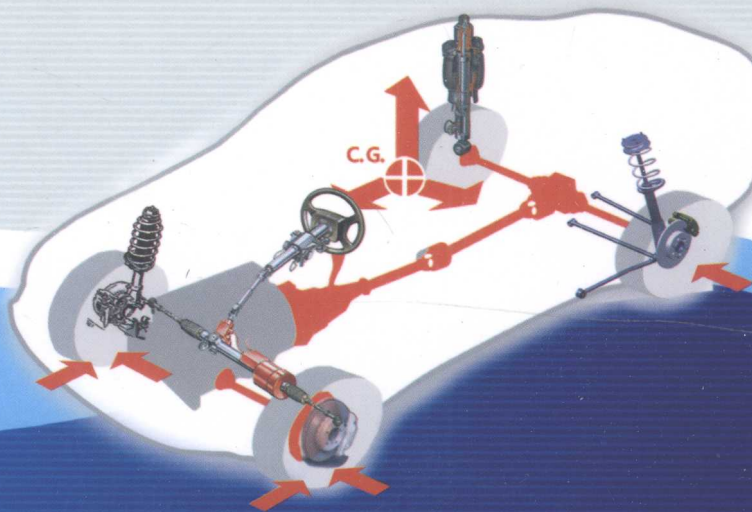


汽车工程学 II: (英文版)

汽车垂向和侧向动力学

Automotive Engineering II:

Vertical and Lateral Dynamics of Vehicles



(德) Henning Wallentowitz 著

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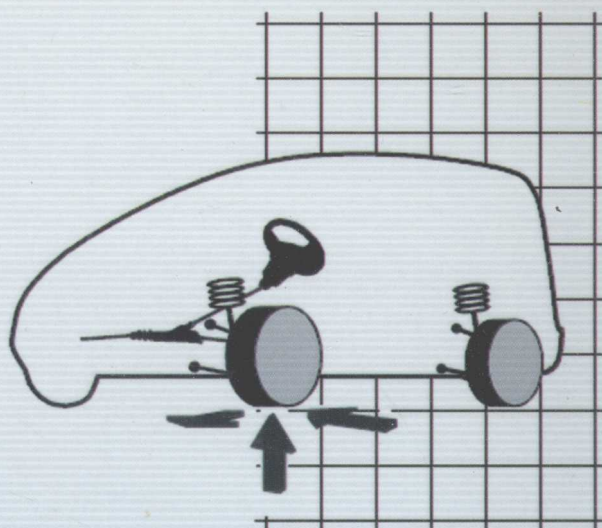
机械工业出版社
CHINA MACHINE PRESS

Automotive Engineering II:

Vertical and Lateral Dynamics of Vehicles

◆ Suspension Systems

◆ Driving Behaviour



○ 策划编辑: 赵爱宁

○ 封面设计: 王伟光

编辑热线: **010-88379217**

地址: 北京市百万庄大街22号

联系电话: (010)68326294

(010)68993821

购书热线: (010)88379639

(010)88379641

(010)88379643

邮政编码: 100037

网址: <http://www.cmpedu.com> (机工教材网)

E-mail: cmp@cmpedu.com

网址: <http://www.cmpbook.com> (机工门户网)

E-mail: cmp@cmpbook.com

ISBN 978-7-111-26402-6

定价: 80.00元

ISBN 978-7-111-26402-6



9 787111 264026 >

汽车工程学Ⅱ： 汽车垂向和侧向动力学

**Automotive Engineering II : Vertical
and Lateral Dynamics of Vehicle**

(英文版)

(德) 亨宁·瓦伦托维兹 (Henning Wallentowitz) 著
李克强 编注



机械工业出版社

本书分为汽车垂向动力学和侧向动力学两部分。

垂向动力学内容包括悬架特性分析、各种不同的悬架结构及其力学特性分析、单轮悬架模型、单体悬架模型和双体悬架模型。

该部分从汽车垂向动力学概念出发,首先分析了悬架对车辆及驾驶员的影响,针对这些影响详细介绍了悬架各个组成构件,并从理论上讨论了这些机械结构的设计思想和工作原理;详细介绍了单轮模型、单体模型等用于分析评估垂向动力学特性的经典模型。不仅如此,本部分还针对当前汽车垂向动力学领域研究的前沿课题进行了初步探讨。

侧向动力学内容包括汽车侧向运动稳定性概念、轮胎静态和动态特性分析、线性二自由度汽车模型及其动力学特性、四轮汽车模型及其动力学特性分析、转向系及其对汽车侧向动力学特性的影响、悬架系统动力学特性及其对侧向动力学特性的影响以及各种不同的悬架结构介绍等。

该部分从侧向动力学的概念、分析方法和影响因素出发,对汽车侧向动力学进行了全面的描述。其内容丰富,不仅对线性二自由度汽车模型、简化悬架系统模型等经典的用于分析汽车侧向动力学特性的内容进行了详细介绍,还对现代汽车侧向动力学的前沿课题进行了初步探讨。

本书除了可用于高等学校车辆工程专业的双语教学外,还可作为汽车工程师的参考书。

图书在版编目(CIP)数据

汽车工程学 = Automotive Engineering. II, 汽车垂向和侧向动力学: 英文/ (德) 瓦伦托维兹 (Wallentowitz, H.) 著; 李克强编注. —北京: 机械工业出版社, 2009. 4

ISBN 978-7-111-26402-6

I. 汽… II. ①瓦…②李… III. ①汽车工程—双语教学—高等学校—教材—英文②汽车—动力学—双语教学—高等学校—教材—英文— IV. U46

中国版本图书馆 CIP 数据核字 (2009) 第 025050 号

机械工业出版社 (北京市百万庄大街 22 号 邮政编码 100037)

策划编辑: 林 松 赵爱宁 责任编辑: 赵爱宁 版式设计: 霍永明

封面设计: 王伟光

责任印制: 洪汉军

北京瑞德印刷有限公司印刷 (三河市胜利装订厂装订)

2009 年 8 月第 1 版第 1 次印刷

184mm×260mm · 16.5 印张 · 3 插页 · 407 千字

标准书号: ISBN 978-7-111-26402-6

定价: 80.00 元

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Preface

This book is used for lecturing the basics of automotive engineering at RWTH Aachen University. The automotive lectures in total are organized according to the different degree of freedom in the vehicle movement. Thus this book is one out of the series and it is covering vertical and lateral vehicle dynamics.

The vertical dynamics is determined by road roughness. Therefore the road characteristics are determined first. A mathematical description for different road qualities is given. The components of suspension systems are described in the next chapter. Starting with the tire, different suspension systems and the performance of shock absorbers are described. The mathematical equations for the suspension design are mentioned in addition. A description of seat performance starts the discussion of the human feeling for vibration. The co-operation of the suspension elements are discussed in a mathematical way, when the so called single wheel suspension model is described. Here above all the two-mass model can be used to determine the right data for designing suspensions. The application is shown by some parametric studies. The influences of tires, spring stiffness and shock absorber characteristic are becoming obvious.

Single and two track suspension models and some remarks about the suspension investigation methods are closing this part of vehicle dynamics discussion.

The other very essential degree of freedom covers the lateral vehicle dynamics. Starting with the demands on vehicle behaviour and the control loop between car and driver, the technical discussion starts with the tire again. The longitudinal performance with brake and traction slip and the lateral performance with side slip is discussed in detail. Afterwards the co-operation between longitudinal and lateral tire forces is made clear. For the understanding of vehicle dynamics, the single track vehicle model is essential. This is also the basis for the vehicle-control systems of today. The steering behaviour of vehicles is defined and the evaluation of those characteristics is described. The mathematical equations support the understanding. Then the four wheel vehicle model with wheel load changes is discussed. Now the vertical dynamics and the lateral dynamics are combined and it becomes obvious, that engineers have several possibilities to influence the driving behaviour of the cars. Examples are used to make these relationships clear.

The lateral dynamics of cars depend heavily on the steering systems, therefore a special chapter deals with steering systems. The kinematical data and the steering lay out of cars are explained. As the wheel behaviour is also essential for the vehicle



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dynamics, different suspension types and the specific performance is dealt with. The evaluation of important kinematical data for the different axles is shown. So the theoretical discussion with the four wheel model can be transferred to different axle.

As a summary, this book contains a lot of basic automotive knowledge and hints for practical application. Thus the book is also beneficial for the engineers during their practical work in the companies. With this knowledge in mind it will be possible to solve a lot of other problems. This volume of the series is designed as a reference book for the lecture "Automotive Engineering II" and it was compiled by a team at the institute.

H. Wallentowitz

Univ. -Prof. Dr. -Ing. Henning Wallentowitz

前 言

Foreword

竞争的全球化带来了教育的国际化。教育国际化使得高等学校有机会了解、引进和使用国外的优秀教育资源,有利于拓展学生的国际视野,加强与国际同行的交流。亚琛工业大学(RWTH Aachen University)是德国知名的工科大学之一,在汽车工程领域享有很高的国际声誉。2001年中德双方教育部门共同推出了“清华—亚琛联合硕士生培养项目”,双方共同制订联合培养方案,确定以英语作为教学语言,双方教师共同授课。双方每年互派学生、交换教师,进行教学交流和科研合作。通过开展合作研究以及对研究生的联合培养,借鉴国际先进的教育理念和做法,引进优秀的教育资源,提高了研究生培养质量、教师的教学水平和学术水平,同时提升了学科水平和清华在国际上的影响力。清华学生得益于德国世界先进的工程教育,加强了工程实践能力,在国际环境中的学习研究经历也有利于学生了解异国文化。清华—亚琛联合研究生培养项目是对我国教育部“强强联合”倡导的具体实施。该项目在清华联合培养方面为双方延伸出一个广阔的合作平台,是建设“综合性、研究型、开放式”的世界一流大学中的重要举措。截止2008年6月,清华大学汽车工程系已选派54名研究生和12名教师赴亚琛工大学习和交流,其中28名研究生已获得亚琛工大硕士学位;同期内,亚琛工大汽车研究所向清华选派了39名研究生,其中30名已返回亚琛。2007年11月,清华大学顾秉林校长赴亚琛向8名亚琛工大的学生授予清华大学硕士学位。

本书编者 Henning Wallentowitz 教授是“清华—亚琛联合硕士生培养项目”发起者,具有丰富的工程实践背景、很深的学术造诣和教学经验。Wallentowitz 教授 1978~1985 年在 Daimler Benz (戴姆勒奔驰) 汽车公司担任实验室工程师开发部负责人,1985~1992 年在 BMW (宝马) 汽车公司悬架开发部担任主任工程师,1992~1993 年担任 BMW (宝马) 技术股份有限公司董事长,1997~2004 年担任亚琛工业大学副校长,2004 年被聘为清华大学兼职教授。他在亚琛工业大学主要承担了 Automotive Engineering (汽车工程), Structural design of motor vehicles (汽车结构设计), Mechatronics in Vehicle Engineering (车辆机电一体化系统), Motorbikes (摩托车), Plastics in Automotive Application (汽车中的塑料应用), Interior and Exterior Vehicle Noise (汽车内外噪声) 等课程的教学。

本书不同于传统的“汽车理论”和“汽车设计”,反映了德国汽车工程的教学内容和教学模式以及汽车先进技术,具有全新的教材体系,共分三部分。

《汽车工程学 I》比较全面地介绍汽车的行驶性能和制动性能,即汽车的直线行驶性能。内容主要包括:交通系统与汽车;汽车的经济方面;车轮阻力;空气动力阻力;爬坡阻力;加速阻力;汽车发动机;电驱动;混合动力;离合器;变速器;差速器;制动器;制动传动系统;传动系振动;车辆的驱动性能;车辆的燃油消耗;传动系布置;



驱动极限；摩擦限制的制动能力；制动力分配；制动力控制；制动力调节。该书内容比较新，覆盖面宽，视角独特，对汽车专业的本科生、研究生以及汽车工业的工程师都具有比较大的参考价值。

《汽车工程学Ⅱ》介绍了汽车侧向动力学和垂向动力学的相关基础理论和典型的应用系统。以动力学为基础，分析了轮胎、转向系、悬架等结构的建模、动力学方程及结构参数与系统动力学特性的相互关系，并提出了相关系统参数的设计准则。其内容丰富，理论与实际结构参数的设计方法紧密联系，具有较高的工程应用价值。

《汽车工程学Ⅲ》比较全面地介绍了与汽车安全相关的理论和各模块系统。在汽车横向、纵向动力学控制理论的基础上，介绍了碰撞事故分析和碰撞前后分析的方法，详细阐述了传动总成、照明系统、空调系统、汽车玻璃、驾驶员视线控制、驾驶辅助系统等车内模块的安全特性，并基于生物力学论述了乘员约束系统和行人碰撞安全。内容新颖，在系统介绍汽车安全理论的同时，还涵盖了与汽车安全相关的最新技术成果，并预测了汽车安全技术发展的方向。

清华大学汽车工程系车辆工程专业相关教师在 Wallentowitz 教授编写的汽车工程讲义的基础上，对该系列汽车工程双语教材进行了编注，在每一章结尾着重对其中的专业词句和疑难字句进行了中文注释。其中王霄锋副教授负责编注《汽车工程学Ⅰ》，李克强教授负责编注《汽车工程学Ⅱ》，周青教授和桂良进副研究员带领研究生对《汽车工程学Ⅲ》进行了编注。

本书三部分内容具有很强的工业背景，对于我国借鉴德国的教材编写方式和教学模式具有比较大的参考价值。本书除作为高校汽车专业教学用书外，还可以作为汽车工程师的参考书。

清华大学
汽车工程系

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Resume of Prof. Henning Wallentowitz

1 Vertical Dynamics (Suspension)



1.1 Suspension—Demands and Possibilities of Implementation

The roads commonly used by motor vehicles are uneven. This unevenness causes vertical movements of the vehicle and the passengers during the driving process.

The vehicle is connected to the road by the tire. Small unevenness in comparison to the tire contact patch size can be compensated by the tire elasticity, whereas larger unevenness entails a vertical acceleration or deflection of the wheels. In order not to transfer these accelerations into the vehicle structure, a length compensating element [1.1-1] has to be placed between the wheel and the vehicle structure.

Steel springs are the technologically most simple elements with variable length. Due to this fact it is also the most common length compensating element, whose force is a function of the length variation. It is usually used in the suspensions of motor vehicles. Different parts connected with springs generate oscillating systems. So there has to be added an energy absorbing element, the damper [1.1-2].

The suspension's job in the motor vehicle is to reduce these vertical movements. The essential criteria specifying the quality of a suspension can be listed as follows:

- 1) Suspension comfort for the passengers (Effective acceleration affecting the passengers).
- 2) Forces affecting the load (Effective value of structure acceleration) [1.1-3].
- 3) Wheel load variation (Effective value of the dynamic wheel load), which influences the grip between tires and road (driving safety) and the load application upon the road surface [1.1-4].

The further demands on the suspension in a motor vehicle are various and partially contradicting (Fig. 1.1-1)

Before dealing with the technical details of the spring and absorber elements, first the road and the mathematical description of its unevenness are presented.

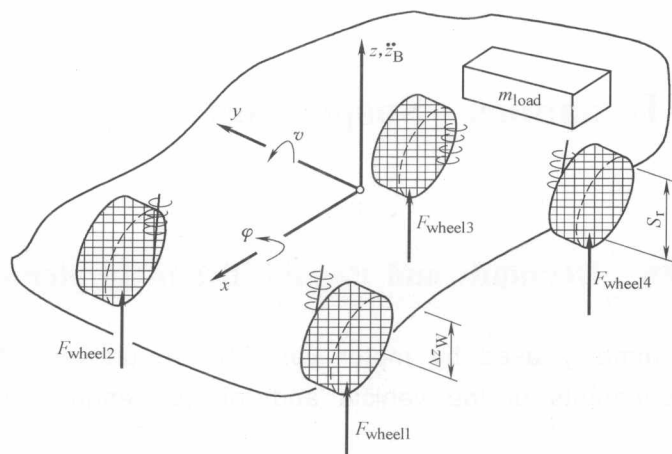


Fig 1.1-1 Demands on a vehicle suspension

\ddot{Z}_B —little body accelerations m_{load} , S_r —vehicle level and vibrational behaviour independent from load
 F_{wheel1} , F_{wheel2} , F_{wheel3} , F_{wheel4} —wheel load distribution according to good driving behaviour
 ΔZ_w —spring travel limited by required space φ , v —little pitch-and rolling motions

中文注释 Notes

[1.1-1] length compensating element 悬架垂向补偿元件

(也可以认为, 作用于汽车车轮和车身之间的减振元件必须具有纵向长度上的可变性。这是弹簧和阻尼减振器应用于此的原因)

[1.1-2] Steel springs are the technologically most simple elements with variable length. Due to this fact it is also the most common length compensating element, whose force is a function of the length variation. It is usually used in the suspensions of motor vehicles. Different parts connected with springs generate oscillating systems. So there has to be added an energy absorbing element, the damper.

由于钢板弹簧是最简单的长度方向上可变形的元件, 其径向变形量是力的函数, 所以它作为最常用的垂向补偿元件应用于汽车的悬架系统中。各零部件之间通过弹簧连接, 因而产生振荡现象。为此, 汽车必须增设减振器这种能量吸收元件来进行补偿。

(此段文字解释了为什么弹簧和减振器必须同时使用, 需要理解两者在汽车纵向动力学结构中的不同作用。弹簧降低了由于路面不平而带来的冲击强度, 而由此造成的车身振动需要由减振器来补偿)

[1.1-3] 2) Forces affecting the load (Effective value of structure acceleration)

2) 车身加速度, 代表了悬架对车身的影响。

[1.1-4] 3) Wheel load variation (Effective value of the dynamic wheel load), which influences the grip between tires and road (driving safety) and the load application upon the road surface.

3) 车轮动载, 反映了车轮的抓地能力, 即安全性能。



1.2 The Road as the Source of Excitation

The unevenness of the road represents the most intensive source of excitation for the vibratory system of the motor vehicle in the frequency range up to approximately 30Hz. The road's unevenness causes vertical movements of the vehicle structure, and as a consequence, the road is affected by tire load variation itself [1.2-1].

Generally road unevenness appears as an excitation with different amplitudes and wavelengths at irregular periods of time. This is called a stochastic excitation of the vehicle. In order to be able to examine the effect of the road unevenness on the vibratory motor vehicle system (see Chapter 1.4), this unevenness first has to be described mathematically [1.2-2].

Considering the most simple case of a harmonic (sinusoidal) radiation, where the road unevenness excites an amplitude 'h' at equal distances L, you will get a characteristic unevenness as a result (Fig. 1.2-1).

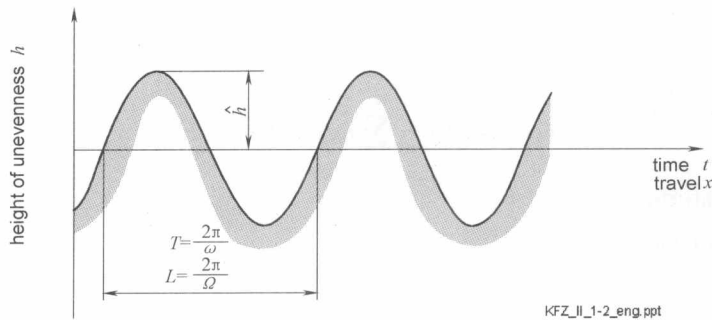


Fig. 1.2-1 Sinusoidal pattern of unevenness

The unevenness height can be described as follows:

$$h(x) = \hat{h} \sin(\Omega x + \varepsilon) \quad (1.2-1)$$

including: $\Omega = \frac{2\pi}{L}$ [1.2-3] as track-dependent circular frequency and ε as phase shift.

When driving on this roadway with constant velocity v this distance-dependent unevenness can be changed into a time-dependent relation:

$$h(t) = \hat{h} \sin(\omega t + \varepsilon) \quad (1.2-2)$$

with ω as time-dependent circular frequency.

The equality of $h(x)$ and $h(t)$ entails $\omega t = \Omega x$, and with the relationship $x = vt$ the time-dependent circular frequency follows:

$$\omega = v \Omega = 2\pi \frac{v}{L} \quad (1.2-3) \quad [1.2-4]$$



The next step in the description of the road unevenness is the transition to a non-sinusoidal, but still periodic, unevenness (Fig.1.2-2) [1.2-5].

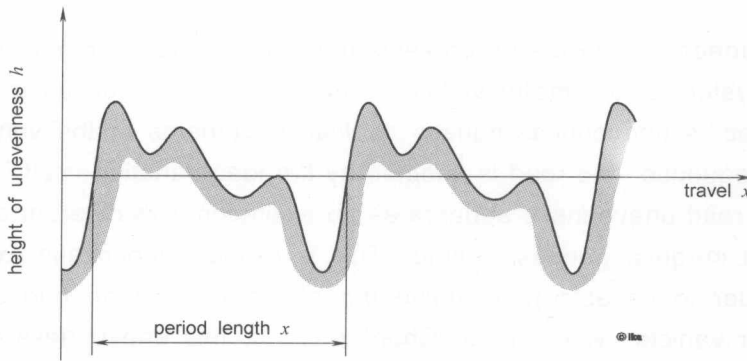


Fig. 1.2-2 Periodic pattern of unevenness

This unevenness can be represented as Fourier series as follows [1.2-6]:

$$h(x) = h_0 + \sum_{k=1}^{\infty} \hat{h}_k \sin(k \Omega x + \varepsilon_k) \quad (1.2-4)$$

or:

$$h(t) = h_0 + \sum_{k=1}^{\infty} \hat{h}_k \sin(k \omega t + \varepsilon_k) \quad (1.2-5)$$

with: \hat{h}_k —amplitude;
 ε_k —phase shift;

$$\Omega = \frac{2\pi}{x}, \quad \omega = v \Omega;$$

x —period length.

When the individual amplitudes \hat{h}_k of the Fourier series are plotted versus the frequency, a discrete amplitude spectrum (line spectrum), belonging to the periodic unevenness, results (Fig.1.2-3).

For the description of real roads there has to be made another step, because they do not show a periodic type of unevenness, but a varying (stochastic) type of unevenness [1.2-7]. The representation of the Fourier series in a complex equation, results as following:

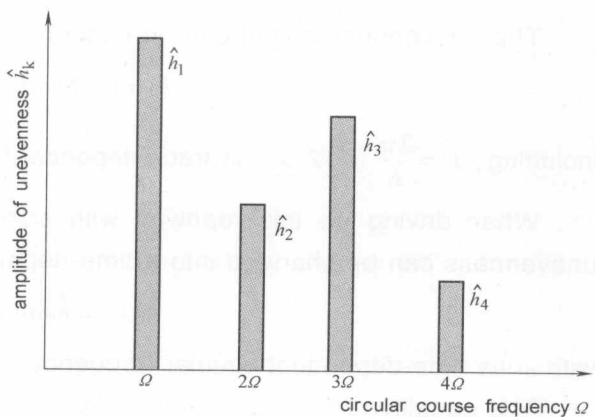


Fig. 1.2-3 Line spectrum of a periodic pattern of unevenness



$$h(x) = \sum_{k=1}^{\infty} \hat{h}_k e^{jk\Omega x} \quad (1.2-6)$$

$$h(t) = \sum_{k=1}^{\infty} \hat{h}_k e^{jk\omega t} \quad (1.2-7)$$

with:

$$\hat{h}_k = \frac{\Omega}{2\pi} \int_{-\frac{x}{2}}^{\frac{x}{2}} h(x) e^{-jk\Omega x} dx \quad (1.2-8) \quad [1.2-8]$$

On the assumption that the regarded period length is very large, the distance between the frequencies in the amplitude spectrum $\Delta\omega$ becomes very small as a consequence [1.2-9].

In the boundary condition $x \rightarrow \infty$ implies that $\Delta\Omega \rightarrow 0$ and the Fourier transformation changes into a Fourier integral as:

$$h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{h}(\Omega) e^{j\Omega x} d\Omega \quad (1.2-9)$$

with the continuous amplitude spectrum

$$\hat{h}(\Omega) = \int_{-\infty}^{\infty} h(x) e^{-j\Omega x} dx \quad (1.2-10)$$

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{h}(\omega) e^{j\omega t} d\omega \quad (1.2-11)$$

$$\hat{h}(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \frac{1}{v} \hat{h}(\Omega) \quad (1.2-12)$$

1.2.1 Spectral density of the road unevenness

For theoretical investigations of the vehicle oscillations caused by road unevenness, the knowledge of the unevenness as a function of time or the distance travelled is usually not very important. It is much more interesting to find out which kinds of excitation appear statistically during the process of travel on a roadway with a characteristic unevenness [1.2-10]. That means which kinds of amplitudes and which kinds of frequencies are excited by road unevenness at certain fixed distances.

The resulting square average value is defined as follows:

$$\bar{x}^2(t) = \frac{1}{T} \int_0^T x^2(t) dt \quad (1.2-13)$$

The characteristic line of unevenness results as follows:

$$\bar{h}^2(x) = \frac{1}{x} \int_0^x h^2(x) dx = \int_0^{\infty} \lim_{x \rightarrow \infty} \frac{|\hat{h}(\Omega)|^2}{x} d\Omega \quad (1.2-14)$$



or:

$$\bar{h}^2(t) = \frac{1}{T} \int_0^t h^2(t) dt = \int_0^\infty \lim_{T \rightarrow \infty} \frac{|\hat{h}(\omega)|^2}{T} d\omega \quad (1.2-15)$$

The limit values occurring here indicate that these simple expressions give a result only for very large time intervals T or distances X . The expressions

$$\Phi_h(\Omega) = \lim_{X \rightarrow \infty} \frac{|\hat{h}(\Omega)|^2}{X} \quad (1.2-16)$$

and

$$\Phi_h(\omega) = \lim_{T \rightarrow \infty} \frac{|\hat{h}(\omega)|^2}{T} \quad (1.2-17)$$

are called circular path frequency or periodical spectral power density (power density spectrum).

With $X = vT$ (see above) similarly the following connection results for the unevenness spectra:

$$\Phi_h(\omega) = \frac{1}{v} \Phi_h(\Omega) \quad (1.2-18)$$

If one measures the power density spectra $\Phi_h(\Omega)$ of different roads and applies this on a double-logarithmic scale, then similar characteristic curves result for all types of roads (Fig. 1.2-4).

In this presentation the power density spectra can be approximated by straight lines, which then can be described by the following equation:

$$\Phi_h(\Omega) = \Phi_h(\Omega_0) \left(\frac{\Omega}{\Omega_0} \right)^{-w} \quad (1.2-19)$$

Here $\Phi_h(\Omega_0)$ represents the power spectral density dependent on Ω_0 (reference circular path frequency), which is usually selected as $\Omega_0 = 10^{-2} \text{ cm}^{-1} = 1 \text{ m}^{-1}$. This corresponds to a reference wavelength of $L_0 = 2\pi/\Omega_0 = 6.28 \text{ m}$. $\Phi_h(\Omega_0)$ is also called the degree of unevenness of the road.

w defines the gradient of the straight

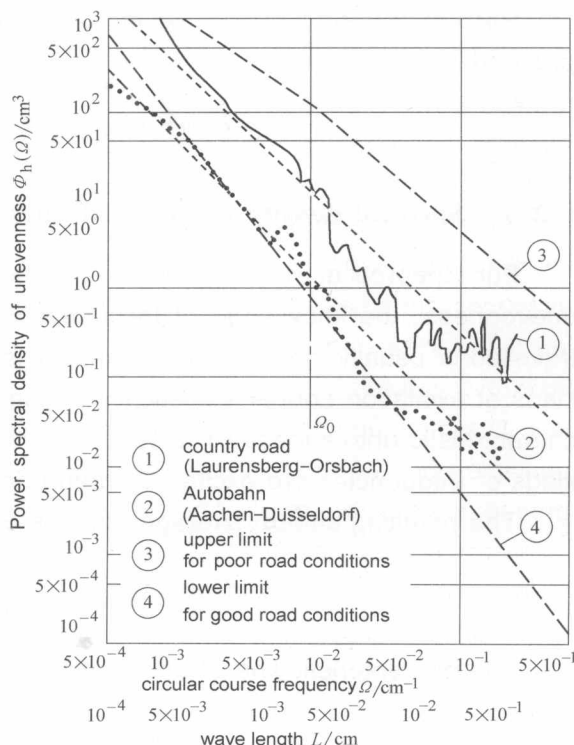


Fig. 1.2-4 Power spectral density of unevenness dependent on circular course frequency